Multiphoton absorption in narrow-band semiconductors in crossed fields

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The behavior of a semiconductor with a narrow forbidden band in a strong magnetic field crossed with a periodic electric field (e.g., a laser field) is investigated theoretically. The two-band approximation equation employed is of the Dirac type. A method for solving this equation approximately is proposed. The quasienergy states are obtained. The spectrum of the quasienergies is investigated near the parametric resonance. The electron states that are identical with the stationary states in the valence band at the instant when the periodic field is turned on are found, using the quasienergy states as the basis system. The probabilities of one- and two-photon interband transitions are calculated. The two-photon transition probability is found to differ from zero and to have a finite value at only one definite frequency of the external field. The results are compared with the theoretical ones obtained by using other band models and with the available experimental data on two-photon magnetoabsorption in a PbTe crystal.

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The advent of powerful optical-radiation sources has stimulated experimental and theoretical investigations of semiconductors in strong electromagnetic fields. These investigations are carried out in many cases in the presence of an external constant magnetic field that plays a major role in the study of the electronic states in semiconductors. Semiconductor crystals with wide forbidden bands (E_g large compared with the intraband motion energy) in an alternating field is the subject of Refs. 1–4. The influence of a constant magnetic field was taken into account in Refs. 5 and 6.

The higher intensities of laser radiation and of the constant magnetic field, as well as the increased interest in the physics of narrow-band semiconductors, call for further development of the theory. The influence of a strong alternating electric field on crystals with narrow forbidden band of width comparable with intraband motion energy in external fields was first considered by Keldysh.[7] This question was later investigated by others.[8,9]

The interpretation of the experimental results[10] was hindered by the absence, until recently, of a consistent theory that takes into account the joint action of a strong constant magnetic field and of an alternating field on a narrow-band semiconductor. In particular, the theory of multiphoton magnetoabsorption in the independent-band model[5,6] has predicted the appearance of resonant maxima (oscillations) in parallel fields only for transitions with odd number of photons, whereas absorption maxima are clearly observed in experiments on two-photon absorption in narrow-band semiconductors both in the case of parallel and in the case of crossed fields.[10,11] This contradiction was completely eliminated by the results of Ref. 12, where a theory of multiphoton magnetoabsorption in parallel fields was proposed in a two-band approximation based on the use of a two-band equation of the Dirac type. On the other hand, the interpretation of the experiments[10,11] performed with crossed fields have not been clearly interpreted to date, in view of the lack of a general theoretical analysis of nonstationary electronic states in a narrow-band semiconductor in crossed fields.

The present paper is therefore devoted to a theoretical study of a semiconductor with a narrow forbidden band in a constant magnetic field crossed with a periodic strong electric field. The two-band approximation equation, just as in Ref. 12, is taken to be the Dirac equation,[13] whose solution is used to calculate the probability of the direct interband transitions. The analysis is within the framework of the general ideas of the quasienergy method of describing periodic systems.[14] An investigation, which in our opinion is of general interest, was made of the quasienergy states and of the spectrum of the quasienergies that appear in the course of the solution of a system of equations of the Mathieu type, which describe the time dependence of the nonstationary states. The presence of singularities (breaks) of two types, due to the action of the electric field, is observed on the plots of the quasienergies against external-field frequency. The breaks of one type are already known, 1–4, 14 and occur in the values of the quasienergy. The breaks of the other type constitute external-frequency regions (at fixed quantum numbers of the electrons and at fixed other parameters of the problem), in which there are no real values of the quasienergy. The breaks of the first type occur near frequencies of odd-photon transitions (resonances), while those of the second type occur near frequencies of even-photon transitions.

The dependence of the spectrum of the two-photon absorption on the frequency, obtained in the present paper, is substantially different compared with the conclusions of Refs. 5 and 6. On the basis of the analytically obtained probabilities of the one-and two-photon transitions we present an interpretation of the results of Refs. 10 and 11.

GENERAL ANALYSIS

It is known that the equation of the two-band model of the effective-mass approximation for isotropic orbitally nondegenerate bands agrees formally with the Dirac equation, in which the speed of light c is replaced by the parameter $s=(E_g/2m)^{1/2}$ (E_g is the width of the forbidden band and m is the effective mass).[15] In an external electromagnetic field the equation takes the form

$$\left[s\left(\alpha,\mathbf{p}+\frac{e}{c}\mathbf{A}\right)+\gamma^0 ms^2\right]\psi=i\hbar\frac{\partial\psi}{\partial t}+e\Phi\psi, \qquad (1)$$

where α and γ^0 are Dirac matrices in the standard representation[16] while Φ and $\mathbf{A}$ are the potentials of the external field. The appearance in (1) of two parameters, c and s, rather than c alone as in the usual Dirac equation, leads to difficulties in its solution; in particular, the known exact solutions of the Dirac's equations cannot be used when a constant magnetic field and a plane-wave field act simultaneously.[17]

Next, neglecting the photon momentum, we assume

$$\Phi=0,\quad A_x=-\frac{c}{\omega}\mathscr{E}_0\sin\varphi,\quad A_y=Hx,\quad A_z=0,\quad \varphi=\omega t. \qquad (2)$$

This corresponds to a constant magnetic field directed along the z axis, and to an alternating electric field of frequency ω directed along the x axis.

Our task is to find approximate solutions of Eq. (1) in the case when the magnetic field at any instant of time is strong compared with the electric field, i.e., when magnetic quantization is preserved. In our model this requirement is equivalent to the condition

$$c\mathscr{E}_0/sH\ll 1. \qquad (3)$$

We change in the usual manner from Eq. (1) to the corresponding second-order equation[16] whose solution Ψ we seek in the form

$$\Psi(\mathbf{r},t)=\exp\left[ip_y y+ip_z z+\frac{ie}{\hbar\omega}\mathscr{E}_0 a_{H'}\eta\sin\varphi\right]\Phi_n(\eta)\chi(\varphi), \qquad (4)$$

where

$$\varphi=\omega t,\quad \eta=\frac{1}{a_{H'}}[x+(\varkappa a_H)^2 p_y]=\frac{1}{a_{H'}}(x-\bar{x}), \qquad (5)$$

$$\varkappa=H\left(H^2-\frac{1}{2}\mathscr{E}_0^2c^2/s^2\right)^{-1/2},\quad a_H^2=\hbar c/eH,\quad a_{H'}=a_H\varkappa^{1/2}.$$

For the function χ we obtain the equation

$$\chi''(\varphi)+2i\frac{e\mathscr{E}_0}{\hbar\omega}a_{H'}\eta\cos\varphi\chi'(\varphi)+\left\{\frac{2ms^2}{(\hbar\omega)^2}W\right.$$
$$-i\frac{e\mathscr{E}_0}{\hbar\omega}a_{H'}\eta\sin\varphi+\left(\frac{e\mathscr{E}_0}{\hbar\omega}\right)^2\left[\bar{x}^2\cos^2\varphi-\frac{1}{2}(\bar{x}+a_{H'}\eta)^2\cos2\varphi\right]$$
$$\left.+\frac{ms^2}{(\hbar\omega)^2}\hbar\Omega_0\hat{\sigma}_z-i\frac{ms^2}{(\hbar\omega)^2}\frac{e\hbar\mathscr{E}_0}{ms}\hat{\alpha}_x\cos\varphi\right\}\chi(\varphi)=0, \qquad (6)$$

where

$$W=\frac{\hbar^2}{2m}p_z^2+\frac{1}{2}ms^2-\frac{1}{2}ma_H^4\Omega_0^2(\varkappa^2-1)p_y^2+\hbar\Omega\left(n+\frac{1}{2}\right), \qquad (7)$$
$$\Omega_0=eH/mc,\quad \Omega=\Omega_0\varkappa^{-1}.$$

Of course, since the variables do not separate, the function χ depends also on the variable η. However, taking into account the terms $\sim\chi''_{\eta^2}$ and χ'_η discarded in (6), we can show that $\chi(\eta)\sim\exp[-c\mathscr{E}_0\eta^2/sH]$. Because of condition (3), this dependence is weaker than that of the function $\Phi_n\sim\exp(-1/2\,\eta^2)$, and will henceforth be neglected. This enables us to obtain an equation for the function $\chi(\varphi)$ by averaging the coefficients in Eq. (6) over the functions Φ_n. The terms linear in η then vanish, and the terms $\propto\eta^2$ are subsequently taken into account and their contribution is of the same order as the last term in the left-hand side of (6). We note that this method of taking the electric field into account ensures the correct dependence on this field in the limiting results as $\omega\to0$. After performing this averaging and discarding the terms $\sim(c\mathscr{E}_0/sH)^4$, of the coefficients, we get

$$\chi''(\varphi)+\left\{\frac{2ms^2}{(\hbar\omega)^2}\left[\frac{\hbar^2p_z^2}{2m}+\frac{1}{2}ms^2+\hbar\Omega\left(n+\frac{1}{2}\right)+\frac{1}{2}\hbar\Omega_0\hat{\sigma}_z\right.\right.$$
$$\left.\left.-\frac{1}{2}\left(\frac{e\mathscr{E}_0a_{H'}}{\hbar\omega}\right)^2\left(n+\frac{1}{2}\right)\cos2\varphi\right]-i\frac{ms^2}{(\hbar\omega)^2}\frac{e\hbar\mathscr{E}_0}{ms}\cos\varphi\hat{\alpha}_x\right\}\chi(\varphi)=0. \qquad (8)$$

We seek $\chi(\varphi)$ in the form

$$\chi(\varphi)=a_1[v_+(\varphi)w_++v_-(\varphi)w_-]+a_2[v_+(\varphi)\tilde{w}_+-v_-(\varphi)\tilde{w}_-], \qquad (9)$$

where

$$w_+=\begin{bmatrix}1\\0\\1\\0\end{bmatrix},\quad w_-=\begin{bmatrix}0\\1\\0\\1\end{bmatrix},\quad \tilde{w}_+=\begin{bmatrix}1\\0\\-1\\0\end{bmatrix},\quad \tilde{w}_-=\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}. \qquad (10)$$

The coefficients a_1 and a_2 are so far arbitrary.

Substitution of (8) and (9) in (7) leads to the following system of equations for the functions $v_+(\varphi)$ and $v_-(\varphi)$:

$$v_+''+[\alpha_1^2-4p\cos2\varphi]v_+-4iq\cos\varphi v_-=0,$$
$$v_-''+[\alpha_2^2-4p\cos2\varphi]v_--4iq\cos\varphi v_+=0; \qquad (11)$$

here

$$\alpha_{1,2}(n,p_z)=(\hbar\omega)^{-1}\left[(ms^2)^2+(\hbar sp_z)^2+2\hbar\Omega ms^2\left(n+\frac{1}{2}\pm\frac{1}{2}\varkappa\right)\right]^{1/2}$$
$$=(\hbar\omega)^{-1}|E_{1,2}(n,p_z)|, \qquad (12)$$

$$q=\frac{1}{4}\frac{e\hbar\mathscr{E}_0}{ms}\frac{ms^2}{(\hbar\omega)^2},\quad p=\frac{1}{2}q\left(n+\frac{1}{2}\right)\frac{c\mathscr{E}_0}{sH}. \qquad (13)$$

Knowing the solutions $\Psi(r,t)$ of the second-order equation we can obtain in the usual fashion also the solutions $\psi(r,t)$ of Eq. (1):

$$\psi(\mathbf{r},t)=\left[ms^2+\gamma^0 i\hbar\frac{\partial}{\partial t}-\gamma\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)\right]\Psi(\mathbf{r},t). \qquad (14)$$

The problem reduces thus to finding the approximate solutions of the system (11).

QUASIENERGY STATES

We assume next that the dimensionless parameters p and q^2 (which are taken to be of the same order) are small: $p,q^2\ll1$, and obtain the solutions of the system (11), which correspond to states with definite quasienergy,[14] i.e., solutions satisfying the condition

$$v_\pm(\varphi+2\pi k)=\exp(-i\lambda2\pi k)v_\pm(\varphi), \qquad (15)$$

λ is the quasienergy in units of h.

Equations (11) describe parametric resonance in a system consisting of an electron and photons. At small p and q^2 the functions v_+ and v_- are almost always close to the exponentials corresponding to stationary states in the magnetic field.[1)] The perturbation by the alternating electric field becomes large near the parametric-resonance points. An investigation of Eqs. (11) has shown that in our case this is the region of frequencies close to those for which, as $\mathscr{E}_0\to0$, the following conditions are satisfied: $\alpha_1 = -\alpha_2 + 1$—region of one-photon resonance, $\alpha_1 = 1$; $\alpha_2 = 1$—region of two-photon resonance; $\alpha_1 = -\alpha_2 + 3$—region of three-photon resonance, $\alpha_1 = 2$; $\alpha_2 = 2$—region of four-photon resonance, and so forth.

We present the results of an investigation of the quasienergy states in the vicinity of the one- and two-photon resonances.

We seek the solution of Eqs. (1) in these regions in the form of the expansions

$$v_+(\varphi) = e^{-i\lambda\varphi}(A_0 + A_2 e^{2i\varphi} + \ldots),$$
$$v_-(\varphi) = e^{-i\lambda\varphi}(A_1 e^{i\varphi} + A_{-1} e^{-i\varphi} + A_3 e^{3i\varphi} + \ldots). \tag{16}$$

To obtain a solution near the one-photon resonance $\alpha_1 = 2$; $\alpha_2 = 2$ accurate to terms of order q^2 it suffices to take into account in (16) only the first terms. The solution of the system of equations for A_0 and A_1 yields the quasienergies

$$\lambda_{1,2} = {}^1/_2 + {}^1/_2[\alpha_1(n, p_z) - \alpha_2(n, p_z)] \mp \xi(n, p_z), \tag{17}$$
$$\xi(n, p_z) = [{}^1/_4(1 - \alpha_1 - \alpha_2)^2 + 4q^2]^{1/2}, \tag{18}$$
$$A_1 = -\frac{2iq}{\alpha_2{}^2 - (\lambda - 1)^2} A_0.$$

With deviation of the frequency ω in the solution $(v_+^{(1)}, v_-^{(1)})$, corresponding to λ_1 from resonance, we get $|A_0^{(1)}| \gg |A_1^{(1)}|$, which corresponds to a state close to the stationary state of an electron with a positive energy $|E_1(n, p_z)|$. For λ_2 we have correspondingly $|A_1^{(2)}| \gg |A_0^{(2)}|$, and the solution $(v_+^{(2)}, v_-^{(2)})$ corresponds, with increasing distance from the resonance region, to a negative electron state with energy $-|E_2(n, p_z)|$. Near the resonance we have for both roots of (17) $|A_0| \approx |A_1|$.

It is easily seen that two other independent solutions of the system (11) are $(v_+^{(3)}, v_-^{(3)}) = (v_+^{(1)*}, -v_-^{(1)*})$ and $(v_+^{(4)}, v_-^{(4)} = (v_+^{(2)*}, -v_-^{(2)*})$, which correspond respectively to the quasienergies $\lambda_{3,4} = -\lambda_{1,2}$, and on deviation from the resonance they go over into stationary electron states with energies $|E_1(n, p_z)|$ and $|-E_2(n, p_z)|$.

For each of the four independent solutions of Eqs. (11) in the vicinity of the resonance we can obtain with the aid of (4), (9), and (14) the corresponding solutions $\psi^{(k)}$ of Eq. (1). The coefficients a_1 and a_2, which have so far not been defined, will be taken to be $a_1 = a_2 = 1/2$. This choice is equivalent to stating that on going off resonance (or as $\mathscr{E}_0 \to 0$), and with increasing width of the forbidden band, the $\psi^{(k)}$ with $k = 1$ and 2 are transformed respectively into stationary states. These states are $X_{n1}^{(+)}$, with large upper components, which describe an electron with positive spin in the conduction band, and $X_{n2}^{(-)}$ with large lower components, describing an electron with negative spin in the valence band.

In the region of two-photon resonance defined by the condition $\alpha_1 = 1$ there is resonant dipole coupling of noninteracting electron states having equal Landau quantum numbers and equal signs of the effective spin. To solve (11) approximately it is necessary here, generally speaking (if no additional relations between p and q^2 are made), to take into account in the expansions (16) all the written-out terms. The solution of the system obtained for the coefficients A_j for states at resonance yields quasi-energies

$$\lambda_{1,2} = 1 \pm \nu, \tag{19}$$
$$\nu = \{[\alpha_1 - 1 - (-p + {}^2/_3 q^2)][\alpha_1 - 1 - (p - {}^{10}/_3 q^2)]\}^{1/2}. \tag{20}$$

This shows immediately the difference between the obtained situation and one-photon resonance. In fact, it follows from (20) that in the region of two-photon resonance (α_1 close to unity) ν vanishes at two values of $\alpha_1(n, p)$:

$$\alpha_1^{(l)}(n, p_z) = 1 - p + {}^2/_3 q^2,$$
$$\alpha_1^{(r)}(n, p_z) = 1 + p - {}^{10}/_3 q^2. \tag{21}$$

These equations, together with (12) and (13), determines at fixed n and p_z the two external-field frequency at which exact resonance takes place. If the external field frequency lies between $\omega^{(l)}(n, p_z)$ and $\omega^{(r)}(n, p_z)$, then the quasienergy turns out to be imaginary. This must be taken to mean that in this narrow external-electric field frequency interval (at $p, q^2 \ll 1$) our approximate solution, given by Eqs. (4)–(14), is incorrect for the given n. Even if the conditions (3) are satisfied (as is assumed by us in all cases), the character of the motion is determined not by the magnetic field (magnetic quantization), but by the electric field.

We note that the mathematical treatment becomes much simpler, although all the qualitative conclusions remain unchanged if it is assumed that $p \gg q^2$. In this case, near even-photon resonances, the terms $\sim q$ in (11) can be neglected completely. In particular, as seen from (21), in the two-photon case the exact-resonance points are symmetrically arranged relative to the frequency corresponding to $\alpha_1 = 1$.

In the functions $(v_+^{(1,2)}, v_-^{(1,2)})$, corresponding to the quasienergies (19) we have $-A_2^{(1,2)} \to A_0^{(1,2)} \approx 1$ and $A_2^{(1,2)} \to A_0^{(1,2)} \approx 1$ as $\omega \to \omega^{(l)}$ and $\omega \to \omega^{(r)}$, respectively. The coefficients A_{2j+1} always remain of the order of q. Just as above, we put in (10) $a_1 = a_2 = 1$. With increasing distance from resonance ($\alpha_1 - 1 \gg p, q^2$) the perturbation by the alternating field becomes small and the states $\psi^{(1,2)}$ corresponding to $(v_+^{(1,2)}, v_-^{(1,2)})$, turn into stationary states with energies $|E_1(n, p_z)|$ and $-|E_1(n, p_z)|$.

Two other linearly independent solutions of the system (11) are $(v_+^{(3)}, v_-^{(3)}) = (v^{(1)*}, -v_-^{(1)*})$ and $(v_+^{(4)}, v_-^{(4)}) = (v^{(2)*}, -v^{(2)*})$. They correspond to the quasienergies $\lambda_{3,4} = -\lambda_{1,2}$.

Perfectly analogous results are obtained from an analysis of the two-photon resonance $\alpha_2 = 1$ when the states at resonance have the unperturbed energies $|E_2(n, p_z)|$ and $-|E_2(n, p_z)|$. The regions of the two-photon resonances with the same value of n differ in frequency by an amount on the order of the spacing between neighboring Landau levels, i.e., much larger than the width of each line.

Approximately the same conclusions result from a study of resonances in which a large number N of photons participate, although the analytic treatment becomes very cumbersome. At odd N there appears a forbidden gap with width on the order of q^N on the quasienergy axis, and at even N there appears a "forbidden" gap of width $\sim(q^N, p^{N/2})$ on the α_1 axis, with λ imaginary inside this gap. In the latter case the behavior of the

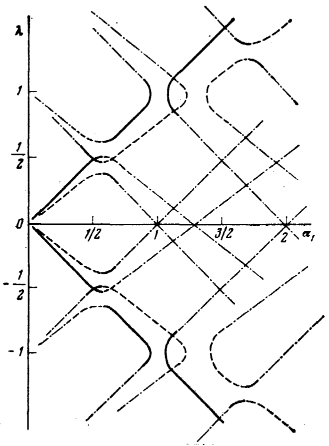

FIG. 1. Dependence of the quasienergy λ (in units of $\hbar\omega$) on α_1. The solid lines are the quasienergies of states that go over in the limit as $\mathscr{E}_0 \to 0$ into stationary states with energies $\pm |E_1(n, p_z)|$; dashed—quasienergies of states that go over in the limit $\mathscr{E}_0 \to 0$ into stationary states with energies $\pm |E_2(n, p_z)|$. The entire pattern is periodic along the λ axis with unity period.

$\lambda(\alpha_1)$ curve is similar to that of the plot of the amplitude of a classical oscillator against ω_0/ω near the boundaries of the parametric-resonance region. A plot of λ against α_1 is shown in the figure.

MAGNETO-OPTICAL ABSORPTION

The purpose of this section is to find the probabilities of the interband transitions due to an intense alternating electric field turned on at the instant of time $t=0$. We assume that before the field is turned on, at $t<0$, the semiconductor was in the ground state, i.e., all the states of the valence band (negative states) are occupied, and the states of the conduction band (positive states) are free.

After turning on the alternating electric field, any one-electron wave function that satisfies Eq. (1) can be represented by an expansion in the quasienergy states[14]:

$$\psi(\mathbf{r}, t)=\sum_{k,n'} b_k(n')\psi^{(k)}(n'; \mathbf{r}, t), \tag{22}$$

where the sum over k denotes summation over states with given n, p_z, and p_y.

We choose the coefficients $b_k(n')$ such that at $t=0$ the initial condition

$$\psi(\mathbf{r}, 0)=X_{nl}^{(s)}(\mathbf{r}, 0; p_y p_z), \tag{23}$$

is satisfied, where $X_{nl}^{(s)}(\mathbf{r}, t; p_y p_z)$ is the stationary state at $\mathscr{E}_0=0$ for the Landau level n, for the sign of the effective spin $l=1, 2$ and for the sign of the energy $s=\pm$. In our case s in (23) should correspond to a negative state.

On the other hand, for the function that satisfies the condition (23) we can write

$$\psi(\mathbf{r}, t)=\sum_{\sigma'} c(\sigma, \sigma'; t)X_{n'l'}^{(s')}(\mathbf{r}, t; p_y, p_z), \tag{24}$$

where $\sigma \to (n, l, s)$ is the aggregate of the quantum numbers of the stationary state in the magnetic field.

The probability of transition, under the influence of the electric field, from the state σ into the state σ' is determined by the square of the modulus of the coefficient c:

$$c(\sigma, \sigma'; t)=\int \psi(\mathbf{r}, t)X_{n'l'}^{(s')*}(\mathbf{r}, t)\,d\mathbf{r}. \tag{25}$$

The total probability of exciting an electron into the conduction band is obtained after summing over the states:

$$w=\sum_{n,n';l,l'}(2\pi\varkappa a_H)^{-2}\int_{-\infty}^{\infty}|c(\sigma, \sigma'; t)|^2 dp_z. \tag{26}$$

To obtain in practice, in the resonance regions, a function $\psi(\mathbf{r}, t)$ satisfying the condition (23) at $p_z \approx 0$ (singularity or maximum of the state density), accurate to terms p and q^2, it was sufficient to take account in expansion (22) only the terms with quantum number $n'=n$, i.e., take only the four described solutions of the Dirac equation (1) at a given frequency ω. The function $\psi(\mathbf{r}, t)$ takes in this case a general form specified by Eqs. (4), (9), and (14), but contains in place of the functions $v_\pm(\varphi)$, corresponding to some quasienergy state, a linear combination $V_\pm(\varphi)$ of all four independent quasienergy solutions of the system (11) in the specified frequency region; this combination is determined by the initial condition (23).

The coefficients $c(\sigma, \sigma'; t)$ differ from zero, generally speaking, for any pair of stationary states σ and σ'. However, if this pair is perfectly arbitrary, then the coefficient $c(\sigma, \sigma')$ yields for $w(t)$, in the vicinity of the resonance of interest to us, a value that oscillates at with frequencies that are multiples of ω. Only for some definite pairs of states near the given resonance will the coefficients c be such that $w(t)$ contains in addition to terms that oscillate or vanish as $t\to\infty$ also a part that is proportional to the time in the limit as $t\to\infty$. In this case we can speak of a definite probability of a transition with absorption of a definite number of photons (corresponding to the given resonance).

In the one-photon resonance-frequency region the transitions allowed in the indicated sense are those between positive and negative states at $\Delta n=0$ and with reorientation of the effective spin. For both transitions $\sigma(n, 2, -) \to \sigma'(n, 1, +)$ and $\sigma(n, 1, -) \to \sigma'(n, 2, +)$ the described procedure yields, accurate to terms $\propto q^2$

$$|c(\sigma, \sigma'; t)|^2=\frac{4q^2(D+{}^1/_2\hbar\omega)^2}{\hbar\omega(ms^2+{}^1/_2\hbar\omega)}\frac{\sin^2\xi\varphi}{\xi^2}, \quad \varphi=\omega t \tag{27}$$

where q and ξ are determined by (13) and (17), and

$$D=ms^2+[2(\hbar s/a_H)^2 n-(\hbar s p_z)^2](ms^2+{}^1/_2\hbar\omega)^{-1}. \tag{28}$$

Changing in (26) to integration with respect to the variable ξ, we obtain

$$w=4\sum_n\left(\frac{2\mu(n)\omega}{\hbar}\right)^{1/2}\int_{\xi_0}^{\infty}\frac{\xi}{(\xi^2-4q^2)^{1/2}}\frac{|c(\sigma, \sigma'; t)|^2}{[1-\alpha_1(n, 0)-\alpha_2(n, 0)+2(\xi^2-4q^2)^{1/2}]^{1/2}}. \tag{29}$$

Here

$$\mu(n)=\mu_1\mu_2/(\mu_1+\mu_2), \quad \mu_{1,2}(n)=s^{-2}[(ms^2)^2+2\hbar\Omega ms^2(n+{}^1/_2\pm{}^1/_2\varkappa)]^{1/2}, \tag{30}$$

ξ_0 is the root of the expression under the radical sign in the last factor of (29):

$$\xi_0=[{}^1/_4[\alpha_1(n,0)+\alpha_2(n,0)-1]^2+4q^2]^{1/2}. \quad (31)$$

If we can assume in the expression for ξ that $|1-\alpha_1-\alpha_2|\gg 4q$, then as $t\to\infty$ we have $\sin^2(\xi\varphi)/\xi^2 \to \pi\omega\delta(\xi)$ and for the number of transitions per unit volume and unit time we get at sufficiently small $|1-\alpha_1(n,0)-\alpha_2(n,0)|$

$$w_1=w/t=\sum_n \frac{2^{3/2}\omega q^2}{\pi a_H{}^2\hbar}\mu^{1/2}(n)\frac{(D+{}^1/_2\hbar\omega)^2}{ms^2+{}^1/_2\hbar\omega}(\hbar\omega-|E_1(n,0)|-|E_2(n,0)|)^{-1/2}. \quad (32)$$

It is seen that the singularity at the frequency of the one-photon transition has the usual square-root character observed in magneto-optics. In the limit of a wide forbidden band and not too strong a magnetic field we have $(\hbar s/a_H)^2\ll ms^2$, $2ms^2\approx\hbar\omega$, and (32) goes over into the well known result of perturbation theory.

In the two-photon resonance region $\alpha_1=1$ the time-independent transition probability occurs for two transitions

$$\sigma(n+1,2,-)\to\sigma'(n,1,+),\quad \sigma(n,1,-)\to\sigma'(n+1,2,+). \quad (33)$$

We note that in contrast to the one-photon transitions, the initial and final states do not interact resonantly here, although, accurate to terms $(c\mathscr{E}_0/sH)^2$, the condition $\alpha_1(n)+\alpha_2(n+1)=2$ is satisfied.

In the general expression (26) it is convenient to change near resonance from integration with respect to p_z to integration with respect to the variable ν defined in (20). This change yields

$$w=2s^{-1}(2\pi a_H\varkappa)^{-2}\sum_{n,n';ll'}\omega\int_{\nu_0}^{\infty}\left(\frac{1-{}^4/_3q^2}{|(\beta^2+\nu^2)^{1/2}|}+1\right)$$
$$\times\frac{\nu|c(\sigma,\sigma';t)|^2d\nu}{[(1-{}^4/_3q^2+|\beta|)^2+((1-{}^4/_3q^2)/|\beta|+1)\nu^2-\Delta^2(n)]^{1/2}}, \quad (34)$$

where

$$\beta=-p+2q^2,$$
$$\Delta(n)=(\hbar\omega)^{-1}[(ms^2)^2+2\hbar\Omega ms^2(n+1)]^{1/2}\equiv E_0(n)/\hbar\omega, \quad (35)$$

$\nu_0(\omega)>0$ is the root of the expression under the radical sign in the last factor of the right-hand side of (35).

For the two transitions of (33), calculation yields near the exact-resonance points $\omega^{(l)}$ and $\omega^{(r)}$ defined in (21)

$$|c(\sigma,\sigma';t)|^2=2\beta^2\left(\frac{\hbar s}{a_H}\right)^2\frac{(n+1)}{(ms^2+\hbar\omega)^2}\frac{\sin^2\nu\varphi}{\nu^2}[1+O(p,q^2)]. \quad (36)$$

The second term in the square brackets of (36) denotes the terms $\propto p$ and q^2 (both independent of time and oscillating at frequencies $2\omega, 4\omega, \ldots,$). These terms are different near the points $\omega^{(l)}$ and $\omega^{(r)}$.

As $t\to\infty$ we have $|c(\sigma,\sigma';t)|^2\sim\delta(\nu)t$ and can readily see that at an arbitrary frequency ω of the external field the integral in (34) vanishes. Obviously, the a nonzero but finite probability of the two-photon transition is obtained only at a field frequency such that $\nu_0(\omega)=0$. This frequency is defined by the condition

$$\Delta(\omega)=1-{}^4/_3q^2+|\beta|. \quad (37)$$

Hence, using the explicit forms of the parameters β, p, and q^2, we find that at $p<2q^2$ there are absorbed two photons of frequency

$$\omega(n)=\frac{E_0(n)}{\hbar}+\frac{1}{8\hbar E_0(n)}\left(n+\frac{1}{2}\right)(e\mathscr{E}_0a_H)^2\left[1-\frac{1}{3}\frac{\hbar\Omega_0ms^2}{(n+{}^1/_2)E_0{}^2(n)}\right] \quad (38)$$

and at $p>2q^2$, two photons of frequency[2)]

$$\omega(n)=\frac{E_0(n)}{\hbar}-\frac{1}{8\hbar E_0(n)}\left(n+\frac{1}{2}\right)(e\mathscr{E}_0a_H)^2\left[1-\frac{5}{3}\frac{\hbar\Omega_0ms^2}{(n+{}^1/_2)E_0{}^2(n)}\right]. \quad (39)$$

Thus the frequencies of the photons drawn by the semiconductor from the field differ only by small increments from the frequency that might be expected from perturbation theory. The main conclusion of our analysis is, first, that, the probability of a two-photon transition in a magnetic field differs from zero at only one definite frequency; second, the irregularity produced in the spectrum by the two-photon transition is not of the singularity or step type, but is a narrow maximum of finite height. This somewhat unusual frequency dependence is due, of course, to the peculiarities of the quasienergy spectrum in the vicinity of the two-photon resonance.

According to (34)–(36) the total number of the two-photon transitions per unit volume and per unit time is

$$w_2=\frac{1}{16\sqrt{2}\pi a_H{}^2}\frac{1}{\hbar^2 s}\left(\frac{e\hbar\mathscr{E}_0}{ms}\right)^2\frac{e\hbar H}{mc}\sum_n\left(\frac{ms^2}{\hbar\omega}\right)^4$$
$$\times\frac{n+1}{(\hbar\omega+ms^2)^2}\left|1-a_H{}^2\frac{\omega^2}{s^2}\left(n+\frac{1}{2}\right)\right|^{1/2}[1+O(p,q^2)], \quad (40)$$

where ω is specified by (38) or (39), depending on whether the quantity under the absolute-value sign is positive or negative.

We present a few estimates. We put $E_g=2ms^2=0.2$ eV, $m=0.01m_0$, $H=10^5$ Oe, $\mathscr{E}_0=10^4$ V/cm. For the parameters used above we get $c\mathscr{E}_0/sH=0.075$, $p=1.25\cdot10^{-4}(n=0)$, $q=6.6\cdot10^{-3}$. The two-photon absorption coefficient is

$$\gamma=\frac{4\pi\hbar\omega}{c\mathscr{E}_0{}^2n_0(\omega)}w_2\approx\frac{4}{n_0(\omega)}, \quad (41)$$

where $n_0(\omega)$ is the refractive index of the semiconductor in the considered frequency region.

We have not considered magnetoabsorption of more than two photons. A study of the structure of the quasienergy spectrum, however, allows us to propose that the results of the present paper for one-photon absorption are qualitatively valid also in the case of any odd number of photons, while the results for two-photon absorption remain qualitatively in force for transitions with absorption of any even number of photons. This agrees with the general conclusions of a number of papers, where other models are used, both in the presence and in the absence of a magnetic field.[2-6,8,12]

We examine now, on the basis of the expression (40), obtained for the probability of two-photon absorption, the experimental results of Refs. 10 and 11. There they obtained the dependence of the photoconductivity on the magnetic field in the narrow-band semiconductor PbTe illuminated by laser light of wavelength $\lambda_1=10.6$ μm and

$\lambda_2 = 9.6\ \mu m$, at $\vec{\mathscr{E}}_0 \pm \|\mathbf{H}\|\mathbf{C}_4$. It is noted, first, that the indicated dependence has clearly pronounced maxima at definite values of the magnetic field; second, the positions of the maxima do not change when the field orientation is changed to $\vec{\mathscr{E}}_0 \|\mathbf{H}\|\mathbf{C}_4$. Both circumstances can be explained on the basis of expressions (35) and (38)–(40). In fact, it follows from these formulas that the probability of the two-photon transition, and consequently also the photoconductivity, if the condition (38) or (39) is satisfied, has maxima that agree, accurate to the small quantities $\sim\mathscr{E}_0^2$, with the corresponding condition in parallel fields.[12]

To make expressions (35)–(40) applicable to optical anisotropic crystals, it is necessary to redefine in these expressions the magnetic field and the effective mass in accord with the rules cited in Refs. 15. In the case of the PbTe crystal, which has equal-energy surfaces in the form of an ellipsoid of revolution around threefold axes, with effective masses $m_\parallel = 0.25m_0$, $m_\perp = 0.028m_0$ and $E_g = 0.19$ eV, we can calculate, following the above redefinitions, the magnetic fields H that satisfy the conditions (38), (39), and (35). Since the resonance conditions for parallel and crossed fields are practically the same, the values of the resonant magnetic fields turn out to be the same as in the case of parallel fields.[12] The agreement between the resonance conditions at different field geometries is due to the fact that in parallel fields the two-photon transitions proceed with selection rules $\Delta n = 0$ without a change of the spin state, whereas in crossed fields they proceed with selection rules $|\Delta n| = 1$ and with reorientation of the effective spin.

Equation (40), according to which $w_2 \propto H^2$, explains also the experimentally observed, especially in Ref. 10, decrease of the intensity of the absorption peaks with decreasing magnetic field H. This seems to explain also the failure to observe in the experiment the theoretically predicted peak corresponding to the smallest of the magnetic-field values.

The two-photon transition probability obtained in the isolated band model (wide forbidden band) is

$$w_2 \approx H^2\mathscr{E}_0^4[2\hbar\omega - (E_{n,\,n+1} + e^2\mathscr{E}_0^2/4\mu\omega^2)]^{-1/2},$$

$$E_{n,n+1} = E_g + \frac{eH}{m_e c}\left(n + \frac{3}{2}\right) + \frac{eH}{m_h c}\left(n + \frac{1}{2}\right), \quad \mu^{-1} = m_e^{-1} + m_h^{-1}. \quad (42)$$

From a comparison of (4) and (42) we see that they yield substantially different frequency dependences of the absorption spectrum.

It follows from (42) that absorption in a wide-band semiconductor should correspond to a square-root singularity, whereas the two-band model leads to finite minima. Also different are the dependences of the transition probabilities (42) and (40) on the electric and magnetic fields. Whereas in the wide-band semiconductor $w_2 \propto H^2\mathscr{E}_0^4$, according to (42), in a narrow-band semiconductor, as seen from (40), the dependence is more complicated and is determined by the ratio of the fields and by the width of the forbidden band.

It follows from the foregoing that the results of the present paper can be used both to study the general properties of quasienergy states in narrow-band semiconductors, and to interpret the magneto-optical experimental data.

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[1)] As $\omega \to 0$, Eqs. (11) yield, accurate to terms $(c\mathscr{E}_0/sH)^2$, the correct energy eigenvalues of the exactly solvable problem of the motion of an electron in constant cross fields.

[2)] If $p = 2q^2$, then it is necessary to take into account from the very outset the terms of higher order in the parameters p and q^2.

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