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Translated by J. G. Adashko

## Observation of the refraction of conduction-electron trajectories by an intercrystal boundary in aluminum

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A radio-frequency size effect (RSE) was observed in bicrystalline aluminum plates with a twin boundary parallel to the outer surface. The positions and amplitudes of the RSE lines attest to a high transparency of the twin boundary to electrons, and indicates that the tangential component of the quasimomentum is conserved when this boundary is crossed.

PACS numbers: 61.70.Ng, 72.15.Qm

The interaction of conduction electrons with intercrystal boundaries was previously revealed in experiment by its averaged effect on the electric conductivity of polycrystals of pure metals (see, e.g., Refs. 1 and 2). It is obviously of interest to develop experimental methods with which to observe the scattering and diffraction of various groups of electrons by intercrystal boundaries of various types.

We have attempted for this purpose to observe the radio-frequency size effect (RSE) in the geometry indicated above. It can be assumed that in this case one can observe the lines of the ordinary RSE in each of the crystals, as a result of diffuse scattering of the electrons by the boundary as well as lines of the RSE due to the presence of electron trajectories that are refracted on passing through the boundary.

Planar intercrystalline boundaries were obtained by annealing (one hour at 650 C) aluminum samples mea-

suring  $1 \times 1 \times 2$  cm with a resistance ratio  $R_{293K}/R_{4,2K} = 3 - 5 \cdot 10^3$ . Prior to annealing the samples were cooled in liquid nitrogen and subjected to 3-4% deformation. In approximately half the samples etching has revealed, besides the more or less bent boundaries between arbitrarily oriented crystals, also twin boundaries directed along the (111) planes of bordering crystals obviously produced from one seed. Measurements under a microscope revealed no visible deviations of these boundaries from a plane (accurate to 3-5  $\mu$ m) over the entire perimeters of the boundaries, which in some cases passed through the entire sample.

A sample containing a twin boundary was mounted on an electric-spark cutting machine in such a way that the cutting plane was parallel to the twin plane, and single-crystal and bicrystal plates of equal thickness were cut from the crystal. After chemical polishing we measured the thickness of the single crystal and the bicry-

stal, respectively  $d_1 = 225 \mu\text{m}$  with an approximate scatter of  $5 \mu\text{m}$  from point to point, and  $d_2 = 215 \pm 2 \mu\text{m}$ . The twin plane was located about  $90 \mu\text{m}$  from the surface of the plate, a fact checked after the work with the sample was finished. The RSE was observed by the usual method<sup>3</sup> at an approximate temperature 1.2 K, both immediately after the polishing and after a second annealing in a vacuum of  $10^{-5}$  Torr at a temperature that decreased from 600 C to room temperature in 10 hours.

In the case of the bicrystal, distinct RSE lines were observed, some even more intense than the lines for the single crystal. After annealing, the line intensities in both samples increased by approximately a factor of 2. Figure 1 shows the registered RSE for a single crystal (I) and a bicrystal (II) after annealing, with the magnetic field  $\mathbf{H}$  directed along  $[1\bar{1}0]$ . The ordinates show, in the same scale for both samples, the derivative  $\partial f/\partial H$  of the frequency  $f$  of the measuring generator in whose tank circuit the sample was placed. The abscissas are the values of

$$k = \frac{e}{c} H d \frac{a}{4\pi\hbar}$$

( $a$  is the period of the aluminum lattice), i.e., the momentum-space dimensions of the Fermi-surface trajectories that cause the appearance of the line in the field  $\mathbf{H}$ , divided by the dimension  $4\pi\hbar/a$  of the Brillouin zone. The positions of the lines  $\alpha_1$  and  $\alpha_2$  or  $\beta_1$  and  $\beta_2$  practically coincide, and the small deviations are apparently due to the error in the measurement of the sample thickness. A noticeable difference is observed in the positions of the most intense lines  $\gamma_1$  and  $\gamma_2$ . These facts have a simple explanation if it is assumed that the lines in the bicrystal are due to the presence of electron trajectories that cross the boundary of the crystals with no change in the tangential component of the quasimomentum. It is easily seen that if the crystals are twins the position and shape of the lines corresponding to the extremal transverse dimension of the closed trajectories should be the same for a single crystal and a bicrystal having the same thickness.

RSE lines can be observed, however, also if a segment of an extremal trajectory with breaks on the ends is contained between the samples surfaces.<sup>4</sup> The positions of these lines may not be the same in the bicrystal and the single crystal. Figure 2 shows, for a sample with surfaces  $AB$  and  $CD$  and a twin boundary  $EF$ , the central sections of the Fermi surface of aluminum in the second band with the  $(1\bar{1}0)$  plane, corresponding to

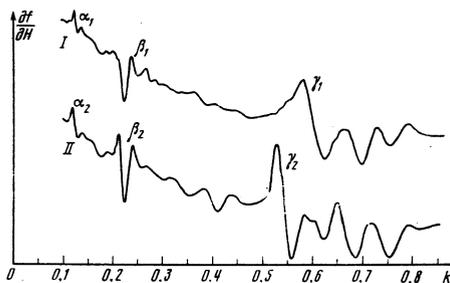


FIG. 1.

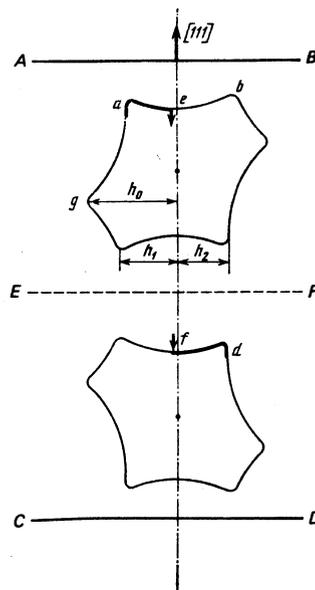


FIG. 2.

the case of Fig. 1. The shape of the section is shown in accord with the data of Ref. 5. The values  $h_0 = 0.44$ ,  $h_1 = 0.28-3$ , and  $h_2 = 0.25$  are referred to the dimension of the band. The line  $\gamma_1$  corresponds apparently to motion of the electron over the Fermi surface from the point  $a$  to the point  $b$  and should therefore be observed at  $k = h_1 + h_2 = 0.53 - 0.55$ . In the bicrystal, the electron goes over halfway between the points  $a$  and  $b$  from the point  $e$  to the point  $f$  on the Fermi surface of the other crystal, continues to move to the point  $d$ , and reaches at that instant the outer surface of the sample in real space. The lines  $\gamma_2$  should correspondingly be observed at  $k = 2h_2 = 0.5$ .

The position of the left wings of the lines  $\gamma_1$  and  $\gamma_2$  agree thus satisfactorily, within the limits of errors, with the model. The lines  $\beta_1$  corresponds apparently to electron motion from the point  $g$  to the point  $a$ , which yields  $k = h_0 - h_2 = 0.19$ . In this case a transition to another Fermi surface when the twin boundary is crossed should not lead, as can be easily seen, to a displacement of the line, in accord with the observations. The point  $a$  on the Fermi surface is in our case the most "effective," since in a considerable region around it the electron velocity is almost parallel to the sample surface, a fact corresponding to relatively large amplitudes of the observed lines  $\beta_1$ ,  $\beta_2$ ,  $\gamma_1$ ,  $\gamma_2$ . No line with  $k = 2h_0$  were observed in our experiments, obviously because of the low effectiveness of points of the type  $g$ . The lines  $\alpha_1$  and  $\alpha_2$  are apparently due to the presence of tubes in the third zone of aluminum.

It is obvious that the twin boundaries in aluminum have an appreciable transparency to conduction electrons. To observe weaker RSE lines due to diffuse scattering of the electrons by the boundary it is obviously necessary to detect the RSE using a measuring-generator coil placed on one side of the sample. In this case the lines due to the refracted trajectories will not be observed if the sample is thick enough.

We are sincerely grateful to L. K. Fionova for a dis-

discussion of the procedure of obtaining flat intercrystalline boundaries, to V. F. Gantmakher for a discussion of the results, and to P. L. Kapitza for interest in the work.

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Translated by J. G. Adashko

## Quantum effects in small ferromagnetic particles

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It is shown that at temperatures  $T < 1/2(\epsilon_a \epsilon_e)^{1/2}$  ( $\epsilon_a$  and  $\epsilon_e$  are the anisotropy energy and the exchange energy per magnetic electron) effects related with coherent quantum magnetization fluctuations should appear in sufficiently small ferromagnetic particles.

PACS numbers: 75.10. - b, 75.30.Cr

### 1. INTRODUCTION

It is well known (see, for example, Ref. 1) that a sufficiently small particle of ferromagnetic material consists of a single magnetic domain whose magnetization whose orientation is determined by the magnetocrystalline anisotropy of the particles and by the external magnetic field  $H$ . In the absence of a field, the easiest magnetization of a ferromagnetic particle has several orientations separated by energy barriers. If the temperature of the particle  $T$  is comparable with the size of the barrier  $U_a$ , then transitions between favorable orientations of the magnetization arise in the particle.

The existence of experimental data indicating that transitions between different orientations of the magnetic moment in small ferromagnetic particles do not disappear completely with a decrease in temperature to absolute zero was noted as far back as in the review by Bean and Livingston.<sup>2</sup> Nevertheless, as far as we know, quantum fluctuations in the magnetization of small particles has not been studied theoretically.

Turning to the study of this problem, we note first that a good approximation to the problem can be obtained if we limit ourselves to coherent magnetic-moment quantum fluctuations in which only the direction and not the magnitude of magnetization changes. This approximation is valid if the energy  $E_a$  necessary for the spin flip of one of the magnetic electrons in the particle significantly exceeds the energy  $u_a = \epsilon_a N$  ( $\epsilon_a$  is the magnetic anisotropy energy per magnetic electron,  $N$  is the number of magnetic electrons in the small particle) required to rotate the particle magnetic moment as a whole

$$\epsilon_a N \ll \epsilon_e. \quad (1)$$

Because of the smallness of the ratio  $\epsilon_a / \epsilon_e \sim 10^{-4} - 10^{-6}$  this condition is satisfied for ferromagnetic par-

ticles that are small but still contain a macroscopically large number of atoms.

The magnetic anisotropy energy is the average energy of the relativistic interaction of the electrons. In taking this interaction into account, the operator of the spin angular momentum does not commute with the Hamiltonian and, in general, the ferromagnetic particle is not in a state with a definite angular momentum. This circumstance should manifest itself by the appearance of a nonzero probability for a transition between  $n$  easiest-magnetization directions. Such transitions lift the  $n$ -fold degeneracy of the ground state with respect to the orientation of the angular momentum, and the ferromagnetic particle goes to a state with a lower energy, in which

$$\langle M \rangle = 0, \quad \langle M^2 \rangle = M_0^2 \quad (2)$$

( $M_0$  is the saturation magnetic moment). The second equality in (2) corresponds to strict ferromagnetic correlation of relative orientations of the spins of different magnetic electrons of the particle.

### 2. QUANTUM FLUCTUATIONS AT $H = 0$ AND $T = 0$

We consider a single-domain ferromagnetic particle with a magnetic anisotropy of the easy axis type (for example, a particle of hexagonal cobalt) rigidly fixed in a nonmagnetic matrix. The dependence of the energy  $E_a$  of the particle on the angle  $\theta$  between the direction of its magnetic moment  $M$  and the anisotropy axis  $z$  is given by the equation

$$E_a = \epsilon_a N \sin^2 \theta. \quad (3)$$

The presence of two energy minima at  $\theta = 0$  and  $\theta = \pi$  corresponds to the two easiest magnetization directions,