Contribution to the theory of weak superconductivity in SNINS systems

A. D. Zaikin and G. F. Zharkov

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted 16 July 1979)
Zh. Eksp. Teor. Fiz. 78, 721–732 (February 1978)

A general relation is obtained for the excitation-energy spectrum in an SNINS junction. The influence of the external magnetic field on the quantization of the excitation enenergy in an INS system is investigated. It is shown that the magnetic field shifts the energy levels of such a system, but is not the cause of the instability of the spectrum (in contrast to the previously investigated behavior of the spectrum of an SNS junction in a magnetic field). General formulas are derived that make it easy to find the Green's functions of an arbitrary superconducting system containing a dielectric barrier with arbitrary transparency, if the Green's functions of this system without this barrier are known. A microscopic calculation is made of the stationary Josephson current in an SNINS junction. The magnitude and temperature dependence of this current are different for different positions of the dielectric barrier inside the normal-metal layer.

PACS numbers: 74.50. + r

1. INTRODUCTION

It is well known that in a superconductor–normal metal–superconductor (SNS) system the energy of the excitations localized in the normal layer is spatially quantized. This phenomenon is due to the weak reflection of the electron excitations from the *NS* boundary, first pointed out by Andreev.[1] Excitations having an energy lower than the energy gap Δ of the superconductor cannot penetrate from the normal metal into the superconductor. However, because of the small value of Δ (compared with the Fermi energy) the excitation momentum is left practically constant by the reflection (the excitation momentum is of the order of the Fermi momentum). In an SNS system this leads to formation of a discrete spectrum at excitation energies lower than Δ, and the wave functions of the incident and reflected excitations have a spinor structure of the "electron-hole" type, which is peculiar to superconductivity. Kulik[2] has shown that in the presence of current in an SNS system the energy levels shift by an amount proportional to the phase difference between the order parameters of the two superconductors. The formula for the spectrum of the SNS junction (under the condition $E \ll \Delta$) is[2]

$$E_n = \frac{\pi v_x}{d}\left(n + \frac{1}{2} \pm \frac{\varphi}{2\pi}\right). \tag{1}$$

Here v_x is the absolute value of the projection of the excitation velocity on the direction of the normal to the boundary between the normal metal and the superconductors, and d is the thickness of the normal metal. We assume $\hbar = c = 1$ throughout. The presence of a discrete excitation spectrum of the type (1) determines in many respects the properties of SNS systems. Thus, the temperature and phase dependences of the Josephson current in an *SNS* junction (see, e.g., Refs. 2–6) differ noticeably from the analogous dependences for an *SIS* junction[7] (I is an insulator layer).

In the present paper we investigate certain properties of *SNINS* junctions, i.e., spatially-inhomogeneous superconducting systems whose "weak spot" is a combination of normal-metal and an insulator layers. A study of these systems is of interest both from the theoretical point of view and in connection with the possibility of performing the appropriate experiments. In Sec. 2 we obtain the excitation spectrum of such junctions. In addition, we discuss the influence of an external magnetic field on the quantization of the excitation energy. In Sec. 3 we calculate the stationary Josephson current in an SNINS system.

2. EXCITATION SPECTRUM, INFLUENCE OF MAGNETIC FIELD

Let a normal-metal layer placed between two superconductors have a width d. We direct the x axis perpendicular to the *NS* boundaries, and let $x=0$ be the midplane of the layer of the normal metal. We choose the order parameter of the system in the form

$$\Delta(x) = \begin{cases} \Delta e^{i\varphi_1}, & x < -d/2 \\ 0, & |x| < d/2. \\ \Delta e^{i\varphi_2}, & x > d/2. \end{cases} \tag{2}$$

This is the universally accepted model if the inequality

$$d \gg \xi_0 \tag{3}$$

is satisfied (ξ_0 is the coherence length of the superconductor). Assume that the thin insulator layer (usually ~10 Å thick) is located inside the normal-metal layer and is thin enough to permit electron tunneling. The small thickness of the insulator layer allows us to simulate its presence in the system by a potential of the form $V\delta(x-x_0)$. Here x_0 is the coordinate the insulator layer, $|x_0| < d/2$.

We write down the Bogolyubov–de Gennes equations[8,9] for a two-component wave function of the excitation in an SNINS system. They take the form

$$\begin{pmatrix} \hat{\mathcal{H}}_e(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -\hat{\mathcal{H}}_e^*(\mathbf{r}) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}. \tag{4}$$

Here $\hat{\mathcal{H}}_e = [-i\nabla - e\mathbf{A}(\mathbf{r})]^2/2m + V\delta(x-x_0) - \mu$, $\hat{\mathcal{H}}_e^*$ is its complex conjugate, $\mathbf{A}(\mathbf{r})$ is the vector potential, μ is the chemical potential, and m is the electron mass.

Assume that at first there is no magnetic field, i.e., $\mathbf{A}(\mathbf{r}) = 0$. We take the Fourier transforms in (4) with re-

 0038-5646/80/020364-06$02.40

spect to the coordinates y and z. We then get

$$\left[-\frac{1}{2m}\frac{d^2}{dx^2}-\xi_x+V\delta(x-x_0)\right]u(x)+\Delta(x)v(x)=Eu(x),$$
$$\left[\frac{1}{2m}\frac{d^2}{dx^2}+\xi_x-V\delta(x-x_0)\right]v(x)+\Delta^*(x)u(x)=Ev(x), \tag{5}$$

where $\xi_x=\mu-k_\perp^2/2m[k_\perp=(k_y^2+k_z^2)^{1/2}$ is the momentum of the excitation in the junction plane]. We seek solutions of Eqs. (5) in the form

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix}=\begin{cases} (A_{i+}e^{ik_0x}+A_{i-}e^{-ik_0x})\begin{pmatrix}1\\0\end{pmatrix}+(B_{i+}e^{ik_1x}+B_{i-}e^{-ik_1x})\begin{pmatrix}0\\1\end{pmatrix}, \\ i=1 \quad \text{if} \quad -d/2<x<x_0, \quad i=2 \quad \text{if} \quad x_0<x<d/2; \\ C_+e^{i\lambda_+(x-d/2)}\begin{pmatrix}e^{i\varphi_2}\\ \gamma\end{pmatrix}+C_-e^{-i\lambda_-(x-d/2)}\begin{pmatrix}e^{i\varphi_2}\\ \gamma^*\end{pmatrix}, \quad x>\dfrac{d}{2}; \\ D_+e^{i\lambda_-(x+d/2)}\begin{pmatrix}\gamma\\ e^{-i\varphi_1}\end{pmatrix}+D_-e^{-i\lambda_+(x+d/2)}\begin{pmatrix}\gamma^*\\ e^{-i\varphi_1}\end{pmatrix}, \quad x<-\dfrac{d}{2}. \end{cases} \tag{6}$$

We use the notation of Ref. 2. Thus,

$$\gamma=\Delta[E+i(\Delta^2-E^2)^{1/2}]^{-1}, \quad \lambda_\pm\approx mv_x\pm i(\Delta^2-E^2)^{1/2}/v_x,$$
$$k_0\approx mv_x+E/v_x, \quad k_1\approx mv_x-E/v_x.$$

The functions $U(x)$ and $v(x)$ should be continuous at the points $x=\pm d/2$ and $x=x_0$, and the derivatives of these functions should be continuous at $x=\pm d/2$ and have a discontinuity at the point $x=x_0$:

$$du/dx|_{x=x_0+0}-du/dx|_{x=x_0-0}=2mVu(x_0),$$
$$dv/dx_{x=x_0+0}-dv/dx|_{x=x_0-0}=2mVv(x_0).$$

After straightforward but rather cumbersome calculations, we obtain the dispersion equation

$$\cos\frac{2Ed}{v_x}+\frac{\cos(4Ex_0/v_x)}{1+v_x^2/V^2}+\frac{\cos\varphi}{1+V^2/v_x^2}=0. \tag{7}$$

This relation determines the energy spectrum of the excitations in the SNINS junction.

Let us examine some particular cases. At $V=0$ we obtain from (7) the spectrum of an SNS junction (1). Putting $x_0=0$ in (7) we get the spectrum of a symmetrical SNINS junction:

$$\cos\frac{2Ed}{v_x}=-\frac{\cos\varphi+(V/v_x)^2}{1+(V/v_x)^2}.$$

We note that at $\varphi=0$ the excitation spectrum is in this case completely independent of V. At $x_0=\pm d/2$ we obtain from (7) the spectrum of the SINS system[10]

$$\cos\frac{2Ed}{v_x}=-\frac{\cos\varphi}{1+2(V/v_x)^2}.$$

We consider now the case when there is no tunneling between the half-spaces $x<x_0$ and $x>x_0$ (i.e., $V=\infty$). It is seen from (7) that in this case there exist two isolated systems of levels (see the figure):

$$E_{1n}=\frac{\pi v_x}{d+2x_0}\left(n+\frac{1}{2}\right), \quad x<x_0,$$
$$E_{2n}=\frac{\pi v_x}{d-2x_0}\left(n+\frac{1}{2}\right), \quad x>x_0. \tag{8}$$

We have in mind here, of course, the condition $d/2-|x_0|\gg\xi_0$. On the other hand, if the insulator layer is located very close to one of the NS boundaries ($d/2-|x_0|\lesssim\xi_0$), then one of the level systems will simply "not fit" in the well made up by the pairing potential.

We assume now for simplicity $x_0=0$ and consider the

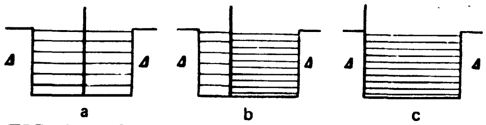


FIG. 1. Schematic representation of the structure of the energy levels of quasistatic excitations without allowance for tunneling: a—symmetric SNINS junction, b—asymmetric SNINS junction, c—SINS junction.

half-space $x>0$ (the INS system). This is equivalent in fact to considering a system in which the insulator occupies the region $x<0$ and the normal metal is in the region $0<x<d/2$, while the superconductor is in the region $x>d/2$. Obviously, the excitation spectrum in such a system is of the form

$$E_n=\frac{\pi v_x}{d}\left(n+\frac{1}{2}\right). \tag{9}$$

A spectrum of the form (9) was indicated for INS systems back in Ref. 10, which was devoted to an investigation of the behavior of SINS junctions. We discuss here the properties of INS systems in greater detail.

We have already established that the excitation spectrum of INS systems does not depend on the phase of the order parameter of the superconductor [see (9)]. Therefore, without loss of generality, we can set it equal to zero. The wave functions of the excitations in the INS system take the form

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix}=\begin{cases} A\sin k_0x\begin{pmatrix}1\\0\end{pmatrix}+B\sin k_1x\begin{pmatrix}0\\1\end{pmatrix}, & 0\leqslant x<\dfrac{d}{2}, \\ C_+e^{i\lambda_+(x-d/2)}\begin{pmatrix}1\\ \gamma\end{pmatrix}+C_-e^{-i\lambda_-(x-d/2)}\begin{pmatrix}1\\ \gamma^*\end{pmatrix}, & x>\dfrac{d}{2}. \end{cases} \tag{10}$$

We recall that in the SNS system there exist two energy-level systems (1) (which are degenerate at $\varphi=0$). This corresponds to two opposite directions of excitation motion (along the x axis). The two types of excitations ("+" and "−") are not intermixed because the momentum is almost completely conserved upon reflection from the NS boundary, i.e., the momentum $p_x=mv_x$ is in this case a "good" quantum number. In the INS system the reflection of the excitation from the IN boundary obeys the laws of specular reflection (the momentum p_x reverses sign), and on the NS boundary we have the usual Andreev reflection with momentum conservation. We see that in this case the two types of excitations already become intermixed (in the sense that the same excitation can have a momentum projection p_x as well as $-p_x$), and a single system of levels (9) is produced.

We investigate now the behavior of the spectrum of the excitations of the INS system in an external magnetic field. Assume the presence in the system of a magnetic field with the following configuration:

$$\mathbf{H}(x)=\{0,0,H(x)\}, \quad H(x)=H\Theta(d/2-x),$$

where $\Theta(x)$ is the Heaviside function, i.e., we assume that the magnetic field does not penetrate into the superconductor. We choose the following gauge for the vector potential:

$$\mathbf{A}(x)=\{0,A(x),0\}, \quad A(x)=H(x-d/2)\Theta(d/2-x).$$

The phase of the order parameter of the superconductor can be regarded as before to be equal to zero everywhere. Equations (4) for the region $0<x<d/2$ take in this case the form (after taking the Fourier transforms with respect to the coordinates y and z)

$$\left[\frac{1}{2m}\left(-\frac{d^2}{dx^2}-2k_yeH\left(x-\frac{d}{2}\right)+e^2H^2\left(x-\frac{d}{2}\right)^2\right)-\xi_x\right]u(x)=Eu(x),$$
$$\left[\frac{1}{2m}\left(\frac{d^2}{dx^2}+2k_yeH\left(x-\frac{d}{2}\right)-e^2H^2\left(x-\frac{d}{2}\right)^2\right)+\xi_x\right]v(x)=Ev(x). \tag{11}$$

In the superconducting region, at the chosen gauge of the vector potential, the magnetic field does not enter at all into the equations, meaning also that the form of the wave functions remains unchanged at $x>d/2$. In the solution of (11) we use the strong inequality $k_0\sim k_1\sim p_F \gg eH_cd>eHd$ (H_c is the critical field of the superconductor, $p_F=mv_F$ is the Fermi momentum). As a result the wave functions in the region $0\leqslant x<d/2$ take in the principal approximation the form

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = A\sin\left\{k_0x+\frac{1}{k_0}\int_0^x ek_y\left(x'-\frac{d}{2}\right)H\,dx'\right\}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$+B\sin\left\{k_1x-\frac{1}{k_1}\int_0^x ek_y\left(x'-\frac{d}{2}\right)H\,dx'\right\}\begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Joining together, as usual, the wave functions and their derivatives at the point $x=d/2$, we obtain the usual formula

$$E_n=\frac{\pi v_x}{d}\left(n+\frac{1}{2}\right)+\frac{e}{4}v_yHd. \tag{12}$$

We see that the energy spectrum in the considered situation now depends not only on the single excitation-velocity component v_x (as was the case at $H=0$), but also on the projection of the velocity on the y axis. Thus, the influence of the external magnetic field on the spectrum of the excitations in INS system reduces to a shift of all the energy levels by an amount proportional to the magnetic field and to the velocity v_y, and this result is valid in fact for all values of H, since in the derivation of (12) we made no assumptions whatever concerning the value of the magnetic field (apart from the obvious inequality $H<H_c$).

The question of the influence of an external magnetic field on the quantization of the energy of the excitations in SNS junctions was discussed in Refs. 11–13. It turned out (as first indicated by Galaĭko[11]) that application of a magnetic field leads to a substantial restructuring of a spectrum of the type (1). Such a spectrum is unstable, and the levels of (1) are replaced by energy bands. An important fact in this case is that the excitation spectrum (1) depends on the phase difference φ. Generally speaking, the phase difference of the order parameters of two superconductors can always be eliminated by a gauge transformation. However, not all the equations that determine directly the measured physical quantities (energy, current, etc.) should contain the gauge-invariant quantity

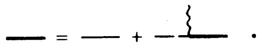
$$\varphi=\varphi_2-\varphi_1-2e\int_1^2 \mathbf{A}\,d\mathbf{l},$$

which is frequently called the gauge-invariant phase difference. This quantity, naturally, can no longer be eliminated by a gauge transformation. In the presence of a magnetic field it varies along the NS boundary, i.e., $\varphi=\varphi(y)$, and it is this which causes the instability of the spectrum (1). If the magnetic field is homogeneous along the y axis, then the function φ is a linear function of this coordinate. The Bogolyubov–de Gennes equations (4) constitute then a system of differential equations with periodic coefficients, so that one can speak of the presence of energy bands for the excitation system.[13]

The excitation spectrum (7) of the SNINS system also depends on the phase difference φ, meaning that it should be unstable when a magnetic field is turned on. However, the phase dependence of the spectrum (7) is determined by the transparency of the dielectric barrier. In the case of small transparency this dependence is weak, and consequently the smearing of the discrete levels of (7) is small. We emphasize once more that this result holds for arbitrary (not only small) values of the magnetic field. It is seen thus that the dependence (or lack of it) of the excitation energy on the phase of the order parameter of the superconductor in the presence of a magnetic field determines the qualitative differences in the behavior of the spectrum of the system. Whereas in the SNS system turning on the magnetic field "mixes" the levels and smears out the singularities in the state density, in the INS system the magnetic field only shifts the energy levels (12) but does not lead to instability of the spectrum.

3. STATIONARY JOSEPHSON EFFECT IN AN SNINS JUNCTION

A nondissipative current (Josephson current) can flow through the considered SNINS system, just as through any other weakly coupled superconducting system. For a microscopic calculation of this current we must know the Green's functions of the system. It is convenient to use for this purpose the following general method. We consider arbitrary (in the general case, inhomogeneous) superconducting system. Assume that we know the Green's functions[14] of such a system. We introduce in this system an external potential $U(x)$. To find the Green's functions of such a system in the presence of a potential it is not at all obligatory to solve the Gor'kov differential equations.[14] These functions can be obtained also with the aid of integral equations of the following form (the Dyson equations):



(13)

The thick lines in (13) correspond to the sought function $g_\omega(x,x')$:

$$g_\omega(x,x')=\begin{pmatrix} G_\omega(x,x') & F_\omega(x,x') \\ F_\omega^+(x,x') & -G_{-\omega}(x',x) \end{pmatrix}.$$

Here $G_\omega, F_\omega, F_\omega^+$ are the Fourier components of the normal and anomalous Green's functions. The thin lines denote the known matrix Green's function of the system $g_\omega^{(0)}(x,x')$ in the absence of a potential, the wavy line is used to introduce the potential $U(x)$ [more accurately $U(x)\tau_3$, since it necessary to take into account the matrix character of the vertex; τ_3 is one of the Pauli matrices].

Equations (13) can be easily solved for a superconducting system containing a thin dielectric interlayer, in other words, $U(x)=V\delta(x-x_0)$. In this case Eqs. (13) reduce to algebraic ones, and as a result we have

$$g_\omega(x,x')=g_\omega^{(0)}(x,x')+g_\omega^{(0)}(x,x_0)V\tau_3[1-g_\omega^{(0)}(x_0,x_0)V\tau_3]^{-1}g_\omega^{(0)}(x_0,x'). \tag{14}$$

After simple calculations we obtain the following expressions for the Green's functions:

$$\begin{aligned}G_\omega(x,x')&=G_\omega^{(0)}(x,x')\\&+\frac{V^2}{1+\Pi}\Big\{G_\omega^{(0)}(x,x_0)\Big[G_\omega^{(0)}(x_0,x')\frac{1}{V}-G_{-\omega}^{(0)}(x_0,x_0)G_\omega^{(0)}(x_0,x')\\&-F_\omega^{(0)}(x_0,x_0)F_\omega^{+(0)}(x_0,x')\Big]-F_\omega^{(0)}(x,x_0)\Big[F_\omega^{+(0)}(x_0,x')\frac{1}{V}\\&-G_\omega^{(0)}(x_0,x_0)F_\omega^{+(0)}(x_0,x')+F_\omega^{+(0)}(x_0,x_0)G_\omega^{(0)}(x_0,x')\Big]\Big\},\end{aligned} \tag{15}$$

$$\begin{aligned}F_\omega^+(x,x')&=F_\omega^{+(0)}(x,x')+\frac{V^2}{1+\Pi}\Big\{F_\omega^{+(0)}(x,x_0)\Big[G_\omega^{(0)}(x_0,x')\frac{1}{V}\\&-G_{-\omega}^{(0)}(x_0,x_0)G_\omega^{(0)}(x_0,x')-F_\omega^{(0)}(x_0,x_0)F_\omega^{+(0)}(x_0,x')\Big]\\&+G_{-\omega}^{(0)}(x_0,x)\Big[F_\omega^{+(0)}(x_0,x')\frac{1}{V}-G_\omega^{(0)}(x_0,x_0)F_\omega^{+(0)}(x_0,x')\\&+F_\omega^{+(0)}(x_0,x_0)G_\omega^{(0)}(x_0,x')\Big]\Big\};\end{aligned} \tag{16}$$

$$\Pi=-V^2\,\mathrm{dét}\,g_\omega^{(0)}(x_0,x_0)-V[G_\omega^{(0)}(x_0,x_0)+G_{-\omega}^{(0)}(x_0,x_0)].$$

We note that in the derivations of (14)–(16) we made no assumptions whatever concerning the value of V. In what follows we shall find it convenient to have an expression for the function $G_\omega(x,x')$, but only in the limit of large V (small transparency of the barrier). We expand the expression (14) for the Green's function in powers of the parameter $1/V$. It is then easy to obtain the function $G_\omega(x,x')$ in any order in this parameter, but we restrict the expansion to terms of first order. We thus have

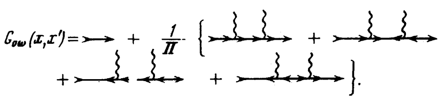
$$G_\omega(x,x')=G_{0\omega}(x,x')+G_{1\omega}(x,x'),$$

where

$$\begin{aligned}G_{0\omega}(x,x')&=G_\omega^{(0)}(x,x')\\&-\{G_\omega^{(0)}(x,x_0)[G_{-\omega}^{(0)}(x_0,x_0)G_\omega^{(0)}(x_0,x')+F_\omega^{(0)}(x_0,x_0)F_\omega^{+(0)}(x_0,x')]\\&+F_\omega^{(0)}(x,x_0)[G_\omega^{(0)}(x_0,x_0)F_\omega^{+(0)}(x_0,x')-F_\omega^{+(0)}(x_0,x_0)G_\omega^{(0)}(x_0,x')]\}\\&\times\{G_\omega^{(0)}(x_0,x_0)G_{-\omega}^{(0)}(x_0,x_0)+F_\omega^{(0)}(x_0,x_0)F_\omega^{+(0)}(x_0,x_0)\}^{-1},\end{aligned} \tag{17}$$

$$G_{1\omega}(x,x')=\frac{1}{V}\,\frac{G_\omega^{(0)}(x,x_0)G_\omega^{(0)}(x_0,x')-F_\omega^{(0)}(x,x_0)F_\omega^{+(0)}(x_0,x')}{G_\omega^{(0)}(x_0,x_0)G_{-\omega}^{(0)}(x_0,x_0)+F_\omega^{(0)}(x_0,x_0)F_\omega^{+(0)}(x_0,x_0)}. \tag{18}$$

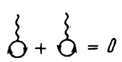
For the sake of clarity we represent the results of the expansion in diagram form:



(17′)

(18′)

We have taken into account here the fact that the inequality



holds for our problem. The fact that the exact Green's functions of the system at arbitrary values of the transparency of the insulating barrier can be determined by solving algebraic (and not differential) equations is a substantial advantage of the method described above. To calculate the current, it remains for us to determine the Green's functions of the SNS junctions. To this end, of course, we must already resort to a solution of differential equations. Such a procedure was used in many papers and is well known. In particular, Kulik[2,6] used a method of expanding the Green's functions in terms of the spectrum of the single-particle problem. This method is lucid, but can be used with sufficient simplicity only for temperatures much lower than critical ($T\ll T_c$). Another method[4] of calculating the Green's functions of an SNS system (the so-called t-representation) is based on a quasiclassical lowering of the order of the Gor'kov equations and "gluing together" the Green's functions from two suitably chosen linearly independent solutions of the homogeneous system. This method [in model (2)] is exact and is used in our calculations. For the sake of brevity we do not present here the calculations, which are quite cumbersome. We note that this procedure of constructing the Green's functions of an SNS junction is described in detail in Refs. 4 and 10. We present below only the final result which we need for the calculation of the tunnel current.

We use the well known equation for the current in an inhomogeneous superconducting system[15]

$$\begin{aligned}j=\frac{em^2}{2\pi i}T\sum_\omega\int_0^{v_F}v_x\,dv_x\iint_{-\infty}^{\infty}dx\,dx'\,(\operatorname{sign}x-\operatorname{sign}x')\\\times\Delta(x)\Delta^*(x')G_{1\omega}{}^n(x,x')G_{1-\omega}(x',x),\end{aligned} \tag{19}$$

which enables us to calculate the current in first order in the transparency of the insulating barrier. The summation is carried out, as usual, over the frequencies $\omega=\pi T(2n+1)$. The function $G_{1\omega}(x,x')$ is determined from (18), while $G_{1\omega}{}^n(x,x')$ is the Green's function of the normal metal in the presence of an insulating interlayer in first order in $1/V$. The expression for it is well known (see, e.g., Ref. 15):

$$G_{1\omega}{}^n(x,x')=-\frac{1}{V}\exp\Big\{\Big(ip_x\operatorname{sign}\omega-\frac{|\omega|}{v}\Big)(|x|+|x'|)\Big\}. \tag{20}$$

Equation (20) can also be easily obtained from (18) if we assume the anomalous mean values to be equal to zero, and replace $G_\omega^{(0)}(x,x')$ by the Green's function of the homogeneous normal metal.

We note also that the second term in the numerator of (18) makes no contribution to the current (this can be easily verified directly). We can therefore replace $G_{1\omega}(x,x')$ by the function $D_\omega(x,x')$, which contains, with allowance for the foregoing, only terms that contribute to the current. We present the final expression for this function, obtained from (18) after suitable computations ($\Omega^2=\Delta^2+\omega^2$)

$$\begin{aligned}D_\omega(x,x')=-\frac{1}{2V}\exp\Big(\frac{\Omega d}{v_x}\Big)\Big\{(\Omega-\omega)^2\exp\Big[\Big(-ip_x-\frac{\Omega}{v_x}\Big)\\\times(|x|+|x'|)-\frac{\omega d}{v_x}\Big]+(\Omega+\omega)^2\exp\Big[\Big(ip_x-\frac{\Omega}{v_x}\Big)(|x|+|x'|)\\+\frac{\omega d}{v_x}\Big]\Big\}\Big[(\Omega^2+\omega^2)\operatorname{ch}\frac{2\omega d}{v_x}+\Delta^2\operatorname{ch}\frac{4\omega x_0}{v_x}+2\omega\Omega\operatorname{sh}\frac{2\omega d}{v_x}\Big]^{-1}.\end{aligned} \tag{21}$$

The poles of the function (21) [or (18)] determine the spectrum of an SNINS junction at zero transparency of the insulating barrier.

Using (19)–(21), we arrive at a final expression for the density of a stationary Josephson current in the SNINS system [$\Delta=\Delta(T)$ throughout]:

$$j=\frac{8\pi\Delta^2}{eR}T\sum_{\omega}\int_0^1 d\alpha\alpha^3\sin\varphi\left[(\Omega^2+\omega^2)\,\mathrm{ch}\frac{2\omega d}{v_F\alpha}+\Delta^2\,\mathrm{ch}\frac{4\omega x_0}{v_F\alpha}+2\omega\Omega\,\mathrm{sh}\frac{2\omega d}{v_F\alpha}\right]^{-1} \quad (22)$$

Here $R=2\pi^2V^2/e^2\mu^2$ is the resistance of the insulating barrier in the normal state,[15] and $\alpha=v_x/v_F$. Expression (22) is suitable for any position of the insulator inside the normal-metal layer. Putting $x_0=\pm d/2$ we obtained directly Eq. (3.7) of Ref. 10 for the current in an SINS junction. Generally speaking, the result (22) is valid in the model at an arbitrary temperature and at an arbitrary value of d. Thus, at $d=x_0=0$ expression (22) yields the well known Ambegaokar–Baratov formula.[7] However, the model (2) itself, can be strictly speaking used only for broad junctions $d\gg\xi_0$. Under this condition greatest interest attaches to the region of low temperatures $T\ll T_c$. In this case expression (22) takes the simpler form

$$j=\frac{8\pi}{eR}T\sum_{\omega}\int_0^1 d\alpha\alpha^3\left[\mathrm{ch}\frac{2\omega d}{v_F\alpha}+\mathrm{ch}\frac{4\omega x_0}{v_F\alpha}\right]^{-1}\sin\varphi. \quad (23)$$

At $T=0$ the summation over the discrete frequencies is replaced by integration. As a result we get for a symmetrical SNINS junction ($x_0=0$)

$$j=\frac{4}{5}\frac{v_F}{eRd}\sin\varphi. \quad (24)$$

At $x_0=\pm d/2$ we obtain the result of Ref. 10 for an SINS junction:

$$j=\frac{\pi}{5}\frac{v_F}{eRd}\sin\varphi, \quad (25)$$

which can be seen to be somewhat smaller than the current (24).

In our case it is easily concluded from (22) [or (23)] that the critical current at any temperature, as a function of x_0, has a maximum at $x_0=0$ (i.e., for a symmetrical *SNINS* junction). Physically this result is quite understandable. In fact, according to Bardeen and Johnson[5] the superconducting properties of a system containing a broad ($d\gg\xi_0$) layer of normal metal determines the gap in the spectrum of the quasiparticle excitations of such a system. Let us compare, for example, a symmetrical SNINS junction and a SINS junction having the same parameters. In the former case the excitation spectrum has at a fixed value of v_x a gap equal to (see Fig. a) $\Delta_e=\pi v_x/2d$. In the latter case this gap in the spectrum is half as large (Fig. b), so that the superconducting properties of this system should be weak, i.e., the critical current through an SINS junction should be less than the current through an SNINS junction, as is in fact confirmed by exact calculation.

At $v_F/d\ll T\ll T_c$ we obtain for an SNINS junction

$$j=\frac{8v_F}{eRd}\exp\left\{-\frac{2d}{\xi_T}\right\}\sin\varphi,\quad \xi_T=\frac{v_F}{\pi T}. \quad (26)$$

Equation (26) is valid in a rather wide range of the parameter x_0:

$$d/2-|x_0|\gg\xi_T.$$

At $|x_0|\sim d/2-\xi_T$ the tunnel current in the system begins to decrease somewhat and at $x_0=\pm d/2$ we arrive at the expression[10] for the current in an SINS junction, which turns out to be half the value (26) at the same temperatures. This, in our opinion, is a rather interesting property of SNINS systems. This effect [just as the presence of an exponential factor in (26)] is due to the well known exponential damping of the anomalous Green's function in the interior of a normal-metal layer.[9]

In the temperature region $T\sim T_c$ the parameter Δ is small and Eq. (22) yields

$$j=\frac{2v_F}{\pi^2eRd}\left(\frac{\Delta}{T_c}\right)^2\exp\left\{-\frac{2d}{\xi_T}\right\}\sin\varphi \quad (27)$$

Strictly speaking, the model (2) is not valid at high temperatures. Equation (27), nevertheless, is valid since Δ in this formula should be taken to mean (see also Ref. 4) the order parameter of the superconductor near the *NS* boundary, and not far from it (i.e., $\Delta\sim T_c-T$ and not $[T_c(T_c-T)]^{1/2}$. In fact, the change of Δ takes place over distances of the order of $\xi_0/(1-T/T_c)^{1/2}$ from this boundary (this is much larger than ξ_0 at $T\sim T_c$). Therefore at $T\sim T_c$ the quantity of interest to us has its order parameter Δ not in the interior of the superconductor, but near the boundary with the normal metal. Accordingly we can state that the critical current in the SNINS system at temperatures close to T_c is proportional to $(T_c-T)^2$. We note that a similar dependence of the current on the temperature obtains in an SNS junction (in contrast to the SIS junction, where $j\sim T_c-T$).

The result (27) does not depend on x_0, since the discrete spectrum (7) is in fact absent at $T\sim T_c$. The concrete value of the current is determined only by the width of the normal layer d. In this temperature region we have $\xi_T\sim\xi_0$, i.e., when the inequality (3) is satisfied the current (27) turns out to be exponentially small. We note also that all the results are valid if there are no impurities in the system. The presence of impurities, on the other hand, decreases the amplitude of the Josephson current. This question as applied to SNS junctions was investigated in Ref. 16. On the other hand, in the case of an ordinary SIS junction the impurities do not influence the tunnel current, this being, as is well known, a direct consequence of the Anderson theorem.

Thus, we have investigated in the present study some properties of superconducting systems that contain of normal-metal and insulator layers. We obtained the spectrum of the excitations in such systems. We have shown that in the presence of an arbitrary magnetic field in an INS system the spectrum of the excitations (in contrast to SNS systems) is stable; the energy levels shift by an amount proportional to the magnetic field. An analogous conclusion holds also for SNINS systems if the insulating barrier has low transparency.

In fact, although the spectrum of such junctions depends on the difference of the phases of the order parameter φ, nonetheless at low transparency ($V\to\infty$) this dependence is weak, so that the level smearing in this case is also small (including also in strong magnetic fields). It can consequently be assumed that in SNINS systems the magnetic field does not upset the discrete character of the excitation spectrum even at rather appreciable dimensions of the system (in the y direction), whereas in SNS junctions in a strong magnetic field it is utterly meaningless to speak of discrete excitation-energy levels, and the spectrum in this case is continuous. These conclusions can be reconciled completely with experiment. We have constructed also a microscopic theory of the stationary Josephson current in SNINS junctions. The magnitude and quite unique temperature dependence of this current, which are governed by the presence of a phase-coherent spectrum of discrete states in the system, is strongly influenced also by the position of the insulating barrier inside the normal-metal layer.

The noted singularities of the behavior of SNINS systems can in all probability be used also for further experimental study of the proximity effect.

[1]A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1974); **49**, 655 (1965) [Sov. Phys. JETP **19**, 1228 (1974); **22**, 455 (1966)].
[2]I. O. Kulik, Zh. Eksp. Teor. Fiz. **57**, 1745 (1969) [Sov. Phys. JETP **30**, 944 (1969)].
[3]C. Ishii, Prog. Theor. Phys. **44**, 1525 (1970).
[4]A. V. Svidzinskiĭ, T. N. Antsygina, and E. N. Bratus', Zh. Eksp. Teor. Fiz. **61**, 1612 (1971) [Sov. Phys. JETP **34**, 860 (1971)].
[5]J. Bardeen and J. L. Johnson, Phys. Rev. **5B**, 72 (1972).
[6]I. O. Kulik, Weak Superconductivity, Preprint, Inst. Metal Phys. USSR Acad. Sci., Sverdlovsk, 1973.
[7]V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963); **11**, 104 (1963).
[8]N. N. Bogolyubov, Usp. Fiz. Nauk **67**, 549 (1959) [Sov. Phys. Usp. **1**, 236 (1959)].
[9]P. de Gennes, Superconductivity of Metals and Alloys, Benjamin, 1966.
[10]A. I. Bezuglyĭ, I. O. Kulik, and Yu. N. Mitsai, Fiz. Nizk. Temp. **1**, 57 (1975) [Sov. J. Low Temp. Phys. **1**, 27 (1975)].
[11]V. P. Galaĭko, Zh. Eksp. Teor. Fiz. **57**, 941 (1969) [Sov. Phys. JETP **30**, 514 (1970)].
[12]V. P. Galaĭko and E. V. Bezuglyĭ, Zh. Eksp. Teor. Fiz. **60**, 1471 (1971) [Sov. Phys. JETP **33**, 796 (1971)].
[13]G. A. Gogadze and I. O. Kulik, Zh. Eksp. Teor. Fiz. **60**, 1819 (1971) [Sov. Phys. JETP **33**, 984 (1971)].
[14]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoi teorii polya v statisticheskoĭ fizike (Quantum Field-Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965].
[15]I. O. Kulik and I. K. Yanson, Éffekt Dzhozefsona v sverkhprovodyashchikh tunnel'nykh strukturakh (Josephson Effect in Superconducting Tunnel Structures), Nauka, 1970.
[16]I. O. Kulik and Yu. N. Mitsai, Fiz. Nizk. Temp. **1**, 906 (1975) [Sov. J. Low Temp. Phys. **1**, 434 (1975)].

Translated by J. G. Adashko

Anisotropy of the magneto-optical properties of cobalt single crystals

E. A. Gan'shina, G. S. Krinchik, L. S. Mironova, and A. S. Tablin

Moscow State University

(Submitted 18 July 1979)
Zh. Eksp. Teor. Fiz. **78**, 733–740 (February 1980)

The magneto-optical spectra of cobalt single crystals were investigated in the energy region 0.2–3.35 eV. Anisotropy of the frequency spectra of the equatorial Kerr effect was observed in the case of magnetization along different crystallographic directions. The off-diagonal components of the dielectric tensor are calculated. The observed anomalies are identified with definite interband transitions when account is taken of the selection rules for a hexagonal crystal. It is shown that the magneto-optical spectra for cobalt agree best with the band structure proposed for ferromagnetic cobalt by Batallan and co-workers.

PACS numbers: 78.20.Ls, 71.25.Pi

INTRODUCTION

Many physical properties of ferromagnetic $3d$ metals and alloys have by now been explained within the framework of the magnetism theory of Slater, Stoner, and Wohlfarth by invoking the concepts of single-electron band theory. Cobalt, just as nickel, is a typical band ferromagnet. Calculations of the electronic structure of cobalt were reported in a number of theoretical papers.[1-8]

Connoly[1] was the first to publish data on the band structure of cobalt in individual high-symmetry points of the Brillouin zone. The calculations were made by the augmented plane-wave method with optimized spin-dependent potential. The exchange splitting in his model was assumed to be 2 eV. Wakoh and Yamashita[2] later calculated the energy bands and the Fermi surface, using the Korringa–Kohn–Rostoker (KKR) method with constant exchange splitting ($\Delta E_{dd}=1.73$ eV). Ishida[3] calculated the band structure of hexagonal cobalt by using amodified Muller interpolation scheme. None of the presented structures provided a satisfactory description of the experimental data obtained later on the