Self-action of electromagnetic waves in a plasma under thermal modulation instability of the upper-hybrid oscillations

A. G. Litvak and G. A. Mironov

Applied Physics Institute, USSR Academy of Sciences
(Submitted 29 May 1979)
Zh. Eksp. Teor. Fiz. 78, 561–572 (February 1980)

Results are presented of an investigation of the nonlinear stage of the weak-field temperature instability of plasma waves excited in the direction of the pump field and "strictly" transverse to the constant magnetic field. It is shown that in the nonlinear regime the initially homogeneous plasma becomes stratified with a characteristic scale much less than the length of the electromagnetic wave. Averaging over the small-scale oscillations yields an equation for the average field in a medium with an effective (nonlinear) dielectric constant; the equation describes the self-action of the pump wave. Stationary solutions of this equation are considered. It is shown that a consequence of the stratification is self-focusing of the wave beam in a transparent medium and nonlinear penetration of the wave into the dense plasma.

PACS numbers: 52.35.Mw, 52.35.Py, 52.35.Hr

1. INTRODUCTION

In the dynamics of the interaction of electromagnetic waves with a dense plasma, an important role is played by processes connected with the development of modulation instability of the plasma oscillations. The theory usually deals with collisionless effects due to striction nonlinearity. It is known at the same time that in an isotropic plasma with infrequent collisions ($\nu \ll \omega$) the mechanism of excitation of the parametric instabilities of the oscillations with a characteristic scale larger than the mean free path is determined by the ohmic heating of the electron.[1-4] The presence of an even weak constant magnetic field in the plasma influences substantially the motion of the particles and leads, generally speaking, to a decrease of the transport coefficient. This results expands the plasma-parameter region in which the thermal nonlinear effects prevail over the striction effects. Thus, obviously, in a magnetized plasma ($\omega_H \gg \nu$, $\omega_H = eH/mc$ is the gyrofrequency of the electrons) the role of the "mean free path" in the calculation of the coefficients of transport across the magnetic field is played by the Larmor radius, and therefore the thermal effects are decisive for instabilities with characteristic scale $L_E \gg \rho_H$. This shows that the thermal parametric instabilities can play an important role in the aggregate of the phenomena that take place in a magnetoactive plasma. A theoretical investigation of these phenomena is of interest also for specific applications to the propagation of high-power radio waves in an ionosphere plasma[5] and to experiments on the interaction of laser radiation with thermonuclear targets, wherein strong quasistationary magnetic fields are generated in the plasma (see, e.g., Ref. 6).

We report below the results of an investigation of the nonlinear stage of small-scale thermal instability of plasma waves excited in the pump-field direction and strictly transverse to the magnetic field. To explain the mechanism of this instability of the oscillations with characteristic scale much less than the electromagnetic wavelength (pump), we examine the behavior of the plasma in the case of a local dependence of the electron-gas temperature on the intensity of the quasistatic field $E = D/\varepsilon_\perp$ (D is the induction; $\varepsilon_\perp = 1 - \omega_p^2/(\omega^2 - \omega_H^2)$, ω_p is the electron plasma frequency). It is seen that at $\varepsilon_\perp < 0$ the minima of the density perturbations coincide with the regions of the maximum field, and consequently the regions of higher gas-kinetic pressure. In other words compression of the plasma in a certain region is accompanied by a decrease of the pressure in this place (as if the medium were to lose elasticity), and this leads to further compression at increased rates. Since the inequality $\varepsilon_\perp < 0$ is satisfied only at $\omega > \omega_H$, to understand the physical picture it suffices to consider the simplest case of a weak magnetic field ($\omega \gg \omega_H \gg \nu$), in which the high-frequency dispersion characteristics of the plasma differ insignificantly from the characteristics of an isotropic plasma, and it is important to take into account the constant magnetic field only in the equations for the low-frequency motions.

The results of the linear theory of thermal modulation instability of upper-hybrid oscillations are given in Ref. 3. The nonlinear stage of this instability was considered in Refs. 7 and 8 in a quasilinear approximation, in which the phases of the growing oscillations were assumed to be random. This assumption is justified in an inhomogeneous plasma, in which the convective transport produces waves with different frequencies at a given point of space.

The dissipative-instability mechanism described above is similar to the mechanism of the modulation instability of a collisionless plasma. Therefore in a sufficiently homogeneous plasma, just as in the collisionless case, phased spatial harmonics of plasma oscillations can be generated and can lead to formation of essentially nonlinear soliton distributions. (This circumstance was noted in Ref. 4 and was confirmed by numerical calculations in Ref. 9.)

Obviously, parametric excitation of plasma oscillations leads to the appearance of nonlinear corrections to the imaginary and real parts of the refractive index

 0038-5646/80/020282-07$02.40

of the electromagnetic pump wave. Usually principal attention is being paid to the determination of nonlinear absorption, characterized by an effective electron-collision frequency ν_{eff}.[7,8,10] The change of the real part of the refractive index becomes particularly substantial if soliton distributions are produced. In fact, the electric characteristics of a plasma with soliton perturbations of the density can be defined as the characteristics of a mixture of two substances, a homogeneous unperturbed plasma and a soliton gas. The dielectric constant of such a mixture, as is well known,[11] does not reduce to a sum of the dielectric constants of the components. It turns out[12,13] that excitation of solitons of even moderate amplitude can lead to a substantial change in the refractive index of the electromagnetic wave and consequently to nondissipative self-action effects—self-focusing and waveguide propagation of the radiation. It is important that these changes of the refractive index settle within a characteristic time of the order of the time of development of the modulation instability, i.e., much faster than the ordinary thermal nonlinearity due to the redistribution of the plasma density in the region occupied by the electromagnetic wave. It is therefore natural to call these self-action processes "fast."

The purpose of the present paper is to investigate the self-focusing of waves and the nonlinear penetration of the field, which are connected with the development of thermal modulation stability of upper-hybrid plasma oscillations. We consider the simplest case of transverse propagation of electromagnetic waves in a homogeneous weakly magnetized plasma. In Sec. 2 we present the initial equations and formulate for their solution a method that is a modification of the averaging method. In Secs. 3 and 4 we consider the effects of self-action in a "postcritical" and a transparent plasma, respectively.

2. BASIC EQUATIONS. GROWTH RATES AND THRESHOLD FIELDS OF THE INSTABILITIES

The initial system of equations describing the thermal stratification of the plasma in the field of an electromagnetic wave consists of the equations of diffusion and heat conduction for slow perturbations of the density and temperature of the plasma, which arise under the influence of the field, and the parabolic equation for the slow complex amplitude of the wave $\mathscr{E}=E(\mathbf{r},t)e^{i\omega t}$. In the latter we take into account the spatial dispersion due to the thermal motion of the electrons. In our case of transverse propagation in a homogeneous magnetized plasma with $\omega_H \ll \omega$ we have

$$\frac{\partial n}{\partial t}=D_\perp\frac{\partial^2}{\partial x^2}(n+\Theta), \tag{1}$$

$$\frac{\partial\Theta}{\partial t}=-\delta\nu\Theta+\rho_H^2\nu\frac{\partial^2\Theta}{\partial x^2}+\frac{\delta\nu|E|^2}{E_p^2}, \tag{2}$$

$$-\frac{2i}{\omega}\frac{\partial E}{\partial t}+3r_d^2\frac{\partial^2 E}{\partial x^2}+\frac{\Delta E}{k_0^2}+\left(\varepsilon_0-n+i\frac{\nu}{\omega}\right)E=0, \tag{3}$$

where ε_0 is the dielectric constant of the unperturbed plasma, n and Θ are the relative perturbations of the concentration and of the temperature, $E_p=(3m\delta T\omega^2/e^2)^{1/2}$ is the plasma field of the thermal nonlinearity, δ is the relative fraction of the energy transferred by the electron to the heavy particle in one collision, $k_0=\omega/c$, r_d is the Debye radius, and $D_\perp$ is the coefficient of diffusion across the magnetic field, defined by the relations[14]

$$D_\perp=v_s^2\frac{\nu_{im}}{\nu_{im}^2+\Omega^2},\quad L_\parallel\ll L_\perp\left[\frac{M}{m}\frac{\nu_{im}}{\nu_{em}}\left(1+\frac{\Omega_{Hi}^2}{\nu_{im}^2}\right)\right]^{1/2}, \tag{4}$$

$$D_\perp=\frac{v_T^2}{\omega_H^2}\nu,\quad L_\parallel\gg L_\perp\left[\frac{M}{m}\frac{\nu_{im}}{\nu_{em}}\left(1+\frac{\Omega_{Hi}^2}{\nu_{im}^2}\right)\right]^{1/2}, \tag{5}$$

where $L_\parallel$ and $L_\perp$ are the characteristic scales of the inhomogeneity of the concentration along and across the magnetic field, respectively, Ω_H is the ion gyrofrequency, and ν_{im} is the ion collision frequency.

Since the characteristic spatial scale λ_{sp} is small compared with the electromagnetic wavelength, it is possible to use in the investigation of the Langmuir-oscillations dynamics, the averaging method proposed in Ref. 12 and to solve the problem in two successive stages. We consider first the excitation of the plasma oscillations in a field of given induction $D=-k_0^{-2}\Delta E$ (actually in the given magnetic field of the electromagnetic wave):

$$-2\frac{i}{\omega}\frac{\partial E}{\partial t}+3r_d^2\frac{\partial^2 F}{\partial x^2}+\left(\varepsilon_0-n+i\frac{\nu}{\omega}\right)E=D, \tag{6}$$

next, averaging (6) over the small-scale oscillations, we obtain an equation for the average (vortical) field $\bar{E}$ that describes the interaction of the pump wave in a medium with a certain effective dielectric constant ε_{eff}:

$$-\frac{2i\omega}{c^2}\frac{\partial\bar{E}}{\partial t}+\Delta\bar{E}+k_0^2\varepsilon_{\rm eff}\bar{E}=0. \tag{7}$$

We consider now the linear stage of the modulation instability (see also Ref. 3). Linearizing (1), (2), and (6) with respect to standing plasma oscillations with wave vector k, we easily obtain the following dispersion equation

$$[i\Omega+(\delta+k^2\rho_H^2)\nu][i\Omega+k^2D_\perp]=-\frac{k^2(\varepsilon_0-3k^2r_d^2)D_\perp}{(\varepsilon_0-3k^2r_d^2)^2+\nu^2/\omega^2}\delta\nu\frac{E_0^2}{E_p^2}. \tag{8}$$

From this it is quite simple to determine, in various limiting cases, the increments, threshold fields, and the optimal instability scales. Thus, in a transparent medium ($\varepsilon_0\gg\nu/\omega$) for a maximum instability increment of the plasma waves in the pump-wave field that does not exceed significantly the threshold value ($\gamma\ll\delta\nu$), we get

$$\Gamma=k_{\rm opt}^2D_\perp\{E_0^2/E_{\rm thr}^2-1\}, \tag{9}$$

where

$$k_{\rm opt}^2\approx\varepsilon_0/3r_d^2,\quad E_{\rm thr}^2/8\pi N_0T_0=2\nu(\delta+k_{\rm opt}^2\rho_H^2)/\omega.$$

When the field intensity exceeds the threshold greatly ($\gamma\gg k^2\rho_H^2\nu$) we get the expression

$$\Gamma=(k_{\rm opt}^2D_\perp\delta\omega E_0^2/2E_p^2)^{1/2}. \tag{10}$$

In the immediate vicinity of the plasma-resonance region ($|\varepsilon_0|\ll\nu/\omega$) we obtain from (8) from the maximum instability growth rate ($\Gamma\ll\delta\nu$) and for the corresponding wave number:

1) in the case of weak collisions, $\nu/\omega\ll 3\delta\omega_H/\omega$,

$$\Gamma=k_{opt}^2 D_\perp\{(E_0/E_{thr})^{4/3}-1\},$$
$$3^{1/2}k_{opt}r_d=(2^{1/2}\nu/\omega)^{1/2}(E_0/E_{thr})^{1/3},\quad E_{thr}=E_p(2^{1/2}\nu/\omega)^{1/2}; \tag{11}$$

2) in the case of strong collisions

$$\Gamma={}^2/_3 k_{opt}^2 D_\perp\{(E_0/E_{thr})^2-1\},$$
$$3^{1/2}k_{opt}r_d=\left(\frac{\nu}{9\omega}\right)^{1/2}\left(\frac{E_0^2/E_{thr}^2-1}{E_0^2/E_{thr}^2}\right)^{1/4},\quad E_{thr}=E_p\nu\frac{3^{1/2}\delta}{\omega_H}. \tag{12}$$

In a transcritical plasma ($\varepsilon_0<0$, $|\varepsilon_0|\gg\nu/\omega$) we obtain in the same approximation ($\Gamma\ll\delta\nu$)

$$\Gamma=k_{opt}^2 D_\perp(E_0/E_{thr}-1), \tag{13}$$

where

$$k_{opt}=\left(\frac{E_0}{E_{thr}}-1\right)^{1/2}\frac{|\varepsilon_0|^{1/2}}{3^{1/2}r_d}\quad\text{at}\quad|\varepsilon_0|^{1/2}\ll\frac{3^{1/2}r_d\delta^{1/2}}{\rho_H}, \tag{14}$$

$$k_{opt}=\frac{\delta^{1/2}}{\rho_H}\left(\frac{E_0}{E_{thr}}-1\right)^{1/2}\quad\text{at}\quad|\varepsilon_0|^{1/2}\gg\frac{3^{1/2}r_d\delta^{1/2}}{\rho_H},\quad E_{thr}=E_p|\varepsilon_0|^{1/2}. \tag{15}$$

The character of the nonlinear stage of the thermal modulation instability and its physical consequences differ principally in the cases of a transparent and a transcritical plasma. We shall therefore consider these cases separately.

3. NONLINEAR STAGE OF MODULATION INSTABILITY IN A TRANSCRITICAL PLASMA

1. Just as in the collisionless case,[1,2] the development of modulation instability leads to a stratification of the plasma and to a homogeneous heating of its electronic component, a heating determined by the electric field averaged over the stratified structure. In a transcritical plasma at a sufficiently small excess of the field above the threshold value, when the spatial dispersion of the plasma waves can be neglected ($k^2r_d^2\ll\nu/\omega$), the nonlinear stage proceeds qualitatively in the same manner as in an isotropic plasma.[4] We consider this very simple case and formulate the assumptions used to obtain the expression for the effective dielectric constant.

The system (1), (2), and (6) has in the case of a spatially bounded wave beam several characteristic times. The longest of them is the time needed to crowd out the plasma out of the entire region heated by the field. As a result of this process, a pressure balance is established in the entire region occupied by the field. We are interested in the quasistationary process and in solutions that are realized over much shorter times and take into account the redistribution of the plasma over a scale on the order of the wavelength of the plasma oscillations. In addition, we assume that ν is constant. As a result we arrive at the equation

$$\frac{d^2n}{d\zeta^2}-n-\left\{\left[(\varepsilon_0-n)^2+\frac{\nu^2}{\omega^2}\right]^{-1}-\overline{\left[(\varepsilon_0-n)^2+\frac{\nu^2}{\omega^2}\right]^{-1}}\right\}\frac{D^2}{E_p^2}=0, \tag{16}$$

where $\zeta=(3\delta)^{1/2}x/\rho_H$, and the superior bar denotes averaging over x. The remaining quantities are defined as follows:

$$\Theta=\bar\Theta-n,\quad E=\frac{D}{(\varepsilon_0-n)+i\nu/\omega},\quad \bar\Theta=D^2\overline{\left[(\varepsilon_0-n)^2+\frac{\nu^2}{\omega^2}\right]^{-1}}. \tag{17}$$

The character of the solution of (16), whose integral is of the form

$$\dot n^2-n^2-2\frac{D^2}{E_p^2}\frac{\omega}{\nu}\left[\operatorname{arctg}\left(\frac{\omega}{\nu}(n-\varepsilon_0)\right)+\operatorname{arctg}\left(\frac{\omega}{\nu}\varepsilon_0\right)\right]+2\bar\Theta n=\text{const}, \tag{18}$$

is easiest to analyze qualitatively by using the phase plane. This plane is shown in the figure for a field exceeding the threshold value.[1)] It is natural to expect that the development of the instability results in a distribution having a period $L=2\pi/k_{opt}$ corresponding to the maximum instability growth rate (13). Thus, in the stationary state the initially homogeneous plasma with $\varepsilon_0<0$ becomes a layered inhomogeneous medium, in which the layers with $\varepsilon\lesssim-\nu/\omega$, having a characteristic dimension $\sim\rho_H/\sqrt{\delta}$ alternate with the layers having $\varepsilon\approx\varepsilon_0$, with a period $L=2\pi/k_{opt}$. In the case of a slight excess of the field above the threshold, the solution of the equation takes the form of a periodic set of solitons in the form

$$n=-|\varepsilon_0|\exp\{-(3\delta)^{1/2}|x|/\rho_H\}, \tag{19}$$

spaced $L=2\pi/k_{opt}$ apart, where k_{opt} is given by expressions (14) and (15). We note that in contrast to the case of modulation instability in a collisionless plasma[12] the "thermal" stratification of the plasma does not produce an alternating-sign structure of the dielectric constant [$\varepsilon(x)<0$ in the entire stratification region].

Using this representation of the nonlinear stage of the modulation instability, we easily obtain an expression for ε_{eff}. This is easiest to do either by assuming that the optimal scale everywhere in the instability region is determined by the maximum value of the field at the initial instant of time $E_{max}(t=0)$, or by regarding the optimal scale as locally connected with the average field $\bar E$ in the given section. In either case, ε_{eff} takes the form

$$\varepsilon_{eff}=\varepsilon_0+i\frac{\nu}{\omega}-\frac{\varepsilon_0}{\pi}\frac{k_{opt}\rho_H}{(3\delta)^{1/2}}\left[\ln\frac{\omega|\varepsilon_0|}{\nu}+i\frac{\pi}{2}\right]. \tag{20}$$

Here k_{opt} is determined by expressions (14) and (15), in which E_0 must be taken to mean either E_{max} at $t=0$, or else $\bar E$.

From (20) it follows first that the stratification makes the plasma more transparent, $|\mathrm{Re}\,\varepsilon_{eff}|<\mathrm{Re}\,\varepsilon_0$, but in contrast to the striction stratification, the thermal stratification does not lead to a nonlinear induced transparency of a transcritical plasma, since the dielectric constant is still of alternating sign [see (19)]. However, the depth of penetration of the field into the plasma can change substantially.

Another important consequence of the onset of rela-

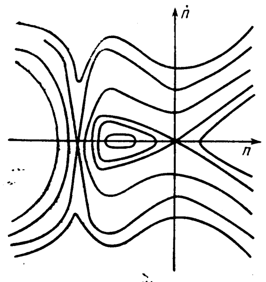

FIG. 1.

tively extended regions with $\varepsilon \lesssim 0$ is additional nonlinear absorption due to ohmic losses in layers with a dielectric constant close to $\varepsilon = 0$. It is seen that a small excess of the field above threshold suffices to make the nonlinear absorption predominant. The result of the plasma stratification is also an appreciable decrease of the threshold field of the striction modulation instability in regions with $\varepsilon \lesssim 0$, and this in turn can lead to slower (collisionless) stratification with all its consequences.[12,13]

2. With increasing intensity of the pump-wave field, regions can be produced in which the dielectric constant is positive. When account is taken of the spatial dispersion in these regions, the stationary distribution of the plasma oscillations is described by the system of equations

$$\frac{\rho_H^2}{\delta}\frac{d^2n}{dx^2} - n = |\mathscr{E}|^2 - \overline{|\mathscr{E}|^2}, \tag{21}$$

$$3r_d^2\frac{d^2\mathscr{E}}{dx^2} + \left(\varepsilon_0 - n + i\frac{\nu}{\omega}\right)\mathscr{E} = D, \tag{22}$$

where $\mathscr{E} = (E^2\delta/8\pi N_0 T_0)^{1/2}$ and the bar denotes averaging over the optimal instability scale.

We consider two limiting cases, in which the effects are qualitatively different. If the plasma wavelength $(\lambda \sim r_d/\sqrt{\varepsilon})$ exceeds greatly the characteristic thermal-conductivity length $\rho_H/\sqrt{\delta}$, the concentration is a local function of the field and we arrive consequently at the equation

$$3r_d^2\frac{d^2\mathscr{E}}{dx^2} + \left(\varepsilon_0 + |\mathscr{E}|^2 - \overline{|\mathscr{E}|^2} + i\frac{\nu}{\omega}\right)\mathscr{E} = D. \tag{23}$$

In the absence of losses this equation was discussed in detail in Ref. 12 for the purpose of obtaining estimates for the parameters of the nonlinear induced transparency of a collisionless plasma (of the threshold field intensity and of the induced-transparency time). Using the results of Ref. 12, we obtain the following expression for the effective dielectric constant:

$$\varepsilon_{\text{eff}}^{-1} = \frac{3^{1/2}}{2}\left\{-\frac{1}{3^{1/2}} + \frac{6^{1/2}k_{\text{opt}}r_d}{|\varepsilon_0|^{1/2}}\right\}, \tag{24}$$

where k_{opt} is determined by Eq. (14). We see therefore that the threshold induced-transparency field practically coincides with the threshold modulation-instability field and amounts to $10/9E_0|\varepsilon_0|^{1/2}$.

To attain analytic expressions in the other limiting case of nonlocal connection between n and ε $(|\varepsilon_0| \gg \delta\omega_p^2/\omega_H^2)$ we use the fact that the distribution of the concentration $n(x)$ is a smooth function of x (over the scale of the plasma wavelength $r_d/\sqrt{\varepsilon}$). The solution of the linear inhomogeneous differential equation (22) can then be written in explicit form, using asymptotic methods (see, e.g., Ref. 16). The integration constants are determined under the assumption that the characteristic stratification dimension $\rho_H/\sqrt{\delta}$ is smaller than the damping length of the plasma wave $\sim v_T\sqrt{\varepsilon}/\nu = l\sqrt{\varepsilon}$ and consequently the regions with positive dielectric constant constitute high-Q resonators $(\omega_H\sqrt{\delta} > \nu)$.[2)]

Let the dielectric constant $\varepsilon(x)$ be positive at $|x| < x_0$ and negative at $x_0 < x < L$. Using the geometrical-optics approximation, we write down the solution of (22) in these regions in the following manner:

$$\mathscr{E} = \frac{B}{(\varepsilon_0 - n)^{1/4}}\cos\frac{1}{3^{1/2}r_d}\int_x^{x_0}\left(\varepsilon_0 - n + i\frac{\nu}{\omega}\right)^{1/2}dx + \frac{D}{\varepsilon_0 - n}, \quad |x| < x_0;$$

$$\mathscr{E} = D/(\varepsilon_0 - n), \quad x_0 < x < L/2. \tag{25}$$

It is easy to estimate the amplitude B of the plasma wave excited in the vicinity of the plasma resonance $(x \approx \pm x_0)$, by assuming that the distribution of the plasma concentration in the transition region is linear with a characteristic dimension l $[\varepsilon \approx (x - x_0)/l]$. Applying Airy functions to the asymptotic integration (22) and recognizing that an integer number of half-wave is spanned by the distance between the turning points, we get

$$B = iD\frac{L_{\text{Airy}}\,\omega}{\varepsilon_m^{1/2}x_0\nu}\left(\frac{l}{r_d}\right)^{1/6}, \tag{26}$$

where $L_{\text{Airy}} = (3r_d^2 l)^{1/3}$, ε_m is the maximum positive value of the dielectric constant reached at $x = 0$.

To calculate the parameters of the self-consist stratified structure we substitute (25) and (26) in (21) and average over the characteristic scale of the plasma waves. We then get

$$\frac{\rho_H^2}{\delta}\frac{\partial^2 n}{\partial x^2} = \begin{cases} -(L - 2x_0)|B|^2/2L, & |x| < x_0 \\ x_0|B|^2/L, & x_0 < x < L/2 \end{cases}. \tag{27}$$

It is seen that the continuous distribution of the perturbed concentration of the plasma under strongly nonlocal nonlinearity is described by segments of parabolas. We shall not present here the algebraic system of equations for the stratification parameter and its solution, but write out immediately the expression that we shall need later for the amplitude of the plasma wave:

$$|B| = \frac{|\varepsilon_0|^{5/4}}{3^{1/2}\delta^{3/4}r_d^{1/2}}\left(\frac{36\cdot 3^{1/2}\nu}{D\omega}\right)^{1/2}\left(\frac{\rho_H}{L}\right)^{3/2}\left(\frac{L}{3^{1/2}r_d}\right)^{1/4}. \tag{28}$$

Finally, averaging (22) over the characteristic stratification scale (13), using the geometrical-optics solution (22), we get an equation for the determination of ε_{eff}:

$$\left(\frac{3}{4}\varepsilon_0 + i\frac{\nu}{\omega}\right)\overline{\mathscr{E}} + \frac{3^{3/2}}{4}i|B|r_d^{1/2}\left(\frac{r_d^2}{2|\varepsilon_0|L^2}\right)^{1/3} = D. \tag{29}$$

It follows from this that both the real and imaginary parts of ε_{eff} vary. At $\text{Im}\,\varepsilon_{\text{eff}} \ll |\varepsilon_0|$ we have $\text{Re}\,\varepsilon_{\text{eff}} = 3/4|\varepsilon_0|$ and we get for the effective collision frequency

$$\nu_{\text{eff}} = \nu\left[1 + \frac{\sqrt{3}}{8}\frac{\delta^{1/4}}{|\varepsilon_0|^{1/4}D^{1/2}}\left(\frac{\omega_H}{\omega}\right)^{5/12}\left(\frac{\omega}{\nu}\right)^{1/2}\right]. \tag{30}$$

It is seen that ν_{eff} decreases with increasing pump-wave amplitude $(\overline{\mathscr{E}} = 4D/3\varepsilon_0)$. This is a direct consequence of the increase of the inhomogeneity gradient with increasing field, and leads to a lowering of the effectiveness of the linear transformation.

4. SELF-ACTION OF ELECTROMAGNETIC WAVES IN A TRANSPARENT PLASMA

To describe the nonlinear stage of thermal modulation instability in a transparent plasma $(\varepsilon_0 > 0)$ we make use of the fact that the instability growth rate has a sharp maximum for plasma waves with wave vector $k \approx \varepsilon_0^{1/2}/3^{1/2}r_d$. Therefore, at least in fields that do not exceed greatly the threshold value, it is natural to assume that in the nonlinear regime there is no enrichment of the spectral or the spatial harmonics,[3] and consequently the distribution of the field and of the plasma parameters takes the form of a standing wave with $k=k_{opt}$. By way of example we consider a case in which the distribution of the temperature can be regarded as quasistationary and an instability with a growth rate (9) is realized in the linear stage. However, the results of a qualitative construction of the stationary picture are valid also in the other limiting case (10). Just as above, we neglect the global distribution of the plasma over the characteristic dimension of the beam. Putting in (1), (2), and (3)

$$E=E_{\text{thr}}(\bar{e}+e_\sim\cos kx),\qquad n=n_\sim\cos kx,\qquad \Theta=\bar{\Theta}+\Theta_\sim\cos kx$$

and averaging over the small-scale oscillations, we easily obtain an equation for the self-action of the average (in the scale $2\pi/k$) field

$$\partial n_\sim/\partial t=-k^2D_\perp(1-|\bar{e}|^2)n_\sim, \tag{31}$$

$$\frac{2i\omega}{c^2}\frac{\partial\bar{e}}{\partial t}+\Delta\bar{e}+k_0^2\left(\varepsilon_0+n_\sim^2\frac{\omega}{\nu}\right)\bar{e}=0. \tag{32}$$

The remaining quantities are determined by the expressions

$$\bar{\Theta}=\frac{\nu}{\omega}\varepsilon_0\left(1+\frac{n_\sim^2\omega}{4\nu}\right)|\bar{e}|^2,\quad \Theta_\sim=-n_\sim|\bar{e}|^2,\quad e_\sim=\frac{\omega}{\nu}\frac{n_\sim\bar{e}}{i-1}.$$

The only mechanism that leads to elimination of the instability within the framework of Eqs. (31) and (32) is the reaction of the growing density perturbations on the pump field. It can lead to establishment of a stationary state ($\partial/\partial t=0$), in which the intensity of the electric field in the entire instability region is equal to the threshold value ($|e|=1$). Consequently, the problem of finding the stationary state reduces to a determination of that plasma-wave field distribution, for which the average field satisfies the equation[4]

$$|\bar{e}|=1\ \text{at}\ n_\sim>0,\qquad |\bar{e}|<1\ \text{at}\ n_\sim=0. \tag{33}$$

The general analysis of the solution of this kind of inverse electrodynamic problem (as well as the answer to the questions of its existence and uniqueness) is quite difficult. We shall attempt to describe qualitatively the stationary interaction of the transverse and plasma waves using by way of a very simple example a wave beam propagating in a homogeneous plasma along the magnetic field. We assume that the characteristic transverse dimensions of the region in which the field exceeds the threshold value ($|\bar{e}|\geq 1$) are much larger than the wavelength, and that the longitudinal dimension $L_\parallel\gg l/\sqrt{\delta}$, so that the processes can be described on the basis of a parabolic equation for the complex amplitude. Obviously, the development of the instability and the associated small-scale stratification change the structure of the beams and lead to a distortion of the initial form of the instability region. The physical factors that ensure constancy of the field amplitude in this region (32) are the reflection of the wave and the refraction-induced bending of the rays. If the wavelength is much less than the characteristic dimension of the inhomogeneities ($\sim l/\sqrt{\delta}$), the reflection can be neglected. Thus, the equation that describes the stationary self-action of the average field takes the form

$$2ik_0\varepsilon_0^{1/2}\frac{\partial\bar{e}}{\partial z}+\Delta_\perp\bar{e}+k_0^2n_\sim^2\frac{\omega}{\nu}\bar{e}=0 \tag{34}$$

under the condition (33). Representing the complex amplitude of the field in the form $\bar{e}=\mathscr{E}e^{i\varphi}$, we obtain from (34) the following equations for the phase of the field in the instability region ($|\bar{e}|=\mathscr{E}=1$):

$$2k_0\varepsilon_0^{1/2}\frac{\partial\varphi}{\partial z}+(\nabla_\perp\varphi)^2-k_0^2n_\sim^2\frac{\omega}{\nu}=0, \tag{35}$$

$$\Delta\varphi=0. \tag{36}$$

In the axially symmetrical case it follows from (36) that the solution bounded on the z axis ($r=0$) has $\partial\varphi/\partial z=0$, i.e., the rays are parallel to the z axis in the instability region. Thus, the amplitude of the perturbations of the density and of the plasma oscillations $n_\sim=\text{const}$ can be determined from the condition that the system of rays entering the instability region should become transformed on the boundary of this region into a beam of rays parallel to the z axis, thus ensuring constancy of the amplitude of the electromagnetic wave $\bar{e}=1$. The calculations will be continued for the case of a paraxial Gaussian beam in which the intensity distribution is described by the expression

$$|e|^2=e_F^2\frac{\exp(-r^2/a^2)}{1+(z-z_F)^2/l_F^2}, \tag{37}$$

where e_F is the maximum intensity at least at the center of the beam ($r=0, z=z_F$); the characteristic radius of the beam is

$$a(z)=[a_F^2+(z-z_F)^2/k^2a_F^2]^{1/2},$$

while a_F and $l_F=ka_F^2$ are the characteristic longitudinal and the transverse dimensions of the focal region (it is assumed that $ka_F\gg 1$).

We represent the system, resulting from Eq. (34) of the paraxial rays in the region where the field below threshold ($n_\sim=0$) in the form

$$r^2=r_F^2(1+(z-z_F)^2/k^2a_F^4), \tag{38}$$

where r_F is the transverse coordinate of the ray at $z=z_F$. Applying Snell's law to the leading boundary of the instability region and recognizing that the system (35) is transformed in this case into a distribution parallel to the z axis, we obtain an equation for the shape of the boundary

$$\frac{dr}{dz}=k^2a_F^4\left[1+\frac{(z-z_F)^2}{k^2a_F^4}\right]\left[1-\left(\frac{\varepsilon_0+n_\sim^2\omega/\nu}{\varepsilon_0^{1/2}}\right)^{1/2}\right]\frac{1}{r(z-z_F)}. \tag{39}$$

It follows therefore that the boundary of the instability region is concave towards the beam ($dr/dz<0$ at $r>0$).

Next, recognizing that the field amplitude in the instability region is constant ($|\bar{e}|=1$), it is easy to estimate from (37) and (39), using the energy-flux conservation law (neglecting reflection) in the case of not too large an excess of the field above the threshold ($e_F\gtrsim 1$), the amplitude of the small-scale stratification:

$$n_\sim^2\frac{\omega}{\nu}\approx\varepsilon_0(e_F^4-1). \tag{40}$$

Thus, the general qualitative picture of the nonlinear interaction of a wave beam with plasma waves is the following. In the region where the initial value of the field intensity exceeds the threshold, instability develops and small-scale stratification of the plasma takes place. With further establishment of the self-consistent distribution of the field and of the plasma, the structure of the rays changes in such a way that the leading boundary of the plasma instability becomes concave in a direction opposite to the initial beam. The initial shape of the real boundary, as can be easily seen from Snell's law, decreases the divergence of the system of rays and by the same token expands the instability region. Thus, radiation that enters into the initial instability region turns out to be trapped in waveguide channel, i.e., self-focusing of the radiation takes place. In contrast to ordinary thermal self-focusing effects,[18,19] which are connected with redistribution of the plasma density over the entire dimension of the wave beam, this process is characterized by an anomalously fast settling time. To realize this process it is necessary only that the density be redistributed over the characteristic scale of the instability (which is of the order of the length of the plasma wave), and not over the entire dimension of the beam. It is important to take note of the stability of the surface of this region of the wave beam relative to perturbation of its form, which follows directly from Snell's law and from relation (33).

We consider now separately the region in which plasma stratification took place. A wave with a flat phase front propagates in this region. We investigate its stability to perturbation of the transverse structure, in analogy with the procedure customarily used in self-focusing theory (see, e.g., Ref. 19). We represent the solution of Eqs. (31) and (32) in the form

$$\bar{e}=(1+e_1)\exp[ik_0(\varepsilon_0+n_0)^{1/2}z],\qquad n_\sim=n_\sim^0+n_1,$$

where $|e_1|\ll 1, n_0=(n_\sim^0)^2\omega/\nu$, $n_\sim^0$ is the equilibrium amplitude of the small-scale stratification of the plasma, $e_1=v_1+iv_2$,

$$v_{1,2}=\mathrm{Re}\,V_{1,2}\exp[ik_0\varepsilon_0^{1/2}(kz+\varkappa\mathbf{r}_\perp)]+i\omega\varepsilon_0\Omega t,$$

and linearize. As a result we obtain the following dispersion equation:

$$[\Omega+2k(1+n_0)^{1/2}]^2+\varkappa^2(\varkappa^2+2in_0/\tau\Omega)=0,$$

where $\tau=(4k^2D_\perp/\omega\varepsilon_0)^{-1}$. It is easy to estimate from this the maximum growth rate of the instability: $\mathrm{Im}\,\Omega_{\max}\approx(2n_0/\tau)^2$, and the optimal transverse and longitudinal wave numbers:

$$k_{\text{opt}}\approx\varkappa_{\text{opt}}^2\approx(2n_0/\tau)^{1/3}.$$

Thus, the field distribution inside the waveguide channel is unstable if its characteristic dimension exceeds

$$2\pi/k_0\varepsilon_0^{1/2}[8k^2D_\perp(e_F^4-1)/\omega]^{1/4}.$$

We have confined ourselves above to stationary models of stratification of a homogeneous plasma under thermal parametric instabilities. The relations obtained show that thermal stratification of the plasma leads not only to additional absorption of electromagnetic waves, but also to a change in the character of the penetration of the field into the plasma, and in particular to anomalously fast thermal self-focusing of the electromagnetic waves. To assess the quantitative role of the considered dynamic nonlinear effects in experiments in which radio waves act on the ionosphere and in experiments on laser-driven thermonuclear fusion it is necessary to carry out additional investigations with account taken of the nonstationary character of the interaction and of the initial inhomogeneity of the unperturbed plasma.

[1)]We note that an analysis carried out without allowance for the conservation of the total number of particles[15] yields also a transition solution (from one value of the concentration to another).

[2)]The case of a low-Q resonance is analyzed in Ref. 10.

[3)]Using the final result (40), we can easily estimate that the enrichment of the spectrum can be neglected at $\varepsilon_0<16\nu/\omega$. In the general case this representation should be regarded as an approximation of the true distribution of the plasma and field parameters.

[4)]From the mathematical point of view the problem considered here is equivalent to that of a stationary gas discharge produced in the field of a beam of electromagnetic waves (see Ref. 17).

[1]V. A. Mironov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 12, 1765 (1969).

[2]A. G. Litvak and V. A. Mironov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 18, 828 (1975).

[3]S. M. Grach and V. Yu. Trakhtengerts, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 18, 1288 (1975).

[4]A. G. Litvak and V. A. Mironov, Thirteenth Intern. Conf. on Phenomena in Ionized Gases, Berlin, GDR, 1977.

[5]Radio Science 9, No. 11 (Special issue), 1974. Izv. Vussh. Uchebn. Zaved. Radiofiz. 18, No. 9 (special issue), 1975 20, No. 11 (special issue), 1977.

[6]G. A. Askar'yan, M. S. Rabinovich, A. D. Smirnov, and V. B. Studenov, Pis'ma Zh. Eksp. Teor. Fiz. 5, 116 (1967) [JETP Lett. 5, 93 (1967)]. J. A. Stamper, K. Papadopoulos, R. N. Sudan, S. O. Dean, E. A. McLean, and J. M. Dawson, Phys. Rev. Lett. 26, 1012 (1971).

[7]N. A. Mityakov, V. O. Rapoport, and V. Yu. Trakhtengerts, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 18, 1273 (1975).

[8]S. M. Grach, A. N. Karashtin, N. A. Mityakov, V. O. Rapoport, and V. Yu. Trakhtengerts, Fiz. Plazmy 4, 1321, 1330 (1978) [Sov. J. Plasma Phys. 4, 737, 742 (1978)].

[9]T. A. Burinskaya and A. S. Volokitin, Pis'ma Zh. Eksp. Teor. Fiz. 27, 321 (1978) [JETP Lett. 27, 301 (1978)].

[10]V. V. Vas'kov and A. V. Gurevich, Zh. Eksp. Teor. Fiz. 69, 176 (1975); 73, 923 (1977) [Sov. Phys. JETP 42, 91 (1975); 46, 483 (1977)]. Fiz. Plazmy 2, 113 (1976) [Sov. J. Plasma Phys. 2, 62 (1976)].

[11]L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continous Media), Gostekhizdat, 1957, Sec. 9.

[12]A. G. Litvak, V. A. Mironov, and G. M. Fraiman, Pis'ma Zh. Eksp. Teor. Fiz. 22, 364 (1975) [JETP Lett. 22, 172 (1975)].

[13]A. G. Litvak, V. A. Mironov, and A. M. Feĭgin, Zh. Eksp. Teor. Fiz. 77, 1360 (1979) [Sov. Phys. JETP 50, 684 (1979)].

[14]A. V. Gurevich and E. E. Tsedilina, Usp. Fiz. Nauk 91, 609 (1967) [Sov. Phys. Usp. 10, 214 (1967)].

[15]A. V. Gurevich and A. V. Shvartsburg, Nelineĭnaya teoriya rasprostraneniya radiovoln v ionosfere (Nonlinear Theory of Propagation of Radiowaves in the Ionosphere), Nauka, 1973, p. 56.

[16]V. A. Fok, Problemy difraktsii i rasprostraneniya elektromagnitnykh voln (Problems of Diffraction and Propagation of

Electromagnetic Waves), Sovetskoe radio, M., 1970, p. 462.
[17]V. B. Gil'denburg and S. V. Golubev, Zh. Eksp. Teor. Fiz. **67**, 89 (1974) [Sov. Phys. JETP **40**, 46 (1974)].
[18]G. A. Askar'yan, Zh. Eksp. Teor. Fiz. **42**, 1567 (1962) [Sov. Phys. JETP **15**, 1088 (1962)].
[19]A. G. Litvak, V. A. Mironov, G. M. Fraiman, and A. D. Yunakovskii, Fiz. Plazmy **1**, 60 (1975) [Sov. J. Plasma Phys. **1**, 31 (1975)].

Translated by J. G. Adashko

Thermodynamic properties of a nonideal argon or xenon plasma

V. K. Gryaznov, M. V. Zhernokletov, V. N. Zubarev, I. L. Iosilevskiĭ, and V. E. Tortov

Division of the Institute of Chemical Physics of the USSR Academy of Sciences
(Submitted 21 June 1979)
Zh. Eksp. Teor. Fiz. **78**, 573–585 (February 1980)

Results are presented on shock-wave dynamic compression and on the investigation of the equation of state of a strongly nonideal argon and xenon plasma. The experiments were performed with explosive propelling devices, using the energy of the detonation of powerful condensed explosives. A considerable increase of the pressure and a decrease of the internal energy are established. This is a consequence of the deformation of the electronic energy levels of the strongly compressed plasma. A quantum-mechanical model of a bounded atom and a pseudopotential model of a plasma are proposed to describe this effect.

PACS numbers: 52.25.Kn

1. INTRODUCTION

The operation of many contemporary technical devices is based on concentration of appreciable energy in dense media,[1-3] which leads to the production of a high-pressure plasma with strong interparticle interaction. The theoretical description of the physical processes in a nonideal plasma entails considerable difficulties of correctly accounting for the varied and structurally complicated multiparticle interactions, and is possible at the present time only in the limit of low density by the methods of perturbation theory,[4,5] or in the case of idealized systems by computer-experiment methods.[6,7] The main source of information on the properties of strongly compressed plasma in this situation is experiment, which makes it possible to establish the limits of applicability of the asymptotic approximation and which yields the required information for the construction of thermodynamic models of a strongly nonideal plasma.

Experiments with a nonideal plasma call for high local concentrations of the energy in dense media and for the use of diagnostics methods that are not traditional in plasma physics. This is due to the patently insufficient number of thermodynamic measurements made in the region of advanced nonideality.[8-13] In particular, there are no systematic investigations of the region of extremely high plasma densities, where strong interparticle interaction manifests itself not only in the continuous spectrum, but also causes distortion of the discrete energy levels of the electrons bound in the atoms and ions.[1,7]

In this paper we present the result of investigations of the thermodynamic properties of a dense nonideal plasma of argon and xenon, obtained by compression and by irreversible heating of the gases in the front of high-power ionizing shock waves produced when powerful condensed explosives are detonated. The combination of electronic-contact and optical basic methods has made it possible to register the kinematic characteristics of the motion of the shock waves and, by using the mass, momentum, and energy conservation laws to obtain the equations of state of a dense shock-compressed plasma.

The experiments yielded shock waves with velocities up to 9.6×10^5 cm/sec. The plasma generated thereby has a high temperature $T\sim(2-6)\times10^4$ K and a pressure $P\sim1-6$ kbar, and its density approaches that of the liquid phase (see Figs. 4a and 4b below). Thus, the plasma investigated in the experiment is nonideal with respect to a broad spectrum of interparticle interactions with substantial participation of excited states of atoms and ions.

2. EXPERIMENTAL TECHNIQUE

Experiments on shock compression of inert gases were performed with generators of rectangular shock waves using explosive driving systems.[14,12,15] In these devices, the metallic flyer plates were accelerated to 5–6 km/sec by the detonator products of charges of powerful condensed explosives. The high uniformity and reproducibility of the dynamic parameters of the generators was attained by using specially shaped detonation lenses, flyer plates of special construction, as well as by using active charges with geometrical dimensions sufficient to establish stationary detonation. The collision of the moving flyer plates with the condensed targets produces in the latter shock waves with maximum pressures of the order of 1 Mbar. The emergence of the wave to the interface between the target and high-pressure inert gas is accompanied by expansion of the target material in the centered release wave

 0038-5646/80/020288-08$02.40