Distribution of partons in a relativistic hadron

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The distribution with respect to the number of slow partons in a hadron is considered as a function of the rapidity of the hadron. The corrections for the recombination of parton chains are ignored in the derivation and finding of an explicit solution for this distribution, which depends on the initial distribution at $y=y_0\approx 1$. Comparison with Regge diagrams yields a connection between the parameters of the parton model and Regge field theory. The Abramovskiĭ-Gribov-Kancheli cutting rules for reggeon diagrams are given a parton interpretation. The parameters of the parton model are estimated numerically, which shows that at the existing energies the slow parton density is apparently close to the saturation value, and allowance for enhanced Regge graphs is therefore in principle important.

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1. INTRODUCTION

The parton model[1,2] arose as a generalization of various field-theory models of the peripheral interaction of hadrons at high energies. By introducing a small number of rules, one can explain many characteristic properties of hadron-lepton and hadron-hadron interactions. Reproducing many results of Regge phenomenology, the parton model also enables one to analyze the space-time structure of the interaction.[2]

Fora long time the parton model existed as a method of qualitative analysis, guiding our intuition without being a rigorous theory. Recently, significant progress has been made in the development of the formalism of the parton model and its applications. Several papers[3-5] have studied the connection between the parton model and Regge field theory,[6] which is the only self-consistent theoretical scheme which describes elastic and inelastic[7] hadron interactions at high energies and not too large momentum transfer from a unified point of view. Grassberger[3] proposed a method for studying the parton wave function when considering the evolution of the parton distribution with respect to displacements of the origin in the rapidity scale. He also showed[5] that despite the apparent inconsistency of the concept of the parton wave function the parton model reproduces all the results of Regge field theory. Therefore, the parton model can be used as a rigorous phenomenological scheme to calculate hadronic processes. The parton model leads to a clear physical interpretation of the graphs of Regge theory field and the parameters contained in the theory. For example, in Ref. 4 a connection was established between the values of the pomeron parameter $\alpha_p(0)$ and the three-pomeron constant. It will be shown below that the actual relationship between these quantities is more complicated.

In the present paper, we study the distribution of slow partons in a relativistic hadron. The need for such information arises in various physical problems, especially when one is considering the interaction of high-energy hadrons with nuclei. The part played by tree Regge diagrams—the analog of a parton cascade in hadron-nucleus interactions—was understood for the first time by Kancheli and Matinyan,[8] who studied these graphs in detail.

The present paper is arranged as follows. In Sec. 2, we introduce a method for studying the parton wave function of a hadron by shifts with respect to the rapidity. Particular attention is devoted to the passive component,[3] the state not containing slow partons. The weight of the passive component was determined earlier in Ref. 9 from data on the total cross sections of hadron-nucleus interactions.

In Sec. 3, we consider the development of the parton configurations under shifts in the rapidity. We make a detailed calculation for the case when the interaction between the parton chains can be ignored. We consider the asymptotic behavior. We show that the neighborhood of the critical point in the values of $\alpha_P(0)$ is distinguished.

In Sec. 4, we compare the parton model with the contributions of Regge graphs. This yields a connection between the parameters of the two approaches. We show that the contribution of the nonenhanced pomeron branchings has an eikonal form only under the condition that the number of fast partons in the hadron has a Poisson distribution. The rules for cutting of Regge graphs[7] is interpreted in the parton model.

In Sec. 5, the parameters of the parton model are estimated numerically. The values of these parameters are found to be very sensitive to the value of P_q, the contribution of the active component to the wave function of the valence quark. An accurate determination of P_q is also important in elucidating the part played by enhanced Regge graphs.

2. THE PASSIVE COMPONENT

The parton wave function of a hadron is not invariant under Lorentz transformations. The generator of a Lorentz transformation along the z axis has the form

$$L_z=K_zt-\int d^3x\, H(x)z. \qquad (1)$$

Here, K_z is the z component of the total momentum, and $H(x)$ is the density of the Hamiltonian.

The generator L_z can be represented in the form

$$L_z=L_z^{(0)}+L_z^{int}, \qquad (2)$$

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where $L_z^{(0)}$ contains only the free part of the Hamiltonian and generates shifts of the partons with respect to the rapidities. The interaction of the partons, i.e., their decay and fusion, is described by $L_z^{\rm int}$. In "soft" field theory, in the spirit of which the parton model is constructed, $L_z^{\rm int}$ has a structure such that its intensity (rate of fusion and decay of partons) decreases rapidly, at least as $1/E$, with increasing parton energy.

As a result, we have the following picture. When the hadron momentum is increased (which is achieved by development under the influence of the "Hamiltonian" K_z), the partons of its wave function that have sufficiently large rapidity, $y \gg 1$, do not interact and behave as free systems, i.e., their rapidities increase uniformly. At the same time, in the region of the slow partons ($y \approx 1$) the shift with respect to the rapidity is accompanied by decays and fusion of partons.

Note that if for some value of Y a given component of the wave function contains only fast partons with $y_i \gg 1$, then if Y is increased there is only a shift in the position of the partons in the rapidity scale without decay and fusion of the partons. Since such a component does not interact with a target at rest, we shall say it "passive." On the other hand, the "active" component, which contains several slow partons, can be transformed into the passive component with increasing Y, since there exists a probability that the slow partons will not decay during the time required to reach the fast part of the spectrum. It can be seen from this that the norm of the passive component w_0 of the parton wave function of the hadron is a monotonically increasing function of its rapidity Y (Ref. 3):

$$dw_0/dY \geqslant 0. \tag{3}$$

If Y is sufficiently large, the processes of multiplication and fusion of slow partons occurs independently of the quantum numbers of the hadrons as Y is increased. But in the region $Y \lesssim 1$ the hadron quantum numbers can influence the growth of w_0, which in this region can occur very rapidly because of the power-law decrease with the energy of the contribution of secondary reggeons.

It follows from what we have said that the passive component of the wave function is distinguished, since its weight w_0 can reach large values. In addition, w_0 depends on the quantum numbers of the hadron.

The existence of a large passive component is manifested, for example, in the interaction of high-energy hadrons with nuclei. In Ref. 9, the total nucleon-nucleus cross sections were analyzed with a view to determining the weight of the passive component of the quarks. Under the assumption of additivity of the quark amplitudes and with neglect of the distribution over the number of slow partons in the active component and mixing of the different components during the passage of the hadron through the nucleus it was shown that the weight of the active component P_q for u and d quarks is only about 0.5. A similar analysis of the experimental data[10] on the total cross sections of K_LA interactions showed that for the strange quark P_s is even smaller: $P_s/P_q \approx 0.5$.

3. CALCULATION OF A PARTON CASCADE

1. Using the qualitative arguments of the preceding section, we obtain here a system of equations that describe the variation with the energy of the distribution with respect to the number of slow partons in a hadron. We denote by $w_n(y)$ the probability that a hadron with rapidity y contains n slow partons. These probabilities have the normalization

$$\sum_{n=0}^{\infty} w_n(y) = 1. \tag{4}$$

As we have noted above, a slow parton may not succeed in decaying during the "time" of transition to the fast part of the spectrum. We shall call this phenomenon breaking of the parton chain. We denote the probability of breaking in the unit of "time" (rapidity) by γ. It may also happen that a parton does succeed in decaying and the chain is not broken. If a parton succeeds in decaying twice during the "time" of transition to the fast part of the spectrum, two independent parton chains can be formed. We denote the probability of this per unit rapidity by λ.

The system of equations describing the evolution of the the parton distribution function has the form[5]

$$dw_0/dy = \gamma w_1, \tag{5}$$

$$dw_n/dy = -(\gamma+\lambda) n w_n + \gamma (n+1) w_{n+1} + \lambda (n-1) w_{n-1}. \tag{6}$$

Equation (6) refers only to the active component. In the system of equations (5)–(6) we have ignored the process of fusion of two parton chains whose wee partons were separated by a short distance in the plane of the impact parameters. We shall return to this question below.

We introduce $F(x,y)$, the generating function for the distribution function w_n:

$$F(x, y) = \sum_{n=0}^{\infty} x^n w_n(y). \tag{7}$$

It follows from (4) that

$$F(1, y) = 1. \tag{8}$$

Using (5) and (6), we obtain

$$\frac{\partial F(x,y)}{\partial y} = (1-x)(\gamma-\lambda x)\frac{\partial F(x,y)}{\partial x}. \tag{9}$$

This equation can be solved by the method of characteristics. The general solution has the form

$$F(x,y) = F\left(\frac{\gamma(1-x)-(\gamma-\lambda x)e^{-\Delta y}}{\lambda(1-x)-(\gamma-\lambda x)e^{-\Delta y}}, 0\right). \tag{10}$$

Here, we have introduced the notation

$$\Delta = \lambda - \gamma. \tag{11}$$

If the initial parton distribution $w_n(0)$ at $y=0$ is given, then from (9) we can find $\langle n\rangle_y$, the mean number of slow partons. We have

$$\langle n\rangle_y = \frac{\partial}{\partial x} F(x,y)|_{x=1} = \langle n\rangle_0 e^{\Delta y}. \tag{12}$$

This growth of $\langle n\rangle_y$ ensures growth of the total hadron cross sections with the energy and can be related to the quantity

$$\Delta=\alpha_P(0)-1. \tag{13}$$

Using (7) and (10), we can readily calculate the distribution function with respect to the number of wee partons. If there was one slow parton at $y=0$, then

$$w_0(y)=\frac{(1-P(\infty))(1-e^{-\Delta y})}{1-(1-P(\infty))e^{-\Delta y}}, \tag{14}$$

$$w_n(y)=P^2(\infty)e^{-\Delta y}(1-e^{-\Delta y})^{n-1}/[1-(1-P(\infty))e^{-\Delta y}]^{n+1}. \tag{15}$$

Here

$$P(\infty)=\Delta/\lambda \tag{16}$$

is the weight of the active component in the asymptotic behavior for the tree graphs.

It can be seen by comparing (14) and (15) that the passive component really is separated from the general distribution, which has the form of a geometric progression. It can also be seen from (11) and (16) that the equality $\lambda=\Delta$ obtained in Ref. 4 is valid only for $\gamma=0$. The relation (16) can be readily obtained by a method analogous to that presented in Ref. 4. For this, it is sufficient to assume that the distribution of the partons in the active component is such that $\langle n^2\rangle_{\text{act}} \approx \langle n\rangle^2_{\text{act}}$ (with accuracy $1/\langle n\rangle$) and that the passive component, which has weight $1-P(\infty)$, is separated from the total distribution.

2. We now discuss the part played by the fusion of parton chains. Allowance for these process in Eq. (6) makes it difficult to solve that equation explicitly. However, appreciable corrections arise here because of this only at a fairly high density of slow partons. We shall therefore assume that the solution (10) is a good approximation for not too large values of y.[1)]

Note also that the expression (14) for the weight of the passive component is changed little by allowance for absorption, since the transfer of the norm in w_1 occurs from states with small values of n.

For $\Delta>0$, the value of $\langle n\rangle_y$ increases rapidly in accordance with (12). The increase in the parton density is stopped by the absorption process, as a result of which an equilibrium density of slow partons is formed in the active component.[3,11] This process is described by the equation

$$\frac{\partial\rho(y,\mathbf{b})}{\partial y}=\Delta\rho-\rho[1-e^{-s\rho}]-\alpha'(\nabla_b)^2\rho. \tag{17}$$

Here, $\rho(y,\mathbf{b})$ is the density of the slow partons in the active component in the plane of the impact parameter $\mathbf{b}$; s is a dimensional parameter which characterizes the "rate" of absorption of partons. The factor $1-\exp(-s\rho)$ is the probability that there is at least one parton in the region of impact parameters of area s. When the saturation density ρ_0 is reached, the last term on the right-hand side of (17) vanishes, and

$$\rho_0=-\ln(1-\Delta)/s\approx\Delta/s. \tag{18}$$

The radius of the disk within which $\rho=\rho_0$ is equal to $R\approx 2(\alpha'\Delta)^{1/2}y$, where α' is the rate of diffusion of partons in the $\mathbf{b}$ plane.

In contrast to the work of Levin and Ryskin,[11] who did not take into account the passive component, we find that a disk with saturation density of partons that impinges on a target at rest is not "black" even in the active state. Indeed, the partial-wave amplitude of such a process is equal to

$$f_{AB}(\mathbf{b})=P_A(\infty)[1-\exp(-\rho_0\sigma_B)]. \tag{19}$$

Here, σ_B is the cross section for the interaction of the slow partons with the target B. We now go over to the center of mass system of the colliding hadrons A and B:

$$f_{AB}(b)=P_A(\infty)P_B(\infty)\left[1-\exp\left(-\int d^2b'\rho_0{}^2\sigma_0\right)\right]. \tag{20}$$

Here, σ_0 is the interaction cross section of two slow partons.

The integration over $\mathbf{b}'$ in (20) is over the region of overlap of the colliding disks, which increases with the energy as y^2. Therefore, the exponential in (20) tends to zero, and, comparing (19) and (20), we find that the amplitude of the interaction in the active state of the parton disk with the target B is

$$f^{act}_{AB}(\mathbf{b})=1-\exp(-\rho_0\sigma_0)=P_B(\infty)<1. \tag{21}$$

The picture described above is valid only far from the neighborhood of the phase transition point Δ_c.[12] If $P(Y)$ for $\Delta>\Delta_c$ tends asymptotically to the constant $P(\infty)$, then $P(Y)\to 0$ (as $Y\to\infty$) for $\Delta=\Delta_c$ as a power of Y, while $P(Y)$ decreases exponentially with Y for $\Delta<\Delta_c$.[9] However, formula (16) does not have these properties. The reason for this is that no allowance was made for absorption on account of the assumption $\rho\ll\rho_0$. For $\Delta\approx\Delta_c$, this assumption is certainly not satisfied; for in the limit $\Delta\to\Delta_c$ the value of $P(\infty)$ must tend to zero and, as follows from the Lorentz invariance of (21), $\rho_0\to 0$. In what follows, we shall assume that the supercritical regime is realized.

4. COMPARISON WITH REGGE GRAPHS

1. The evolution of the distribution with respect to the number of slow partons occurs as is described in the previous section only for sufficiently large values of $y\gg 1$, when the quantum numbers of the fast hadron no longer have an influence. We cannot follow the variation of $w_n(y)$ at small y, or at least not at the level at which the present discussion is conducted. We therefore specify the initial parton distribution $w_n^0(y_0)$ by the generating function $F_0(x,y_0)$ at a rapidity value $y_0 \approx 1-2$.

We shall assume that the quantum numbers of the hadrons influence only $F(x,y_0)$, and that the development of the parton chains for $y>y_0$ occurs in a universal manner. Then, using the solution (10), we can calculate the distribution of the slow partons in a fast hadron and study their interaction with the target, comparing the result with the expressions for Regge graphs.

The contribution of the pole graph to the scattering amplitude of hadrons A and B (Fig. 1) corresponds to the case when only one slow parton has interacted with the target and only one parton chain has been broken up

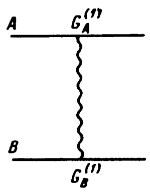

FIG. 1. Diagram corresponding to the pole contribution to the scattering amplitude of hadrons A and B.

(Fig. 2). For this it is necessary to select the parton configurations in which there is no absorption. Comparing these contributions to σ_{tot}^{AB}, we find

$$\sum_n n w_n^A(y) e^{\Delta Y}\sigma_B = G_A^{(1)} G_B^{(1)} e^{\Delta Y},$$
$$\langle n\rangle_A \sigma_B = G_A^{(1)} G_B^{(1)}. \tag{22}$$

Here, we have written down the parton contribution on the left-hand side. The quantity $\langle n\rangle_A$ is the mean number of slow partons in the initial distribution for $y = y_0$; σ_B is the cross section of the interaction of a slow parton with the target B. The factor $\exp(\Delta Y)$ on the left-hand side, which takes into account the multiplication of the partons in each "tree", cancels the same factor on the right-hand side, where it corresponds to pomeron exchange. By $G_A^{(k)}$ and $G_B^{(k)}$ we denote the vertices of emission of k pomerons by hadrons A and B.

The screened two-pomeron graph in Fig. 3 corresponds to the case when two slow partons belonging to different trees collide with the target. This contribution to the elastic amplitude, which has double density of particles in the rapidity scale, is of course positive, but after allowance fort he screening terms in the amplitude with single density of particles the sign is reversed and becomes negative. Comparing the expressions for the corresponding contributions, we find

$$\langle n(n-1)\rangle_A \sigma_B^2 = G_A^{(2)} G_B^{(2)}. \tag{23}$$

From both sides of this equation we have omitted identical factors $\exp(2\Delta Y)$ and $(R^2 + \alpha' Y)$, where R is the radius of the region over which the partons are distributed for $y = y_0$.

The relations for the three-pomeron graphs in Figs. 4a and 4b are obtained similarly:

$$\langle n\rangle_A \lambda \sigma_B^2 = G_A^{(1)} G_B^{(2)} r, \tag{24}$$
$$\langle n(n-1)\rangle_A s \sigma_B = G_A^{(2)} G_B^{(1)} r. \tag{25}$$

Study of more complicated graphs does not lead to new relations between the parameters. From Eqs.

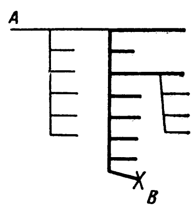

FIG. 2. Interaction of a definite parton configuration of hadron A with target B. Only one wee parton has interacted. The comb indicated by the heavy lines is transformed into a hadron.

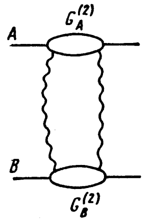

FIG. 3. Screening two-pomeron graph.

(22)–(25) we find

$$r = (\lambda s)^{1/2}. \tag{26}$$

Thus, λ is not equal to the dimensionless three-pomeron constant $r_0 = r/\sqrt{\alpha'}$ (in contrast to the result of Ref. 4), but is connected to it by the relation

$$r_0 = \lambda(\sigma_0/\alpha')^{1/2}, \tag{27}$$

where σ_0 is the cross section for the interaction of two slow partons from different hadrons. To see this, it is sufficient to consider in the center of mass system the screening in the interaction of two clouds of slow partons.

2. It is interesting that the existence of the three-pomeron constant leads to the need to introduce a four-pomeron (two into two) constant t. Indeed, the presence of the inelastic channel in the interaction of two parton chains (two into one) with cross section s leads to the appearance of elastic "scattering" with cross section s as a result of conservation of the total probability. These considerations lead to the following connection between the four-pomeron constant t and the absorption constant s (Ref. 5):

$$t = +s. \tag{28}$$

Further, from the expressions (24) and (25) we can obtain one more relationship between the parameters. In the case when $A = B$ and the distribution over n for $y = y_0$ is a Poisson distribution, we obtain from (24)–(25)

$$\langle n\rangle_A s = \lambda \sigma_A. \tag{29}$$

Note that the expressions (29) and (21) are equivalent. Indeed,

$$P_A(\infty) = \sum_n w_n[1-(1-P(\infty))^n] = 1-\exp\left(-\frac{\Delta}{\lambda}\langle n\rangle_A\right). \tag{30}$$

Here we have used (16). The expression (21) follows from (30), (18), and (29).

3. We now consider the nonenhanced graphs. Comparing (22) and (23), we see that if the initial distribution $w_n(y_0)$ is a Poisson distribution, i.e., if $\langle n(n-1)\rangle_A = \langle n\rangle_A^2$, then $G_A^{(2)} = (G_A^{(1)})^2$ and $G_B^{(2)} = (G_B^{(1)})^2$ and the

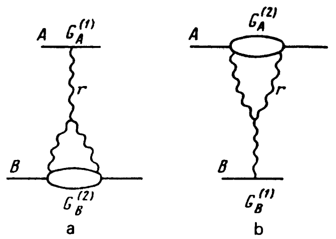

FIG. 4. Three-pomeron diagrams.

contributions to the elastic amplitude from exchanges of different numbers of pomerons have an eikonal form: $G^{(k)}=(G^{(1)})^k$.

This question warrants a more detailed discussion. At the first glance, we have here a contradiction, the essence of which is as follows. It is known[13] that the contribution of inelastic diffraction to the absorptive part to the elastic amplitude changes the eikonal form of the two-pomeron exchange, which corresponds to the contribution to the absorptive part arising from the elastic cross section. On the one hand, although for an initial Poisson distribution of the partons we have obtained an eikonal form of the elastic amplitude, corrections must arise, on the other hand, from inelastic diffraction, whose partial-wave cross section (for given b) is

$$f_{dif}^2=\langle f^2\rangle-\langle f\rangle^2=\exp(-2f\langle n\rangle)[\exp(f^2\langle n\rangle)-1]. \quad (31)$$

In this expression, we have for simplicity ignored the distribution of the partons over b. Since here the nonenhanced two-pomeron graph is under discussion, in (31) we have taken into account only the variance of the amplitudes f_k that arises for partons belonging to different trees, i.e., by f one must understand the partial-wave amplitude for scattering of a system of $\exp(\Delta Y)$ slow partons (the variance of the amplitude within this group leads to the diffraction corresponding to enhanced graphs).

We note that inelastic diffraction is a purely interference phenomenon and arises only in the presence of a variance of the amplitudes for the scattering of different numbers of slow partons. Therefore, the assumption that single-pomeron exchange is dominant in the cross section of inelastic diffraction is completely unjustified. At the same time, in the calculation of the corrections to two-pomeron branching it is necessary to take into account precisely the single-pomeron part of the diffraction, and not the complete cross section, as is usually done. Therefore, we separate the single-pomeron contribution from (31). For this, f_n must be written in the form $f_n^{(1)}$ must be written in the form $f_n^{(1)}=nf$. Then

$$(f_{dif}^2)_P=f^2(\langle n^2\rangle-\langle n\rangle^2)=f^2\langle n\rangle. \quad (32)$$

The circumstance that the diffraction cross section is proportional to $\langle n\rangle$ means that a contribution to $(f_{dif}^2)_p$ is made by only planar graphs, which, as is well known, do not contribute to the elastic scattering amplitude at high energies. Therefore, although the cross section of inelastic diffraction (31) and even its single-pomeron part (32) are nonzero in the considered case, no correction to the eikonal form of the branchings arises. This fact resolves the resulting paradox.

It should be noted that the eikonal form of the nonenhanced graphs is a direct consequence of the assumption of a Poisson distribution over the parton number for $y=y_0$. It is sufficient, for example, to change the weight of the passive component $w_0(y_0)$ to make the dependence $G^{(k)}$ found from (22) and (23) different. Simultaneously, there appear nonplanar graphs, and these make a contribution to $(f_{dif}^2)_p$ and in conjunction with $(f_{el}^2)_p$ lead, as is readily shown, to exactly the same form of $G^{(k)}$.

Note also that the Poisson form of the distribution for $y=y_0$ (and, hence, the eikonal form of the amplitude) certainly cannot be correct for large values of n, when the parton density approaches the saturation value ρ_0 and absorption cuts off the components with large n. The value n_{max} at which the cutoff appears can be estimated in accordance with

$$n_{max}\approx 4\pi\alpha_R' y_0\rho_0. \quad (33)$$

Here, $\alpha_R'\approx 1\ (\mathrm{GeV}/c)^{-2}$ is the slope of the f reggeon trajectory, since the parton distribution in the b plane for $y=y_0$ is determined to a large degree by the diffusion of the valence quark, which corresponds to the f reggeon. A numerical estimate of ρ_0 will be obtained in the following section.

4. To conclude this section, we consider how the rules for cutting the Regge diagrams[7] are interpreted in the parton model. We consider a nonenhanced diagram containing n pomerons, of which m are cut. Its contribution to the inclusive cross section is equal to[7]

$$F_{nm}=(-1)^{n+m}2^{n-m}C_n^m\rho_c^m\rho^{n-m}\frac{1}{n!}[G^{(n)}]^2. \quad (34)$$

Here, $\rho_c=2\rho$ is the Green's function of the cut pomeron. In parton language, such a graph corresponds to the situation when m slow partons (from different trees) in the incident hadron interact with the target. The amplitude of this process is

$$\sum_{k=m}^{\infty} w_k C_k^m f_c^m (1-f)^{k-m}. \quad (35)$$

In (35), f_c is the amplitude for the interaction of a slow parton with the target. The factor $(1-f)^{k-m}$ takes into account the circumstance that in a state with k partons $k-m$ partons do not interact ($f=f_c$). From this factor, we must separate the term $(-1)^{n-m}C_{k-m}^{n-m}f^{n-m}$, and this corresponds to a diagram with n pomerons, of which m are cut. Accordingly, the summation over k must begin with n:

$$F_{nm}=\sum_{k=n}^{\infty} w_k^0 C_k^m f_c^m(-1)^{n-m}f^{n-m}C_{k-m}^{n-m}. \quad (36)$$

This expression can be rewritten as

$$F_{nm}=(-1)^{n-m}C_n^m f_c^m f^{n-m}\sum_{k=n}^{\infty} w_k^0 C_k^n. \quad (37)$$

Writing $f_c=\rho_c\sigma_0$ and $f=2\rho\sigma_0$ and using the definition of $G^{(n)}$ introduced earlier, namely

$$\frac{1}{n!}(G^{(n)})^2=\sigma_0^n\sum_{k=n}^{\infty} C_k^n w_k^0,$$

we obtain the expression (34).

Earlier, the graph cutting rules were interpreted by Levin and Ryskin[14] for the example of the deuteron, which corresponds to the case when only one component with $k=2$ is present in the initial distribution over the parton number.

5. NUMERICAL ESTIMATES OF THE PARAMETERS

1. The magnitude of the passive component of u and d quarks at nucleon energy 240 GeV was determined in

Ref. 9 from data on the total cross sections of neutron-nucleus interactions. Using this result, we can determine the saturation density ρ_0 of the partons.

We find from (14)–(16) that if there is only one parton for $y=y_0$ then

$$P(y)=(\Delta/\lambda)[1-(1-\Delta/\lambda)e^{-\Delta y}]^{-1}. \quad (38)$$

Assuming that the number of partons for $y=y_0$ has a Poisson distribution, we find from (38) the weight of the active quark component P_q:

$$P_q(y)=1-\exp\left[-\frac{\langle n\rangle_q(\Delta/\lambda)}{1-(1-\Delta/\lambda)e^{-\Delta y}}\right]. \quad (39)$$

From (18), using (29), (22), (23), and (39), we obtain

$$\rho_0=\frac{\Delta}{\lambda}\frac{\langle n\rangle_q^2}{[G_q^{(1)}]^2}\approx\frac{\lambda}{\Delta}\ln^2[1-P_q(y)]\left[1-\left(1-\frac{\Delta}{\lambda}\right)e^{-\Delta y}\right]^2\frac{e^{\Delta y}}{\sigma_{tot}^{qq}}. \quad (40)$$

It is here assumed that single-pomeron exchange makes the main contribution to σ_{tot}^{qq}, i.e., that

$$\sigma_{tot}^{qq}\approx[G_q^{(1)}]^2e^{\Delta y}.$$

For numerical estimates, we assume that $\sigma_{tot}^{qq}\approx 8$ mb.

With regard to the value of λ/Δ, it is known only that $\lambda/\Delta\geq 1$. It is however easy to show that the value of ρ_0 is not sensitive to λ/Δ, since the derivative of ρ_0 with respect to λ/Δ vanishes at

$$\lambda/\Delta=[e^{\Delta y}-1]^{-1}.$$

Therefore, ρ_0 hardly changes in the range of reasonable values of λ/Δ. In Table I, we give the values of ρ_0 found from (40) for different values of λ/Δ and $P_q(y)$. The value of the parameter Δ was taken equal to 0.07 in accordance with Ref. 15.

2. It can be seen from Table I that there is weak dependence of ρ_0 on λ/Δ. At the same time, the value ρ_0 is very sensitive to the value of $P_q(y)$. The value of $P_q(y=5)$ found in Ref. 9 is an estimate, but at the achieved accuracy (for different nuclei $P_q\approx 0.4-0.67$) the data of Table I indicate the existence of some problems.

Indeed, let us calculate the density of slow partons corresponding to the emission of one parton "comb":

$$\rho^{(1)}(\mathbf{b})=\frac{1}{4\pi\alpha'y}\exp\left(-\frac{\mathbf{b}^2}{4\alpha'y}\right). \quad (41)$$

For $\alpha'=0.25$ (GeV/c)$^{-2}$, $y=5$, and $b=0$ we have $\rho^{(1)}(0)=0.064$. It can be seen that $\rho^{(1)}(0)$ is of the same order of magnitude as the saturation density ρ_0 calculated for $P_q=0.03-0.7$. This means that the widely accepted point of view (to which we also adhere) according to which the parton combs interact weakly ($\rho^{(1)}\ll\rho_0$) at existing energies, i.e., the contribution of the enhanced pomeron graphs is small, is apparently incorrect. In such a case, the true value of Δ must exceed the value $\Delta\approx 0.07$ found by fitting to the experimental data the formulas of the eikonal approximation,[15] since strong absorption reduces the growth in the number of slow partons, which is proportional to $\exp(\Delta y)$.

TABLE I. Values of the saturated density ρ_0 in units of (GeV/c^2) calculated for different values of the parameters λ/Δ and $P_q(y)$.

P_q ($y=5$)	λ/Δ					
	1	2	4	6	8	10
0.3	0.009	0.008	0.008	0.009	0.011	0.012
0.5	0.034	0.029	0.03	0.035	0.04	0.046
0.7	0.103	0.086	0.091	0.105	0.121	0.138
0.9	0.376	0.316	0.335	0.385	0.442	0.503

TABLE II. Value of the three-pomeron coupling constant r_0 calculated for different values of the parameters λ/Δ and $P_q(y)$.

P_q ($y=5$)	λ/Δ					
	1	2	4	6	8	10
0.3	1.09	2.18	4.36	6.53	8.71	10.89
0.5	0.29	0.58	1.15	1.73	2.31	2.88
0.7	0.10	0.19	0.38	0.57	0.76	0.95
0.9	0.03	0.05	0.10	0.16	0.21	0.26

Nevertheless, the possibility that $\rho^{(1)}\ll\rho_c$ cannot be regarded as ruled out. First, the definition of $P_q(y)$ requires a more accurate analysis. Second, an important assumption in (39) is that of a Poisson distribution with respect to the parton number for $y=y_0$. If this is not the case and the weight of the passive component satisfies $w_0(y=y_0)>\exp(-\langle n\rangle_q)$, the values of ρ_0 will be greater than those given in Table I.

We note further that the results of Table I also enable us to calculate the dimensionless three-pomeron constant $r_0=r/\sqrt{\alpha'}$, where r is given by (27). This value of the unrenormalized constant apparently exceeds considerably[16] the effective constant $r_0^{eff}\approx 0.1$, which is determined from inelastic diffraction data.[17]

The value of r_0 calculated for the same values of P_q and λ/Δ as in Table I is given in Table II.

It is usually assumed that the values of Δ and r_0 are such that the supercritical regime is realized, the condition for which is $\Delta>\Delta_c$ where (Ref. 12) $\Delta_c\approx r_0^2\ln r_0^2$ (it is assumed that $r_0^2\ll 1$). It follows from Table II that by no means all values of r_0 satisfy this inequality. For example, if $P_q=0.5$, then all values of r_0 correspond to the subcritical regime, $\Delta<\Delta_c$. In such a case, all the results of Sec. 3 become inapplicable. This is another aspect of the problem noted above ($\rho^{(1)}\approx\rho_0$). As we have already said, the true value of Δ is larger than the one used in the calculations, and the entire analysis should be redone.

3. Let us attempt to estimate λ/Δ. If we denote by ε the probability of a parton's decaying once during the "time" of transition to the fast part of the spectrum, the condition that the total probability is equal to unity has the form $\varepsilon/(1-\varepsilon)+\gamma=1$. But even if the parton decays twice, the produced partons may be reabsorbed and a branching of the combs need not occur, and therefore $\lambda<\varepsilon^2$. Hence

$$\lambda+\sqrt{\lambda}/(1-\sqrt{\lambda})-\Delta<1.$$

For $\Delta=0.07$, we obtain $1\leq\lambda/\Delta\leq 2.8$. Knowing λ/Δ and ρ_0, we can find

$$\langle n\rangle_q\approx\left[\frac{\lambda}{\Delta}\sigma_{qq}e^{-\Delta Y}\right],$$

which for $P_q(Y)=0.5$ gives

$$0.5\leq\langle n\rangle_q\leq 0.86.$$

Experimentally, $\langle n\rangle_q$ can be determined by studying multiple production of hadrons on nuclei and comparing inclusive spectra on different nuclei in the beam fragmentation region.

6. CONCLUSIONS

Although formally all the results obtained in the parton model can be found from the graphs of Regge field theory, the parton interpretation is frequently simpler and more perspicuous, which makes it possible to obtain new results. Besides direct application to calculations of hadron interactions at high energies, the analysis made in the present paper has shown that the generally adopted point of view concerning the part played by the enhanced Regge graphs at existing energies may be incorrect. The formulas obtained in the present paper enable one to analyze the experimental data on the interaction of hadrons in the framework of the parton model with allowance for all parton configurations. Very important here is the norm of the active quark component P_q, which must be determined more precisely. We note that in such an analysis it is possible to determine the value of the unrenormalized three-pomeron constant.

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[1)]Note that such an approximation corresponds to a restriction to nonenhanced pomeron graphs in a theory with rising cross sections. Moreover, this approximation also includes semi-enhanced graphs, allowance for which is important in the interaction of hadrons with nuclei. Allowance for the fusion of the parton chains in Eqs. (5)–(6) would correspond to the addition of enhanced loop-type diagrams, whose contribution at presently attainable energies is usually assumed to be small. In Sec. 5, we discuss a further possibility when it is necessary to take into account the fusion of the parton chains.

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