Mott exciton in a quasi-two-dimensional semiconductor in a strong magnetic field

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The formation of Mott excitons in quasi-two-dimensional semiconductors (quantized films or layered systems) situated in strong magnetic fields is considered. The exciton energies are calculated for various Landau levels and arbitrary values of the momentum. In all but the ground state, the dispersion laws are found to be nonmonotonic. The interaction between two excitons is considered and it is shown that formation of a biexciton in the system is impossible. The spectrum of an electron on an impurity is calculated and is found to differ qualitatively from the spectrum of an exciton even in the case of a Coulomb impurity.

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1. The problem of a three-dimensional exciton in a strong magnetic field H is dealt with in detail in a number of theoretical papers.[1-3] A magnetic field in which the free-particle spectrum is quasi-one-dimensional alters qualitatively the properties of an electron-hole ($e-h$) pair compared with the usual hydrogenlike case at $H=0$. This paper considers the properties of a Mott exciton in a quantum film in a strong transverse magnetic field. The spectrum of the free particles in such a film is completely discrete (quasi-zero-dimensional) because of the size quantization and of the action of the magnetic field. This is the cause of the singularities of the exciton in the considered system. Thus, the internal motion of the $e-h$ pair can only be finite, and strictly speaking no free e or h state exists. The momentum dependences of the exciton energies of different Landau levels were calculated; in all but the ground state these dispersion relations are nonmonotonic. With increasing momentum, the binding energy of the $e-h$ pair tends asymptotically to zero, and the radius of the state tends to infinity. States with sufficiently large momenta can therefore be disintegrated by any weak perturbation. The interaction of two excitons is considered and it is shown that no biexciton can be produced in the system. The spectrum of an electron on a Coulomb impurity is also calculated and is found to differ qualitatively in this system from the exciton spectrum. We note that the exciton problem is of interest for problems dealing with phase transitions in "quasi-zero-dimensional" $e-h$ systems.[4-6]

We consider here strong magnetic fields

$$r_H \ll a_{e,h} \qquad (1)$$

and sufficiently thin films

$$d^2 \ll a_{e,h} r_H, \qquad (2)$$

where $a_{e,h} = \varepsilon/m_{e,h}e^2$ are the effective Bohr radii of e and h, $m_{e,h}$ are the effective masses (at $H=0$), $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$, $\varepsilon_{1,2}$ are the dielectric constants of the media surrounding the film, $r_H = (c/eH)^{1/2}$ is the magnetic length and d is the film thickness ($\hbar$ is assumed equal to unity). The condition (1) ensures, as will be made clear by the exposition, smallness, of the interaction energy E compared with the distances between the Landau levels, while condition (2) ensures smallness compared with the spacing of the transverse-quantization levels. At the effective masses usual for semiconductors and at $\varepsilon \sim 10$ to 20 the conditions (1) and (2) are satisfied at $H \gtrsim 10^5$ Oe and $d \lesssim 100$ Å. These conditions can be made weaker by choosing a substrate with higher values of ε.

To simplify the exposition, we consider next the strictly two-dimensional problem. It is shown in Sec. 5 that allowance for the quantized motion along the field H does not alter qualitatively the two-dimensional results.

The motion of the mass center of the electron and hole in the magnetic field was separated for the three-dimensional case by Gor'kov and Dzyaloshinskii.[2] They have shown that the operator

$$\hat{\mathbf{P}} = \left(-i\frac{\partial}{\partial\boldsymbol{\rho}_1} + \frac{e}{c}\mathbf{A}(\boldsymbol{\rho}_1)\right) + \left(-i\frac{\partial}{\partial\boldsymbol{\rho}_2} - \frac{e}{c}\mathbf{A}(\boldsymbol{\rho}_2)\right) - \frac{e}{c}[\mathbf{H}\times(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)] \qquad (3)$$

plays in a magnetic field the role of the two-dimensional exciton-momentum operator. The exciton wave function (in a symmetrical gauge) take the form

$$\Psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \exp\left\{i\mathbf{R}\left(\mathbf{P} + \frac{e}{c}[\mathbf{H}\times\boldsymbol{\rho}]\right)\right\}\exp\left(\frac{1}{2}i\gamma\boldsymbol{\rho}\mathbf{P}\right)\Phi(\boldsymbol{\rho} - \boldsymbol{\rho}_0), \qquad (4)$$

where P is the eigenvalue of the operator (3),

$$\boldsymbol{\rho}_0 = \left[\frac{\mathbf{H}}{H}\times\mathbf{P}\right]r_H^2,$$

and the relative-motion wave function $\Phi(\rho)$ satisfies the following Schrödinger equation:

$$\left\{-\frac{1}{2\mu}\Delta_\rho + \frac{ie}{2\mu c}\gamma[\mathbf{H}\times\boldsymbol{\rho}]\nabla_\rho - \frac{e^2}{8\mu c^2}H^2\rho^2 - \frac{e^2}{\varepsilon|\boldsymbol{\rho} + \boldsymbol{\rho}_0|}\right\}\Phi(\boldsymbol{\rho}) = \mathscr{E}\Phi(\boldsymbol{\rho}). \qquad (5)$$

(The notation is that of Ref. 3.)

In the zeroth approximation in the Coulomb interaction, Eq. (5) goes over into the Schrödinger equation for a free particle of mass μ in a field H,[7] provided that $\gamma = (m_h - m_e)/(m_h + m_e) = \pm 1$, i.e., if $m_h = \infty$ or $m_e = \infty$ (this corresponds to motion of one particle of mass $\mu = m_e$ (or m_h) around an immobile impurity. The relative-motion wave function is nevertheless independent of γ and is given by[7]

$$\langle\rho|nm\rangle = \left[\frac{n!}{2^{|m|+1}(n+|m|)!\pi}\right]^{1/2}\frac{e^{-im\varphi}}{r_H}\left(\frac{\rho}{r_H}\right)^{|m|}L_n^{|m|}\left(\frac{\rho^2}{2r_H^2}\right)\exp\left(-\frac{\rho^2}{4r_H^2}\right); \qquad (6)$$

here L_n^m are Laguerre polynomials. The zeroth-approximation energy, however, turns out to be strongly dependent on γ:

$$\mathscr{E}_{nm} = \omega_H(n + {}^1/_2(|m| - \gamma m + 1) \qquad (7)$$

($\omega_H = eH/\mu c$ is the cyclotron frequency). This energy is degenerate in the angular momentum m only at $|\gamma| = 1$. For an exciton (which consists of particles of finite mass), $|\gamma| < 1$, i.e., there is no degeneracy with respect to the angular momentum m of the relative motion. In this representation, however, the exciton energy is degenerate in the momentum **P** (in other words, the exciton mass is infinite in the zeroth approximation).[1]

The unperturbed spectrum (7) is completely discrete, and the Coulomb corrections for it can be obtained under condition (1) by perturbation theory (in contrast to the three-dimensional case, where the free-particle spectrum is continuous in the z-component of the momentum). Since the Coulomb operator is diagonal in the **P** quantum number, with respect to which the energy is degenerate, the Coulomb corrections $E_{nm}(\mathbf{P})$ to the levels (7) are obtained by ordinary perturbation theory for nondegenerate systems):

$$E_{nm}(\mathbf{P}) = -\left\langle nm \left| \frac{e^2}{\varepsilon|\rho+\rho_0|} \right| nm \right\rangle. \tag{8}$$

The spacing of the unperturbed levels (7) is $\sim\omega_H$, and the energy is $E_{nm} \sim e^2/\varepsilon r_H$ [see (11)], so that the higher order corrections are

$$\sim (e^2/\varepsilon r_H)/\omega_H \sim r_H/a_{e,h},$$

i.e., they are small in the parameter (1). At the end of this section we shall consider the situation when the distance between some of the levels (7) is less than E_0 (or—in the limit—these levels are degenerate) and Eq. (8) is not directly applicable.

The dispersion relations (8) can be calculated analytically for all[2] n, m, and **P** but these expressions are extremely cumbersome. We shall present exact expressions only for the dispersion laws for several lower Landau levels; Eq. (8) will be next analyzed qualitatively for arbitrary n and m. In the ground state (at $n = m = 0$) the dispersion law is (see the figure)

$$E_{00}(P) = -E_0 e^{-\beta} I_0(\beta); \quad \beta \equiv P^2 r_H^2/4 \tag{9}$$

with asymptotic forms

$$E_{00}(P) = \begin{cases} -E_0 + P^2/2M_0, & Pr_H \ll 1 \\ -(2/\pi)^{1/2} E_0 (Pr_H)^{-1}, & Pr_H \gg 1 \end{cases}; \tag{10}$$

$I_n(\beta)$ are modified Bessel functions,

$$E_0 = \left(\frac{\pi}{2}\right)^{1/2} \frac{e^2}{\varepsilon r_H} = \frac{e^2}{\varepsilon} \left(\frac{\pi e H}{2c}\right)^{1/2} \tag{11}$$

is the exciton binding energy, and

$$M_0 = \frac{2}{E_0 r_H^2} = \frac{2\varepsilon}{e^2} \left(\frac{2eH}{\pi c}\right)^{1/2} \tag{12}$$

is the effective mass of the exciton at low momenta.

In the states $n = 0, |m| = 1$ the dispersion relation is

$$E_{01}(P) = -{}^1/_2 E_0 e^{-\beta} [(1+2\beta) I_0(\beta) - 2\beta I_1(\beta)]. \tag{13}$$

At $Pr_H \gg 1$ the asymptotic form (13) (as well as of all other dispersion curves) agrees with the asymptotic form of (10) apart from a numerical factor. At $P = 0$ the dispersion law (13) has a maximum (the exciton "mass" is negative at low momenta). The minimum corresponds to a momentum $P_{01} = 1.194 r_H^{-1}$ (see the figure). Near the minimum we have

$$E_{01}(P) = -E_{01} + (P - P_{01})^2/2M_{01}, \tag{14}$$

where

$$\begin{gathered} E_{01} \equiv -E_{01}(P_{01}) = 0.574 E_0, \\ \frac{1}{M_{01}} = \frac{E_0 r_H^2}{8} \left\{ \beta^{1/2} \frac{d^3}{d(\beta^{1/2})^3} [e^{-\beta} I_0(\beta)] \right\}_{\beta=\beta_0} = 0.276 E_0 r_H^2 \\ (\beta_0 = P_{01}^2 r_H^2/4). \end{gathered} \tag{15}$$

In the state $n = 1, m = 0$ the energy

$$E_{10}(P) = -E_0 e^{-\beta} [({}^3/_4 - \beta + 2\beta^2) I_0(\beta) - 2\beta^2 I_1(\beta)] \tag{16}$$

has, besides a maximum at $P_{10} = 2.145 r_H^{-1}$ (see the figure). The parameters of the expansion of the energy (16) near the maxima are

$$\begin{aligned} &E_{10}(0) = -\frac{3}{4} E_0, \quad \frac{1}{M_{10}} = \frac{7}{8} E_0 r_H^2, \quad Pr_H \ll 1, \\ &E_{10}(P_{10}) = -0.402 E_0, \quad \frac{1}{M_{10}'} = 0.282 E_0 r_H^2, \quad (P - P_{10}) r_H \ll 1. \end{aligned} \tag{17}$$

The dispersion laws are nonmonotonic at all n and m (except $n = m = 0$). This can be verified by analyzing Eq. (8), which we rewrite in the form

$$\begin{aligned} E_{nm} &= -\mathrm{const} \int \frac{d^2\rho e^2}{|\rho+\rho_0|} \exp\left(-\frac{\rho^2}{2r_H^2}\right) \left[L_n^{|m|} \left(\frac{\rho^2}{2r_H^2} \right) \right]^2 \rho^{2|m|} \\ &\equiv -\mathrm{const} \int d^2\rho e N_{nm}(\rho) \frac{e}{|\rho+\rho_0|}. \end{aligned}$$

This expression can be interpreted as the energy of the interaction of the charge distribution $eN_{nm}(\rho)$ with a point charge e located at a distance $\rho_0 = Pr_H^2$ from the distribution center. When ρ_0 is varied, the charge goes through n minima of the distribution $N_{nm}(\rho)$, so that n local maxima appear in the interaction energy. If $m = 0$, then $N_{mn}(0) \neq 0$ and the zero momenta correspond to energy minima; at $m \neq 0$ we have $N_{nm}(0) = 0$ and the dispersion curve has a maximum at $P = 0$.

The exciton spectrum is thus a sequence of bands $\mathcal{E}_{nm}(P) = \mathcal{E}_{nm} + E_{nm}(P)$ (of the type shown in the figure)

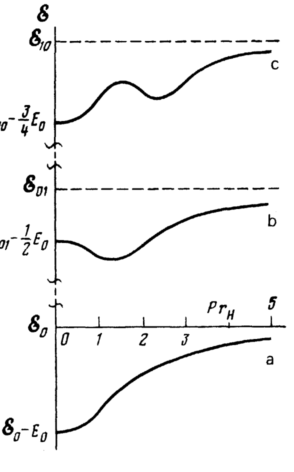

FIG. 1. Dispersion curves of exciton in various states: a) $n = m = 0$; b) $n = 0$, $m = 1$; c) $n = 1, m = 0$. At $\gamma\omega_H \gg E_0$ the curve with $n = 0$ and $m = 1$ coincides in form with curve b, and is shifted relative to it in energy by $\gamma\omega_H$. At $\gamma\omega_H \lesssim E_0$, when the levels $n = 0$ and $m = \pm 1$ are quasi-degenerate, the shapes of both curves remains qualitatively the same as in the figure, albeit somewhat distorted.

with width $\sim E_0$. The dispersion laws in all the bands (except the lowest $n=m=0$) are nonmonotonic: there are $n+1$ minima, with the main minimum at $m=0$ corresponding to a momentum $P=0$, and at $m\neq 0$ to a maximum at the point $P=0$. We note that the exciton binding energy in the ground state was obtained by us earlier[4a,5] (and for the analogous case of anisotropic semiconductors in a magnetic field by Beneslavskii and Buzdin).[8]

We have disregarded so far the fact that at certain values of γ the spacing between some unperturbed levels (7) is less than E_0 (and at rational γ, i.e., when m_e and m_h form a rational fraction, sufficiently high levels are degenerate). Levels with $n=0, m=\pm 1$ are mutually degenerate at $\gamma=0$, and at $\gamma=(|m'|-2)/m'$ the energies of the levels $n=m$, $m=0$ and $n=0$, $m'\geqslant 2$ are equal. Therefore the conditions for the applicability of the equations derived above for the levels $n=0$, $|m|=1$ and $n=1, m=0$ are respectively

$$\gamma\gg r_H/a_{e,h},\quad |(\gamma m'-|m'|)/2+1|\gg r_H/a_{e,h}.$$

If these conditions are not satisfied, then Eqs. (8), (13), and (17) do not hold.[3]

We consider here the case $\gamma=0$ (i.e., $m_e=m_h$). The energy (7) depends now on the quantum number $N=2n+|m|$ and the N-th unperturbed level $(N+1)$ is multiply degenerate. The ground level $N=0$ is not degenerate, so that the dependence of the energy on the momentum is given as before by Eq. (9). The Coulomb interaction lifts (at $P\neq 0$) the degeneracy of the remaining levels, and the corresponding dispersion laws separate. The contribution made to the energy by transitions between levels with different N is small, since the distance between neighboring levels is $\omega_H\gg E_0$.

The dispersion laws for the lowest excited level $N=1$ (i.e., $n=0, m=\pm 1$) is

$$E_1^{(1,2)}(P)=E_{01}(P)\pm\Delta(P),$$
$$\Delta(P)=\langle 01|e^2/\varepsilon|\rho+\rho_0||0-1\rangle=E_0e^{-\beta}[\beta I_0(\beta)-(\beta+{}^3/_2)I_1(\beta)], \tag{18}$$

where $E_{01}(P)$ is given by (13).

The shapes of the dispersion curves (18) do not differ qualitatively from that of $E_{01}(P)$ (curve b in the figure), although the binding energies and the effective masses are somewhat altered. The dispersion law for the level $N=2$ (which corresponds to the quantum numbers $n=1$, $m=0$ and $n=0, m=\pm 2$) is triply split:

$$\begin{gathered}E_2^{(1,2)}(P)=\frac{1}{2}\{(E_{10}+E_{02}+\Delta_2)\pm[(E_{10}-E_{02}-\Delta_2)^2+8\Delta_1^2]^{1/2}\},\\ E_2^{(3)}(P)=E_{02}(P)-\Delta_2(P).\end{gathered} \tag{19}$$

For the sake of brevity we do not present here the analytic expressions for the matrix elements $E_{02}(P)$ and $\Delta_{1,2}(P)$ of the Coulomb operator V. We indicate only that Eq. (19) corresponds to three dispersion curves: one of them (which goes over into the $E_{10}(P)$ curve in the nondegenerate case) has a minimum at $P=0$ and an additional lateral minimum, while the other two are shifted relative to the first upward in energy (by an amount $\sim E_0$) and have coinciding maxima at $P=0$ and two (noncoinciding) lateral minima each. At $Pr_H\ll 1$ the off-diagonal corrections are $\sim(Pr_H)^4$, so that at $P=0$ not only the binding energies but also the effective masses are the same as in the nondegenerate case.

It must be emphasized that in any state the internal motion of the $e-h$ pair is finite: the average radius vector $\langle\rho\rangle$ between e and h is equal to ρ_0 at all n and m. Large momenta $Pr_H\gg 1$ correspond to a low binding energy (10) and to a large exciton size, so that such states are disintegrated by a weak perturbation (impurity, surface inhomogeneity, and others). In this sense, in any real system the exciton momentum has an upper bound (much more substantial than mentioned in footnote 2).

At momenta close to the minima of the dispersion curves, the exciton dispersion laws are quadratic, just as in (10), (14), and (17). The exciton dimension is then

$$\langle\rho\rangle=Ar_H[1+Br_H^2(P-P_M)^2]$$

(P_M is the position of one of the $E_{nm}(P)$ minima, and A and B are dimensionless constants that depend on n and m). In interactions of scale $R\gg r_H$ the wave function of the exciton motion as a whole must be averaged over the internal motion, and the result is a plane wave $\exp(i\mathbf{P}\cdot\mathbf{R}/2)$. Consequently an exciton with momenta $(P-P_M)r_H\ll 1$ can be regarded in such interactions as an ordinary two-dimensional quasiparticle whose motion does not depend on H (but all its characteristics—mass, dipole moment at $P\neq 0$, etc., are determined by the magnetic field[4]).

3. We consider now the interaction of two excitons on the level $n=m=0$ with momenta $P_1, P_2\ll 1/r_H$, located at a distance $R\gg r_H$ from each other. The proper dipole moment of each exciton is

$$\mathbf{d}_{1,2}=er_H^2\left[\frac{\mathbf{H}}{H}\times\mathbf{P}_{1,2}\right]$$

and the corresponding contribution to the energy of their interaction is

$$\sim E_0\left(\frac{r_H}{R}\right)^3P_1P_2r_H^2\sim\left(\frac{r_H}{R}\right)^3\frac{P_1P_2}{M_0}. \tag{20}$$

The sign of (20) depends on the relative orientations of $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{R}$. This energy can also be negative but, as seen from (18) its absolute value is half that of the combined kinetic energies of the excitons. Therefore the total energy of two excitons can have a minimum only at $P_1=P_2=0$.

The energy of the interaction of the excitons at zero momenta (i.e., the energy of the van der Waals attraction) is in this case

$$U(R)=\left(\frac{2}{\pi}\right)^{1/2}E_0\frac{r_H}{a_0}\frac{1}{1-\gamma^2}\left(\frac{r_H}{R}\right)^6. \tag{21}$$

Contributing to this energy are only transitions from the ground state to states with $n=0$ and $m=1$ (the remaining dipole-moment matrix elements, determined with the wave functions (6) in the zeroth approximation in the parameter (1) are equal to zero; with allowance for the corrections, their contribution to (21) would have an additional smallness $\sim(r_H/a_0)^4$). An important factor is the presence in (21) of the small coefficient

r_H/a_0, which makes the interaction energy small compared with exciton self energy E_0 even at $R \sim r_H$. At $R \lesssim r_H$, on the other hand, a strong Pauli repulsion appears between excitons of energy $\sim\omega_H \gg E_0$. Inasmuch as in the potential well made up by the energy $U(R)$ and the barrier at $R = r_H$ the zero-point oscillation energy is

$$\frac{1}{M_0R^2} \sim E_0\left(\frac{r_H}{R}\right)^2 \gg U(R)$$

all the way to $R \sim r_H$, no levels can appear in the well. Consequently, formation of a biexciton in this system is impossible.

We note that the weak van der Waals attraction of the exciton to a charged impurity can produce in principle a bound state, since there is no repulsion in this case (but the binding energy is exponentially small).

4. The interaction of an electron with a charged impurity is described by the Schrödinger equation (5) with $\gamma = 1$. In this case the zeroth-approximation energy (7) is infinitely degenerate in the angular momentum m at $m > 0$. But the electron-impurity interaction operator $Ze^2/\varepsilon\rho$ is independent of angle, i.e., it is diagonal in m ($Ze > 0$ is the charge of the impurity). Therefore the corrections to the degenerate levels (7) are obtained from ordinary perturbation theory:

$$E_{nm} = -\left\langle nm \left| \frac{Ze^2}{\varepsilon\rho} \right| nm \right\rangle \equiv -E_0 Z I_{nm}, \tag{22}$$

where

$$I_{nm} = \frac{\pi^{-1/2} n!}{(n+|m|)!} \int_0^\infty du\, u^{|m|-1/2} e^{-u} [L_n^{|m|}(u)]^2.$$

Integrals of this type are calculated in the book of Landau and Lifshitz[7] [formula (f, 6)]. We present the result in a different form, more convenient for estimates:

$$I_{nm} = \sum_{j=0}^{n} \frac{(2n-2j-1)!!}{(2n-2j)!!} \frac{(2m+2j-1)!!}{(2m+2j)!!} \frac{(2j-1)!!}{(2j)!!}, \tag{23}$$

where the notation $(0)!! = 1$ and $(-1)!! = 1$ is used to simplify the notation.

At small n the coefficients I_{nm} are obtained directly from (21). In particular, $I_{00} = 1$,

$$I_{0m} = \frac{(2m-1)!!}{(2m)!!} \approx [\pi(2m+1)]^{-1/2}. \tag{24}$$

At $n \gg 1$ we can obtain estimates of I_{nm} by using asymptotic expressions similar to (24). For example

$$I_{n0} \approx [8\pi^3(n+1)]^{-1/2}(\ln n + O(1)].$$

Thus, the interaction of the electron with a Coulomb impurity lifts completely the degeneracy in the angular momentum m: The Landau levels (7) acquire the fine structure (22). At $m \gg 1$ the fine-structure components condense in proportion to $E_0 m^{-1/2}$, and the dimension of the electronic state increases in proportion to $r_h m^{1/2}$. Just as in the case of the exciton, there are no non-bound states. On the whole, the spectrum of the electron on the impurity differs qualitatively from the spectrum of the exciton.

We note that the problem of the two-dimensional exciton in a magnetic field was considered also by Grebeshchikov and Korneev,[9] but they used a Schrödinger equation with $\gamma = 1$, which is valid only for electron-impurity interactions. Even in this case, however, the results they obtained by expanding the quasiclassical integral in terms of the parameter (1) differ substantially from the exact expansion we obtained in terms of the same parameter.

Interaction of an electron with a short-range impurity hardly lifts the degeneracy. Thus in the case of an impurity $A\delta(\rho)$ (for which perturbation theory is valid at $(Am_e/2\pi)\ln(L_x L_y/2\pi r_H^2) \ll 1$, where L_x and L_y are the dimensions of the system) there is split off from each Landau level, by an amount $-A/2\pi r_H^2 \sim -Am_e\omega_H$, one level corresponding to a state with angular momentum $m = 0$, while the energies of the states with $m \neq 0$ remain unchanged. If the radius of the interaction of the electron with the impurity is $a \ll r_H$, the degeneracy of the states with $m \neq 0$ is lifted only $\propto (a/r_H)^{2m}$.[10]

5. We discuss now the conditions for the applicability of the employed model. Allowance for the size-quantized motion along the field H does not lead to significant effect. Under the condition (2), the dispersion curves can be calculated as before by perturbation theory (the energy has in this case a nonmonotonic dependence on the momentum also at the higher levels of size quantization). In the formulas of Secs. 2 and 4 (which pertains to the ground level of motion along z) it is necessary to replace $e^2/\varepsilon\rho$ by

$$V(\rho) = \frac{e^2}{\varepsilon} \int \frac{dz}{(z^2+\rho^2)^{1/2}} \Phi_0^*(z)\Phi_0(z), \tag{25}$$

i.e., by the Coulomb potential averaged over the wave functions of the motion along z. A numerical calculation (with $\Phi_0(z) = (2/d)^{1/2}\sin(\pi d/z)$] shows that at $d \sim r_H$ the dispersion curves differ from (10)–(16) by only several percent. At $d \ll r_H$ [a condition stronger than (2)] it can be easily shown analytically that replacement of $e^2/\varepsilon\rho$ by the potential (25) leads in all equations to additive corrections $\propto (d/r_H)$.[2]

The results are applicable also to highly anisotropic semiconductors with $M_z \gg M_{x,y}$ in a strong magnetic field H_z. In this case the problem can be treated adiabatically and Eq. (25) represents the Coulomb potential averaged over the fast motion along the field H. (The problem of the ground state of an electron or an impurity in such a system was considered in Ref. 11; the results agree, in the zeroth approximation in $1/M_z$, with equation (20) in which we put $n = m = 0$.)

Strictly speaking, a Coulomb potential with $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$ can be used only when the state size $\langle\rho\rangle \gg d$. At $n = m = 0$ and $d \sim r_H$ we have $\langle\rho\rangle \sim d$. However, even in this case the replacement of the Coulomb operator leads only to insignificant changes. The qualitative results (as well as the quantitative ones at $d \ll r_H$) remain the same.

Our use of a two-band semiconductor model is not a significant restriction. If the condition (1) is satisfied in a multivalley semiconductor for electrons and holes of all types, then the Landau levels of an e-h pair of any type have the same dispersion laws, which do not depend at all on the masses of e and h. (If the electron and hole bands are separated by P_{eh}, then the dispersion laws become effective starting not with the zeroth momentum, but with P_{eh}.)

One of the possible experimental applications is the study of the dispersion relations of excitons. Besides the traditional methods (e.g., using Raman scattering), observation is possible of dispersion in an electric field $\vec{\mathscr{E}}$ perpendicular to the field H. The point is that, just as in the three dimensional case,[3] in the presence of a field the energy depends on the vector

$$\mathbf{P}'=\mathbf{P}+\frac{Mc}{H^2}[\mathbf{H}\times\vec{\mathscr{E}}].$$

At $P=0$ the momentum characteristic values $P'\sim 1/r_H$ are reached in fields $\mathscr{E}\sim(1/137)(a_0/r_H)H$.

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1) The energy of e or h in the usual representation $|n_1, m_1; n_2, m_2\rangle$ ($n_{1,2}$ and $m_{1,2}$ are respectively the radial and azimuthal quantum numbers of e and h) is given by

$$\mathscr{E}=(n_1+{}^1/_2)\omega_e+(n_2+{}^1/_2)\omega_h,$$

i.e., it is degenerate in m_1 and m_2. Equation (7) corresponds to the cited expression ($\omega_{e,h}=eH/m_{e,h}c$ at $n=\min(n_1, n_2)$, $m=n_2-n_1$.

2) It is implied that $|\mathbf{P}|\ll|\mathbf{G}|$, where G is the reciprocal-lattice vector. This restriction is inessential, since the dispersion curves are nonmonotonic only in the region $P\sim 1/r_H\ll G$.

3) Since the Coulomb operator is diagonal in m at $P=0$, the position of the centers of the exciton bands relative to the unperturbed levels (7) does not change at any value of γ. Nor does the asymptotic form of the dispersion laws change at $Pr_H\gg 1$.

4) In any state, the exciton mass $M_{nm}\sim 1/E_0r_H^2$ does not depend on the effective masses e and h at $H=0$. It is seen, however, from the equality $1/E_0r_H^2\sim(a_0/r_H)M$ (where $a_0=a_e+a_h$ and M is the mass of the exciton as a whole at $H=0$), that $M_{nm}\gg M$.

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NMR investigation of the anisotropy of hyperfine interactions of orthoferrites

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The anisotropy of the hyperfine interactions in a number of rare-earth orthoferrites is investigated experimentally and theoretically on the basis of the singularities produced in the NMR spectrum of ^{57}Fe nuclei in spin-reorientation phase transitions. Both non-substituted orthoferrites and orthoferrites substituted in the rare-earth sublattice are investigated. A phenomenological analysis based on the use of the magnetic symmetry of orthoferrites has made possible a qualitative explanation of all the effects observed in NMR experiments. A microscopic analysis points to a small contribution of the paramagnetic rare-earth sublattice to the anisotropy of the hyperfine interactions (HFI) for the ^{57}Fe nuclei. The most probable mechanism of the HFI anisotropy, explaining all the main features of NMR in the spin-reorientation region is connected with the non-cubic contribution to the crystal field at the Fe^{3+} ions from the next-to-nearest environment, considered within the framework of the point-charge model.

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A few years ago, NMR investigations of phase transitions of the spin-reorientation (SR) type in rare-earth orthoferrites (REO) have led to observation of an interesting phenomenon—violation of the magnetic equivalence of the ions from different sublattices of the iron. This violation consisted in the fact that two values of the local field are produced at the ^{57}Fe nuclei in the SR transitions. In addition, other subtle effects were observed, such as a jump of the local field and the nonsynchronous rotation of the antiferromagnetism vector and the vector of the summary magnetization following the SR.

It was indicated in the first studies that a connection exist between the splitting and shift of the NMR frequencies in the region of the SR transition, on the one hand, and the character of the hyperfine interaction, on the other.[1,2] Allowance for the magnetodipole interactions of the ^{57}Fe nuclei with the surrounding Fe^{3+} ions, however, did not provide a satisfactory quantitative explanation of the observed effect.[3] In addition, no account was taken in the earlier research on the influence of the rare-earth ions. This has made it necessary to explain the experimental data using a model in which the antiferromagnetism and ferromagnetism vectors (**l** and **m**) are not orthogonal in the SR region. Later investigations of mixed orthoferrites containing various rare-earth ions have shown that these effects differ substantially in character, depending precisely on which of the lanthanide-series ion is