Collective relativistic interactions in electron beams

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(Submitted 26 February 1979)
Zh. Eksp. Teor. Fiz. **78**, 119–122 (January 1980)

An interaction law for charges moving in a medium with velocities exceeding the speed of light in the medium (a generalization of Coulomb's law) is derived from Maxwell's equations. It is shown that a charge moving in the wake of a like charge is attracted to the latter. The interaction law is used for a discussion of high-energy electron beams moving in a medium with velocities exceeding the speed of light in that medium. Self-compression of the beam is found, an effect due to the electromagnetic interaction of the relativistic electrons. A one-dimensional model is used to obtain estimates of the characterisic times and of the values of the initial electron density and energy in the beam that are necessary for the existence of the effect.

PACS numbers: 41.80.Dd

In high-power electron beams with energies of the order of 1 MeV and higher, the particle velocities are close to the speed of light and may surpass the speed of light in material media (e.g., in dielectrics). It is interesting to derive the interaction law between charges which move in a medium with velocities exceeding the speed of light ("superluminal" velocities)—an interaction law which generalizes Coulomb's law.

In the present paper, making use of the solution obtained for the field produced by an electron moving with constant velocity, we consider the interaction of superluminal charges with an external field. The result obtained here is used for a study of the collective interaction of superluminal electrons in the beam. We neglect the energy losses of the electrons in their passage through matter, since the losses of each electron are small and of the same order, and do not influence their mutual positions in space. This way of posing the problem is valid for times which are shorter than the lifetime of the directed beam of superluminal electrons in the medium, but allow us to obtain simple estimates.

1. THE PROBLEM OF ONE ELECTRON MOVING IN A MATERIAL MEDIUM

The asymptotic representation of the stationary electromagnetic field of an electron of charge $e<0$ moving with velocity $V<a$ along the z axis, in a comoving frame of reference (a frame attached to the electron), has the form[1,2]

$$E_z=\frac{-2eN^2z}{\varepsilon(z^2-N^2r^2)^{3/2}},\quad E_i=\frac{2eN^4x_i}{\varepsilon(z^2-N^2r^2)^{3/2}},\tag{1.1}$$

$$r^2=x^2+y^2,\quad x_i=x,\,y;$$

$$H_i=\frac{2eVN^2x_{i-1}(1-a^2/c^2)}{(1-V^2/c^2)(z^2-N^2r^2)^{3/2}},\quad H_z=0,\tag{1.2}$$

$$x_{i-1}=y,\,-x,\quad N^2=\frac{V^2/a^2-1}{1-V^2/c^2}=\frac{\mu\varepsilon V^2/c^2-1}{1-V^2/c^2}>0,$$

where E_i and H_i are the components of the electric and magnetic fields, μ and ε are the relative magnetic permeability and the dielectric constant, and $a=c(\mu\varepsilon)^{-1/2}$ is the speed of light in the medium. The solutions (1.1) and (1.2) are defined inside the Mach cone of the superluminal electron, i.e., in the region $z^2-N^2r^2>0, z<0$; outside this cone the proper field of the electron vanishes.

For stationary motion of the electron in the medium ($a<V<c$) the energy dissipation can be represented by two terms: the energy dissipation Γ_0 in the external electromagnetic field, and the losses Γ_V on the wavefront of the proper field of the electron, (Vavilov-Cherenkov radiation). For all beam particles the losses Γ_V will be of the same order of magnitude, and, as was shown by Tamm and Frank,[3] they are negligible compared to bremsstrahlung losses. The energy dissipation Γ_V determines a field in a small neighborhood of the wavefront. Therefore the mutual spatial disposition of the electrons in the beam will be determined by the asymptotic behavior of the field [the solutions (1.1), (1.2)], and the influence of Γ_V may be neglected.

By the method of invariant integrals (just as was done in the case of subluminal velocities[4]) one can obtain from Eqs. (1.1) and (1.2) for the case of superluminal motion of an electron in an external field $\mathbf{E}_0=\{E_{0j}\}$, $\mathbf{H}_0=\{H_{0j}\}$

$$\Gamma_{0j}=eE_{0j},\quad j=x,\,y,\,z.\tag{1.3}$$

Here Γ_{0j} is the irreversible work of the external field when the singularity (the electron) is displaced by one unit of length along the x_j axis. If the external magnetic field vanishes $\mathbf{H}_0=0$, then Γ_{0j} are the components of the force acting on the charge.

We note that the external field is considered in the electron's proper (comoving) coordinate frame attached to the relativistic electron.

2. COLLECTIVE INTERACTIONS

Let another electron e_1 be situated inside the wake (Mach cone) of a first electron e_0 which moves with superluminal velocity $V>a$. For e_1 the external field will now be the field (1.1), (1.2) created by the electron e_0. It is easy to see that e_1 is always attracted to the preceding electron e_0: $\Gamma_{0z}=e_1E_{0z}>0$.

We note that e_1 interacts with the field left behind by the electron e_0, therefore there is no reaction of e_1 on e_0.

One-dimensional system. We consider the behavior of a system of relativistic electrons in a medium, and confine ourselves to a one-dimensional semi-infinite

 0038-5646/80/010059-02$02.40

chain of electrons that at the initial instant are equidistantly separated by intervals b, a model for which a simple analytic solution can be obtained.

In the one-dimensional system there will exist only forces directed along the axis of the chain; we denote by f_{mn} the force acting on the m-th electron and due to the n-th electron ($n < m$). The resultant of all the forces acting on the m-th electron is

$$F_m = \sum_{n=0}^{m-1} f_{mn}. \tag{2.1}$$

According to Eqs. (1.1), (1.3), and (2.1) at the initial instant of the state of the system we obtain:

$$F_m(b) = \frac{2e^2N^2}{\varepsilon b^2} \sum_{n=0}^{m-1} (n+1)^{-2}.$$

It is a known fact that[5]

$$1 \leq \sum_{n=0}^{m-1} (n+1)^{-2} < \frac{\pi^2}{6},$$

hence

$$F_1(b) \leq F_m(b) < \pi^2 F_1(b)/6.$$

As is clear from Eq. (2.2), for any n the forces $F_m(b)$ differ little from $F_1(b)$. Therefore one can obtain a simple estimate of the behavior of a one-dimensional system by considering the motion of a single electron e_1 in the field produced by e_0.

The motion of a charge in the field of a superluminal electron in a medium. For an arbitrary distance $-b < z < 0$ we obtain from Eqs. (1.1), (1.3), (2.1) (a generalization of Coulomb's law to superluminal speeds)

$$F_1(z) = 2e^2N^2/\varepsilon z^2. \tag{2.3}$$

We limit ourselves to the case of small relative particle velocities. Taking (2.3) into account, the relativistic differential equation of motion of the electron e_1 in the moving coordinate frame has the form[6]:

$$\frac{d^2z}{dt^2} = \frac{2e^2(V^2/a^2-1)}{\varepsilon m_0(1-\beta^2)z^2}, \quad \beta = \frac{V}{c}. \tag{2.4}$$

Solving the equation (2.4) with the initial conditions $z = -b$ and $dz/dt = 0$ for $t = 0$, we obtain

$$tK^{1/2} = [-bz(z+b)]^{1/2} + b^{3/2} \arcsin[(z+b)/b]^{1/2},$$
$$K = 4e^2(\beta^2 - \varepsilon^{-1})/m_0(1-\beta^2). \tag{2.5}$$

Let us estimate the characteristic time τ within which the electron e_1 approaches e_0 (and a system of two electrons is formed which is dense enough that quantum interactions not taken into account by the continuous-medium model become decisive for it), setting $z = 0$ in Eq. (2.5),

$$\tau = \pi b^{3/2}/2K^{1/2}. \tag{2.6}$$

We note that the quantities b, t, and τ are considered in the comoving coordinate frame of the first electron. Going over to the laboratory frame $b' = b(1-\beta^2)^{1/2}$, $t' = t(1-\beta^2)^{-1/2}$ we obtain from Eq. (2.6)

$$(\tau')^2 = \frac{\pi^2 m_0 (b')^3}{16e^2(1-\beta^2)^{3/2}(\beta^2-\varepsilon^{-1})}. \tag{2.7}$$

As can be seen from Eq. (2.7) the effect of the electrons approaching each other is most important in a narrow region of energies (velocities) of the particles, where τ' are small. In other words, the velocities V must be significantly larger than a, but not too close to c, when the relativistic contraction of length scales becomes important. For $\beta_m^2 = (3+2\epsilon)/5\epsilon$ the time τ' takes on its minimal value:

$$\tau_m' = \varkappa(1-\varepsilon^{-1})^{-5/4}(b')^{3/2}, \quad \varkappa = \frac{5^{5/4}\pi m_0}{2^{5/2}3^{3/4}|e|}, \tag{2.8}$$

where $\varkappa = 6.437\,10^{-5}$ cm$^{-3/2}$s for an electron beam. For instance, for $b' = 10^{-4}$cm (this is the order of magnitude of distances in pulsed electron beams) we obtain $\tau_m' \sim 10^{-10}$ s.

Electron beams. As was shown with the simple model, relativistic electron beams exhibit a mechanism for internal organization of such systems (self-compression). Since the time T for which the formulation of the problem remains valid is small, this mechanism can manifest itself only for high-intensity beams (small b'). The necessary values of the particle density in the initial beam can be estimated at

$$n \sim (b')^{-3} > \varkappa^2(1-\varepsilon^{-1})^{-5/2}T^{-2}. \tag{2.9}$$

The quantity T is the lifetime of a directed beam of superluminal electrons in the medium and is determined by the interplay of two factors: the deceleration of the electrons to the speed of light in the medium and the losses due to excitation and ionization of the bound electrons of the material medium. For high energies ($\beta \gg \beta_m$) a more precise estimate of the critical density can be obtained from the relation (2.7).

Allowance for the spatial distribution of the electrons in the beam shows that the forces acting inside the system stop being directed along the axis of motion, i.e., there are forces directed towards the boundary of the Mach cone. However, the latter effect occurs only near the forward electron (an end effect), and in the far zone the forces are balanced by the action of the other superluminal electrons. Moreover, electrons situated away from the axis of motion of the forward electron will also be acted upon by a Lorentz force that depends on the relative velocity of motion.

We note that the mechanism of relativistic bunching of electrons in a beam discussed here is in principle valid for arbitrary charged particles of like charge at appropriate energies. A similar phenomenon with very short characteristic lifetimes can be attributed to fields due to the transition radiation of charged relativistic particles in arbitrary (nondielectric) media.[7]

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Translated by M. E. Mayer