Electronic transition of order 2 1/2 in bismuth following simple dilatation

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The effect of deformation of the simple dilatation type on the electric resistance, magnetoresistance, and the Shubnikov–de Haas (SdH) effect in bismuth whisker crystals (1–2 μm thick) was investigated in fields up to 60 kOe at $T = 4.2$ K. The purpose of the investigation was to confirm experimentally a conclusion that follows from the I. M. Lifshitz theory of electronic transitions of order 2 1/2 [Sov. Phys. JETP **11**, 1130 (1960)], that anomalies appear in the properties of a metal when the topology of its Fermi surface changes. The maximum elongation of the samples reached 2%. Results are presented for two tension directions: the first makes an angle 78° with the C_3 axis, and the second an angle 55°. An analysis of the variation of the frequencies of the SdH effect in tension has shown that in the former case the volumes and isotropy of the electron and hole ellipsoids of the Fermi surface of bismuth change greatly. At an elongation of the order of 0.5%, one of the electron ellipsoids vanishes, and the rate of decrease of the electric resistance in the course of tension changes simultaneously, in agreement with Lifshitz's theory. In the second case, a strong anomaly of the dependence of the electric resistance on the tension was observed to set in at an elongation on the order of 0.7%. This anomaly, however, cannot be connected with the topological transition in the Fermi surface, which possibly occurs before the elongation reaches about 2%.

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I. INTRODUCTION

I.M. Lifshitz[1] has shown that anomalies in the behavior of the thermodynamic and kinetic characteristics of a metal can be observed when the topology of the Fermi surface is changed. A detailed analysis was made of anomalies of certain thermodynamic quantities, of the susceptibility, and of the galvanomagnetic characteristics of the metal with change in pressure. The physical nature of the anomalies is connected with the onset of singularities in the state density and with a change in the dynamics and type of electron trajectory (closed orbit $\rightleftarrows$ open orbit) when the Fermi energy ε_F reaches the singular point ε_c at which the Fermi-surface topology changes. It is important that the change of the topology of the Fermi surface must occur with preservation of the lattice symmetry and without a noticable change in the total number of carriers. The anomaly of the phenomena observed when some external parameter is changed and a topological transition occurs on the Fermi surface is customarily called, as suggested by I.M. Lifshitz, an electronic transition of order $2\frac{1}{2}$.

I.M. Lifshitz's theory[1] was further developed in a number of papers,[2,3] in which a detailed analysis was made of the behavior of the critical temperature T_c of superconductors following a toplogical transition of the Fermi surface under the influence of pressure and of impurities. The theory of transitions of order $2\frac{1}{2}$ explains the experimentally observed nonlinear dependence of T_c of a number of superconductors on the pressure[4-6] and on the elastic dilation.[7-9] But in these studies the Fermi-surface topology was not determined experimentally, so that the question of the connection between the observed anomalies of T_c and the actual changes in the topology of the Fermi surface remains open.

A topological transition of the Fermi surface was observed in cadmium[10] in a study of the qualitative change of the anisotropy of the magnetoresistance under pressure. It was found that at a pressure exceeding 15 kbar additional open directions in the (0001) plane appear in the Fermi surface which is open along the [0001] axis.

It was shown in Refs. 11–13 that in pure bismuth the electron transition should occur at a pressure on the order of 25 kbar, whereas in the alloys $Bi_{1-x}Sb_x$($x \leq 0.07$) the critical pressure is smaller. In these cases, however, the transitions must be identified as metal–insulator transitions, inasmuch as at critical pressure both the positive and negative carriers disappear and the onset of the anomalies is obvious.

The first experimental verification of the conclusions of I.M. Lifshitz's theory[1] was undertaken by Brandt and Ponomarev,[14] who investigated in weak magnetic fields, at pressures up to 20 kbar, the galvanomagnetic properties of Bi and of the alloy $Bi_{97.58}Sb_{2.4}$ to which Pb and Sn were added as acceptor impurities. The acceptor impurities made it possible to change the equality of the concentrations of the electrons and holes, and by the same token get rid of the metal–insulator transition when the electron carriers vanished. A change of the components of the galvanomagnetic tensor under pressure was observed, and an analysis by the two-band model shows that this change correlates with the vanishing of the electron carriers in the conduction band.

The paper by Brandt and Ponomarev[14] is the only one in which the anomaly of the properties and the stimultaneous change of the Fermi-surface topology are deduced on the basis of experimental data. This is no

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accident, for when working with high pressures the question of reversibility and reproducibility of the results is very acute. In a number of phenomena the results are not reproducible, since the high pressures obtained at low temperatures are not fully hydrostatic. At the same time, even in his first paper[1] I.M. Lifshitz noted that anisotropic deformations of the crystal lattice alter greatly the geometry of the Fermi surface and require less stresses than hydrostatic compression, and this should make them preferable in the investigation of transitions of order $2\frac{1}{2}$. But the use of homogeneous anisotropic deformations of any type in single crystals is inevitably limited by the elastic strength, which experiment has shown to prevent bulky samples from experiencing strains exceeding 0.01–0.05%. It appears that this difficulty cannot be overcome in principle. Recently, however, a number of methods were proposed for the production of strong anisotropic strains exceeding 1%.[15–17] In these methods, the single-crystal samples are components of complicated systems which are subjected to tension (compression) or to hydrostatic compression. In such systems, the strain in the sample is not uniform, but there are individual regions in which the uniformity is sufficient for the investigation of the change of the topology of the Fermi surface. The new method was used to investigate bismuth and its alloys, and various topological transitions of the Fermi surface were observed.[17,18]

Another possibility of producing large anisotropic strains is provided by whiskers, whose mechanical strength is close to the theoretical limit and makes it possible to obtain under tension relative elongations up to 2–3%. It is known that the strength in this case is attributed to freedom of the volume from defects and to perfection of the surface of the whisker at thicknesses of the order of 1 micron or less.

Theoretical and experimental investigations have shown that under the condition $2r>d$, $\lambda>d$ (r is the Larmor radius, d is the thickness of the sample and λ is the electron mean free path) the galvanomagnetic properties and quantization in thin samples differ from those in bulky samples. At the same time, in magnetic fields, when $2r<d$, the size effect can no longer interfere with any conclusions drawn concerning the topology of the Fermi surface of a metal from galvanomagnetic properties or from quantum oscillations.[19–22] The first attempt to use whiskers to investigate transitions of order $2\frac{1}{2}$ were made by us on zinc whiskers.[23]

The present paper is devoted to an investigation of the influence of tension on the frequencies of quantum oscillations of the resistance (the Shubnikov–de Haas (SdH) effect), on the resistance in the absence of a magnetic field, and on the magnetoresistance of bismuth whiskers. Preliminary results were reported in Refs. 24 and 25.

II. SAMPLES AND MEASUREMENT PROCEDURE

The whiskers were grown by the modified method of deposition from the gas phase with the bismuth evaporated in a vacuum of 10^{-6} Torr. The details of the method are described in Ref. 26. The whisker was mounted in a specially designed rigid[27] tension-producing unit,[27] which made it possible to obtain continuous elongations of the whisker

$$\xi=\frac{\Delta l}{l_0}=\frac{l-l_0}{l_0}$$

(l is the sample length) up to 3%. The elongation was measured accurate to 0.02%. The whisker and the electric four-contact circuit were fastened with silver-filled Epotek-20 epoxy resin.

We investigated 37 bismuth whiskers in the thickness range $d=1-2$ μm and at lengths $l=1-1.5$ mm. With 19 samples we were able to obtain information that permitted generalization of the results. In their crystallographic orientation, physical properties, and dependence on the tension, these 19 samples can be divided into two distinct types. We shall consider hereafter the results obtained for only these two types of bismuth whiskers (arbitrarily type I or type II).

The orientations of the whisker's axes were determined indirectly from the observed frequencies of the quantum oscillations of the resistance and from a comparison of these frequencies with know experimental data on bulky samples.[28,29] The method is based on the fact that any magnetic-field direction relative to the bismuth-crystal axes is determined in the general case uniquely by four SdH oscillation frequencies (three electron and one hole frequency). By orienting the magnetic field along the whisker axis it was possible in principle to determine the orientation of this axis. Most frequently, in fact, not all four frequencies were observed, so that the orientation determined from the measurement in a longitudinal field alone was not single valued. The ambiguities can be eliminated by performing additional measurements at other magnetic-field directions. The most convenient for this purpose are measurements in a field perpendicular to the whisker axis. The accuracy with which the orientation is determined depends completely on the accuracy with which the frequencies of the SdH effect are measured. A practical technique for determining the orientations is described in Ref. 30.

The orientation of the whisker axis is specified by the angles θ and φ, where θ is the angle between the trigonal axis C_3 and the sample axis, while φ is the angle between the bisector axis C_1 and the plane passing through the C_3 axis and the sample axis. In the chosen system, the C_1 axis has angles $\theta=90°$ and $\varphi=0°$, and the long axis of the three electron ellipsoids, at one and the same angle $\theta=84°$, have the following values of $\varphi_1=\pm180°$, $\varphi_2=-60°$, $\varphi_3=+60°$. The subscripts 1, 2, and 3 will henceforth be used throughout to indicate that the frequencies f of the SdH effect pertain to these three ellipsoids (f_1, f_2, f_3). The long axis of the hole ellipsoid of revolution is parallel to the C_3 axis and the frequencies corresponding to this ellipsoid are labeled by the subscript 4(f_4).

The thickness of the whisker samples, which were nearly square in section, was determined from the resistance R at room temperature (without allowance for the size effect) and from the length of the sample between the potential contacts, and was monitored with interference and electron microscopes.

The measurements were made at $T = 4.2$ K in magnetic fields H of two superconducting systems, one of which produced a longitudinal field up to 50 kOe, and the other a transverse field up to 78 kOe. The derivatives of the resistivity with respect to the magnetic field $\partial\rho/\partial H = f(H)$ were recorded by a standard modulation technique at a frequency 22.5 Hz.

The dependence of the resistance on the tension

$$\frac{\Delta R}{R_0} = \frac{R(\Delta l/l_0) - R_0}{R_0}$$

in the absence of a magnetic field could be obtained both in point-by-point measurements and by automatic recording.

III. MEASUREMENT RESULTS

1. Undeformed state

The results of the measurements of the two types of bismuth whiskers indicated above can be conveniently described using as an example two samples for which we were able to perform the most complete program of investigation of the orientations of the magnetic field, the relative elongation, the resistance, and the quantum oscillations. These were the samples Bi-73 (I) and Bi-81 (II) (the numbers in parentheses identify the arbitrary type of the sample, I or II). Some of the other investigated samples were used only to illustrate the reproducibility of the observed phenomena (there were 11 samples of type I and 8 of type II).

For sample Bi-73 (I) ($d = 1.6$ μm) in a longitudinal field $\mathbf{H} \parallel \mathbf{J}$ (J is the measuring current), the following frequencies were observed in units of 10^4 Oe: $f_1 = 1.4 \pm 0.1$, $f_2 = 4.2 \pm 0.3$, $f_4 = 16 \pm 1$. These frequencies correspond to orientation of the whisker axis with angles $\theta = 78 \pm 2°$, $\varphi = 12 \pm 3°$ (for the determination of the frequency f_2 at $\xi = 0$ see below).

Figure 1 shows the dependence of the frequencies of the SdH effect for the sample on the rotation angle ϑ of the magnetic field in a plane perpendicular to the whisker axis (transverse field $\mathbf{H} \perp \mathbf{J}$). The observed anisotropy of the frequencies is determined by the orientation of the whisker axis with angles $\theta = 75 \pm 3°$, $\varphi = 15 \pm 3°$ (sold lines in Fig. 1). We note that, just as in a longitudinal field, no frequency that could be identified as f_3 was discerned here.

Figure 1 shows also the anisotropy of the resistance of the sample in a constant magnetic field $R_H(\vartheta)$. Its form served as a supplementary (not principal) attribute of samples of type I.

For the sample Bi-81 (II) ($d = 1.5$ μm), the following frequencies were observed in a longitudinal field: $f_1 = 1.7 \pm 0.2$, $f_2 = 2.3 \pm 0.3$, $f_4 = 11 \pm 1$ (in units of 10^4 Oe). These frequencies correspond to an orientation of the whisker axis with angles $\theta = 55 \pm 5°$, $\varphi = 3 \pm 3°$. For this orientation there should exist a frequency $f_3 = 2.7 \cdot 10^4$

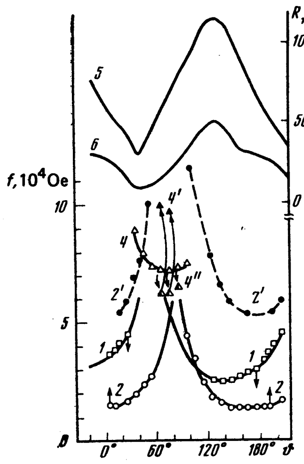

FIG. 1. Anisotropy of the frequencies of the quantum oscillations and of the resistance in a transverse field for the sample Bi-73 (I) ($d = 1.6$ μm). The angle ϑ is reckoned from an arbitrary point. Curve 1—frequency f_1 at elongation $\xi = \Delta l/l_0 = 0$; 2, 2′—frequency f_2 at $\xi = 0$ and 1.2%, respectively; 4, 4″, 4′—frequency f_4 at $\xi = 0$, 0.4, and 1.2%. Points—experiment, solid lines—calculation (see the text). The arrows indicate the direction of the change of the frequencies under tension. Curve 5—resistance in field $H = 5$ kOe and $\xi = 0$; 6—the same at $\xi = 1.2\%$.

Oe. It could, however, not be discerned.

A number of cases showed a frequency $f_2 = (2.5 \pm 0.3) \cdot 10^4$ Oe, meaning an angle $\varphi = 0 \pm 3°$. However, the non-symmetrical character of the anisotropy of the resistance in the magnetic field (Fig. 2) makes it necessary to assume that $\varphi \neq 0$, i.e., the sample axis does not lie in the binary plane. It is possible that the discerned frequency f_2 is a superposition of two close frequencies from two electron ellipsoids 2 and 3. In any case, in a transverse field these frequencies could be observed separately, and their changes under tension were identical—Fig. 2.

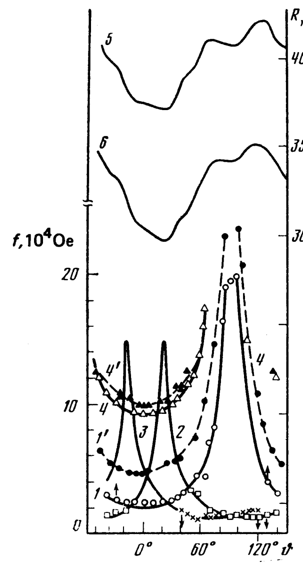

FIG. 2. The same as in Fig. 1 but for sample Bi-81 (II) ($d = 1.5$ μm). Curves 1, 1′, and 4, 4′—frequencies f_1 and f_4 at $\xi = 0$ and $\xi = 1.2\%$; 2, 3—frequencies f_2 and f_3. The arrows indicate the direction of the change of the frequencies under tension up to 1.2%. Curves 5 and 6—resistance in field $H = 10$ kOe at $\xi = 0$ and 1.2%.

In a transverse field we observed oscillations from all four ellipsoids of the Fermi surface. The anisotropy of the corresponding frequencies is shown in Fig. 2. For comparison, the figure shows the calculated angular dependences of the oscillation frequencies for a sample-axis orientation with angles $\theta = 55°$, $\varphi = 0°$.

The most distinguishing attribute of samples of type II is the shape of the plot of the derivative of the resistivity $\partial\rho/\partial H = f(H)$ in a longitudinal field—Fig. 3. For the high frequency f_4 in the strong-field region, the resistivity peaks with the smaller numbers are not equidistant, i.e., the linear connection between the number of the oscillation peak and the reciprocal field is violated. More accurately speaking, this dependence is modulated at a lower frequency $f_2 = 2.5 \cdot 10^4$ Oe. This decreases the accuracy of f_4 in the longitudinal both in field free and deformed samples. In addition, for the same frequency f_4 the resistance peak corresponding to the third and fourth Landau levels (the calculated values of the field are $H_3 \approx 35$ kOe and $H_2 \approx 50$ kOe) are absent. In a transverse field, these singularities of the oscillation curves were not observed for samples of type II.

2. Effect of simple dilatation

A. Dilatation along the direction $\varphi \approx 12°$, $\theta \approx 78°$ (samples of type I)

Under tension, the frequencies of the SdH effect exhibit a great variation in their behavior. Let us note the singularities of the oscillation curves.

In a longitudinal field (Fig. 4), stretching the sample shifted the peaks of the low frequency f_1 (fields up to 15 kOe at $\xi = 0$) towards weaker fields. At $\xi = 0.2\%$ it was still possible to observe three peaks of this frequency, while at $\xi = 0.4\%$ only one peak remained noticable, with the Landau number $n = 1$ ($H = 4$ kOe). It is also seen on the curves that with increasing length of the sample the peaks of the other frequency f_2 become more and more pronounced, and the presence of this frequency is noticed even at zero tension (for example, a peak is clearly seen in a field $H = 10$ kOe). The positions of the peaks of this frequency in the magnetic field do not change with tension up to the maximum possible elongation $\xi = 1.2\%$. It was this which made it possible to determine the frequency f_2 at $\xi = 0$.

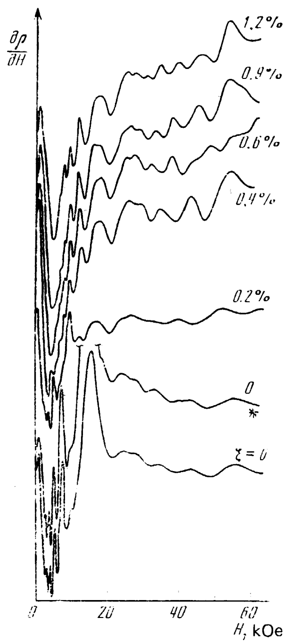

FIG. 4. Sample Bi-73 (I). Change of the form of the oscillation curves of the derivative of the resistance in a longitudinal magnetic field as a function of the consecutive elongation of the sample; the asterisk marks the curve obtained after removing the tension-producing load. The curves are vertically separated by arbitrary intervals.

In the field region $H > 15$ kOe, the oscillation picture is quite complicated. A Fourier analysis with a computer has made it possible to observe here two frequencies, one of which, f_4, can be traced with 10% accuracy at all elongations, while the second (f_3?) can be traced with 20% accuracy starting with elongation $\xi > 15$ kOe, the oscillation picture is quite complicated. A Fourier analysis with a computer has made it possible to observe here two frequencies, one of which, f_4, can be traced with 10% accuracy at all elong-elongations, while the second (f_3?) can be traced with 20% accuracy starting with elongation $\xi > 0.1\%$. Finally, this set is completed by the curve obtained after removal of the tension, and demonstrates the complete reversibility of the observed changes.

The dependence of the SdH oscillation frequencies in a longitudinal field on the elongation is shown in Fig. 5. The orientation of whiskers of the first type should correspond, at zero tension, to a frequency $f_3 = 2 \times 10^4$ Oe. Therefore the frequency f_3? which appeared under tension was extrapolated to this value, although the situation is not quite clear to us. It is seen from the figure that the frequencies f_4 and f_3? increase con-

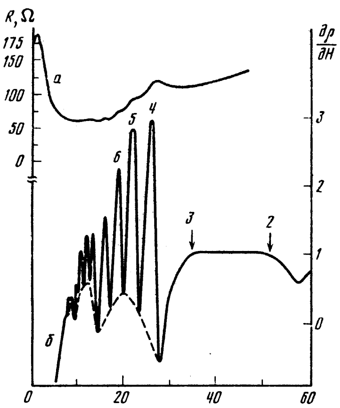

FIG. 3. Longitudinal resistance (a) and its derivative (b) (arbitrary units) against the magnetic field for sample Bi-63 (II) ($d = 1.3$ μm). The curves are marked with the numbers of the Landau levels. The arrows indicate the calculated positions of numbers $n = 2$ and 3.

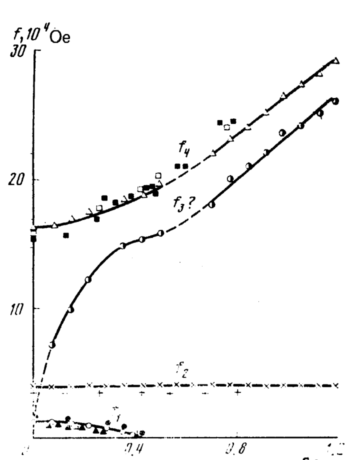

FIG. 5. Dependence of the frequencies of the oscillations of the resistance in a longitudinal magnetic field on the sample elongation. Points for f_1: ●—Bi-73 (I), ○—Bi-94 (I) ($d = 0.9$ μm), ▲—Bi-100 (I) ($d = 1$ μm); for f_2; ×—Bi-73 (I), +—Bi-96 (I) ($d = 1.2$ μm); for f_3: f_3? ◐—Bi-73 (I); for f_4: △—Bi-73 (I), □—Bi-100 (I), ■—Bi-94 (I).

siderably, while the frequency f_1 decreases. Extrapolation of the frequencies f_1 for different samples to zero value shows that this component of the SdH oscillations vanishes completely at $\xi = 0.5 \pm 0.1\%$ (see also Fig. 2 of Ref. 24). Despite the large change in the frequencies f_1, f_4, and f_3?, the most striking is the constancy of the frequency f_2!

In a transverse field (examples of the original plots of the oscillation curves of the resistance are shown for this case in Figs. 6 and 7) the frequency f_2 was observed for all samples in a wide range of directions of the magnetic field at all the obtained elongations. The modification of the resistance oscillation curves with peaks of frequency f_2 can be seen on Fig. 6. The curves of Fig. 6a have also peaks of frequency f_1, which shift upon stretching towards weaker fields, thus indicating a decrease of this frequency. Unfortunately, starting with $\xi = 0.2-0.3\%$ no peaks of frequency f_1 could be observed against the background of the strong dependence of $\partial\rho/\partial H$ on H.

The behavior of the frequency f_4 is unique. In this small interval of magnetic-field direction in which it could be observed (the region of the minimal values of f_4 on Fig. 1, $\vartheta \approx 80^\circ$), the peaks of this frequency shift towards weaker fields up to an elongation of the order of 0.4%, and then towards stronger fields (Fig. 7).

In contrast to the longitudinal field, no signs were observed at all of the existence of the frequency f_3, even at certain isolated values of the tension.

The dependence of the observed frequencies of the ShD effect on the elongation of the samples for certain directions of the transverse magnetic field is shown in Fig. 8. The frequency f_1, just as in a longitudinal field, decreases and should vanish in the region $\xi = 0.4-0.5\%$. The frequency f_2 increases monotonically

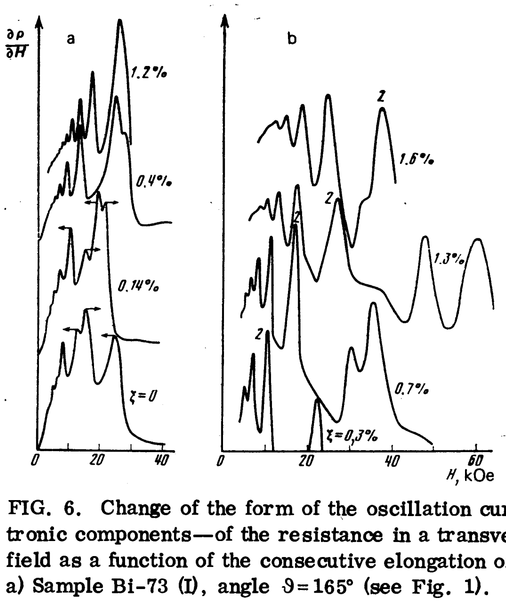

FIG. 6. Change of the form of the oscillation curves—electronic components—of the resistance in a transverse magnetic field as a function of the consecutive elongation of the sample. a) Sample Bi-73 (I), angle $\vartheta = 165^\circ$ (see Fig. 1). The arrows indicate the directions of displacement of the maxima under tension: the peaks of frequency f_1 shift to the left, and those of frequency f_2 to the right. b) Sample Bi-44 (I) ($d = 0.9\ \mu$m), $\vartheta = 160^\circ$; the number 2 pertains to the second Landau level. The curves are separated vertically by arbitrary intervals.

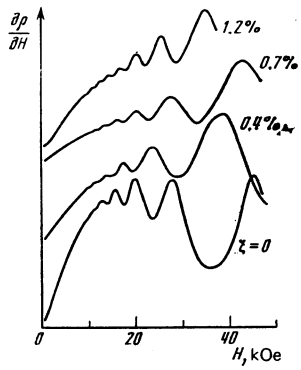

FIG. 7. The same as Fig. 6, but for the hole component. Sample Bi-73 (I), $\vartheta = 75^\circ$.

in the entire wide interval of the angles ϑ in which it could be observed (approximately linearly with the elongation and at a constant rate). At $\xi = 1.2\%$ it increased by factor of four (Fig. 1).

Under tension, the magnetoresistance decreases, and its anisotropy in a transverse field increases somewhat (Figs. 1 and 9). It is interesting to note the change of the character of the dependence of the resistance on the magnetic field at large elongations (Fig. 9), where the curve with saturation is replaced at $\xi = 0$ by a gradual linear dependence modulated by quantum oscillations.

Finally, the dependence of the resistance (without magnetic field) on the tension is quite typical of whisker samples of type I (Fig. 10): the linear variation of the resistance with tension has two sections with different slopes. The change of the slope occurs in the vicinity of $\xi = 0.5$; for some samples the slope is decreased by one-half. At room temperature the resistance also decreases at approximately the same rate as at 4.2 K—Fig. 10.

B. Tension along the direction $\varphi \approx 3^\circ$, $\theta \approx 55^\circ$ (samples of type II)

Plots of the oscillation curves of the resistance in a magnetic field in the case of tension, using as examples the samples Bi-61 (II) and Bi-81 (II), are shown

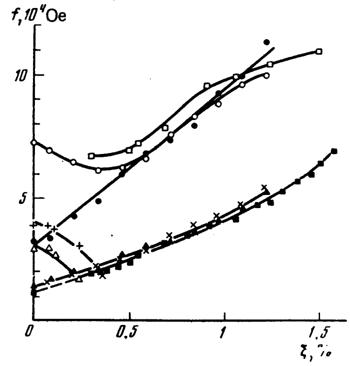

FIG. 8. Dependence of the frequencies of the resistance oscillations in a transverse field on the elongation of the sample. Frequency f_4: ○—Bi-73 (I), $\vartheta = 165^\circ$, □—Bi-44 (I), $\vartheta = 160^\circ$, frequency f_2: ●—Bi-73 (I), $\vartheta = 105^\circ$, ×—Bi-73 (I), $\vartheta = 15^\circ$, ▲—Bi-73, $\vartheta = 165^\circ$, ■—Bi-44 (I), $\vartheta = 160^\circ$, frequency f_1: +—Bi-73 (I), $\vartheta = 15^\circ$, △—Bi-73 (I), $\vartheta = 165^\circ$. (Angle ϑ—see Fig. 1.)

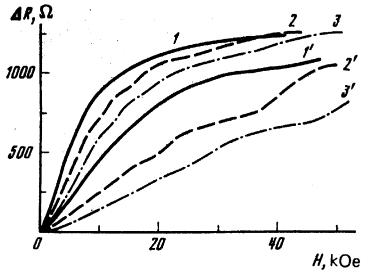

FIG. 9. Dependence on the resistance on the magnetic field: $\Delta R = R(H) - R(0)$. Sample Bi-48 (I) ($d = 2$ μm). Curves 1–3—in a direction of the maximum of the angular dependence of $R(\vartheta)$ ($\vartheta \approx 130°$, Fig. 1); curves 1′–3′—in the direction of the minimum $R(\vartheta)$ ($\vartheta \approx 40°$). Solid, dashed, and dash-dot lines—for sample elongations ξ equal to 0, 0.8, and 1.1%, respectively.

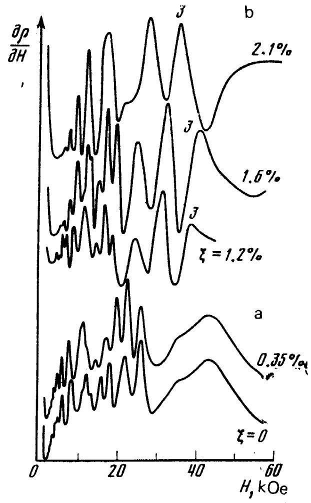

FIG. 11. Change of the form of the oscillation curves of the derivative of the resistance in a longitudinal magnetic field as a function of the sample elongation: a) Bi-61 (II) ($d = 1.6$ μm) b) Bi-81 (II) ($d = 1.5$ μm). The numnumber 3 denotes the third Landau level. The curves are separated vertically by arbitrary intervals.

in Fig. 11. In fields up to 20 kOe one can see a strong dependence of the shape of the curves on the tension, whereas in strong fields the position of the peaks remains practically unchanged (these peaks pertain to the frequency f_4), and only at $\xi > 1\%$ do they begin to shift towards weaker fields. On the curve one can see in this case the peak with Landau number $n = 3$, which did not appear at smaller tensions. Just as at zero tension, the peaks of frequency f_4 are not equidistant in the reciprocal field. Therefore, because of the low accuracy with which f_4 was determined, it is difficult to say anything about its variation up to elongation of the order of 1%. At larger elongations, as shown by the measurements, the influence of the low-frequency component of the SdH oscillations becomes weaker and one can state definitely that f_4 decreases (Fig. 12).

Analysis of the oscillation curves shows that the low frequency f_1 increases monotonically up to elongations $\xi = 1$–1.5%, and then begins to decrease weakly. The frequency f_2 decreases linearly at least up to $\xi = 1.2\%$, after which it was impossible to observe it. If the variation of the frequency f_2 is linearly extrapolated to zero value, then it can be concluded that it vanishes at $\xi \approx 2\%$ (Fig. 12).

These same components of the SdH oscillations behave similarly in a transverse field. The peaks of frequencies f_2 and f_3 shift under tension towards weaker fields (Figs. 12 and 13), while those of frequency f_1 shift, up to elongations of the order of 1%, into the region of stronger fields, and then towards the weaker fields (Fig. 13). Just as in the longitudinal field, extrapolation shows that the frequencies f_2 and f_3 vanish at $\xi \approx 2\%$ (Fig. 12).

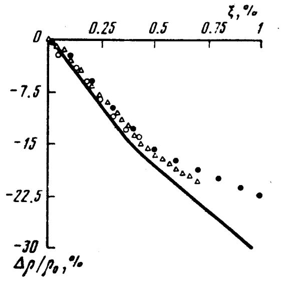

FIG. 10. Dependence of the resistance in the absence of a magnetic field

$$\frac{\Delta R}{R_0} = \frac{R(\Delta l/l_0) - R(0)}{R(0)}$$

on the sample elongation. Temperature $T = 4.2$ K: ●—Bi-47 (I) ($d = 2$ μm), △—Bi-94; solid line—automatic plot for Bi-96 (I) (see also Fig. 2 of Ref. 24). Temperature $T = 295$ K: ○—Bi-80 (I) ($d = 1.7$ μm).

The peaks of frequency f_4 in the reciprocal field are equidistant and no difficulties arise with their determination. Up to elongations $\xi \approx 1\%$, the frequency f_4 increases insignificantly, and then decreases. Figure 14 shows plots of the frequencies of the SdH effect in a transverse field against the elongation at certain fixed field directions.

Figure 2 shows the change of the curves $f_1(\vartheta)$ and $f_4(\vartheta)$ when the elongation changes from zero to the value $\xi = 1.2\%$ at which these frequencies have reached approximately their maximum. It can be seen that the

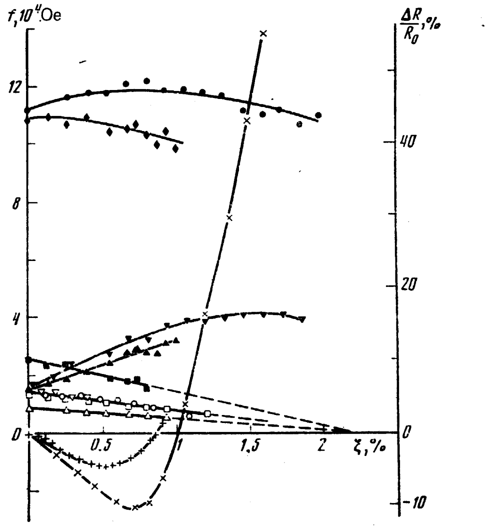

FIG. 12. Resistance-oscillation frequencies, oscillation-peak position, and resistance in the absence of magnetic field vs. sample elongation. Longitudinal field ($\mathbf{H} \parallel \mathbf{J}$), frequency f_4: ●—Bi-81 (II); ♦—Bi-49 (II) ($d = 1.3$ μm); frequency f_1: ▼—Bi-81 (II), ▲—Bi-49 (II); frequency f_2; ■—Bi-49 (II). Transverse field ($\mathbf{H} \perp \mathbf{J}$), $\vartheta = 90°$ (Fig. 2), frequency $f_2 = f_3$: ○—Bi-46 (II) ($d = 2.2$ μm), ▽—Bi-38 (II) ($d = ?$); $\vartheta = 125°$, position of peak with Landau number $n = 1$ of frequency f_2: □—Bi-81 (II). Resistance in the absence of magnetic field: +—Bi-101 (II) ($d = 0.9$ μm), ×—Bi-81 (II).

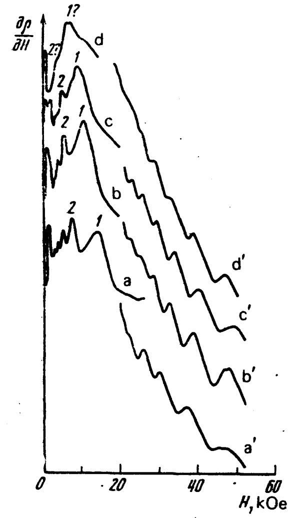

FIG. 13. Change of the form of the resistance-oscillation curves in a transverse magnetic field as a function of the elongation of the sample Bi-81 (II), $\vartheta = 85°$ (see Fig. 2). Curves a, a'—$\xi = 0$; curves b, b'—$\xi = 0.5\%$, curves c, c'—$\xi = 0.8\%$, cu curves d, d'—$\xi = 1.2\%$. For the primed curves the sensitivity is increased by a factor of 4. The numbers 1 and 2 mark the peaks of the corresponding Landau numbers. The curves are separated vertically by arbitrary intervals.

rate of growth of the frequency f_1 depends on the angle ϑ, i.e., the anisotropy of f_1 changes. Thus, the frequency has increased by a factor 2.3 at $\vartheta = 0°$, as against a factor 1.3 at $\vartheta = 85°$. This indicates a decrease of the anisotropy of f_1. The small changes of the frequency f_4 and its low accuracy do not make it possible to draw definite conclusions concerning the change of its anisotropy under tension.

The anisotropy and the magnitude of the magnetoresistance depend little on the tension (Fig. 2). The change of the form of the $R(H)$ dependenceis also small. However, up to $\xi \approx 1\%$ these changes are analogous to the changes observed for whiskers of type I.

The behavior of the resistance without the magnetic field under tension (Fig. 12) up to $\xi \approx 0.5-0.7\%$ is quite peculiar: it decreases approximately linearly by 5–10%, goes through a minimum, and then begins to increase rapidly. The resistance goes through the initial value at $\xi \approx 0.8-1\%$.

IV. DISCUSSION OF RESULTS

We make first two remarks.

The first concerns the relation between the spectrum of conduction electrons in whiskers and in bulky bismuth. The smallest electron momentum in bismuth

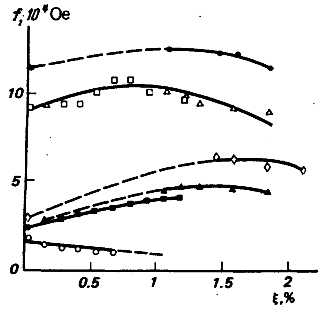

FIG. 14. Dependence of the resistance-oscillation frequencies on the elongation of the sample Bi-81 (II) in a transverse field. Frequency f_4: ●—$\vartheta = 140°$, □—$\vartheta = 0°$, △—$\vartheta = 14°$, frequency f_1: ×—$\vartheta = 320°$, ▲—$\vartheta = 0°$, ■—$\vartheta = 140°$, frequency f_2: ○—$\vartheta = 125°$.

corresponds to a de Broglie wavelength $\lambda_D \approx 10^{-5}$ cm, which is smaller by one order of magnitude than the thicknesses of the investigated whiskers. There are therefore no grounds for fearing a noticeable influence of the quantum size effect on the spectrum. It is also easy to overcome the influence of the classical size effect, which changes the frequencies of the SdH effect only under the condition $2r > d$. Estimates show that in bismuth at $d = 1$ μm the condition $2r = d, H = H_d$ is realized for different sections of the Fermi surface in the field range $H = (0.1-1) \cdot 10^4$ Oe, whereas the corresponding frequencies lie in the range $(1.2-20) \cdot 10^4$ Oe. Consequently, the principally observable number of oscillation peaks of the SdH effect not distorted by dimensions is sufficient for a relatively accurate determination of the frequency, since this number is equal to $f/H_d = 10$. Actually, however, when the condition $2r = d$ is approached from the strong-field side, the amplitude of the oscillation peaks decreases sharply until they vanish completely, and this of course decreases the accuracy with which f is determined. The described phenomenon can be seen in Fig. 3. On curve a, in weak fields, the resistance passes through a maximum which occurs under the condition $r \approx d$.[20,32] These size-effect maxima correspond to singularities of the derivative $\partial\rho/\partial H$. They can be seen on Figs. 4 and 13 (the initial sections of the curves). One can see here also the vanishing of the peaks of the ShD effect when these singularities are approached.

The second remark concerns the connection between the frequencies of the observed resistance oscillations and the areas of the extremal sections of the Fermi surface.

In the quasiclassical region ($\hbar\omega \ll \varepsilon_F$, ω is the cyclotron frequency) the frequencies of the quantum oscillations are proportional to the areas of the sections of the Fermi surface.[19] In the quantum limit ($\hbar\omega \approx \varepsilon_F$) this relation may be violated. Thus, in particular, in the presence of several groups of carriers, the energy for each of them depends on the magnetic field differently, and the quantum limits are reached at different field values. Moreover, the dependence of the chemical potential on the field has an oscillatory character. Equalization of the chemical potential for different carrier groups leads to flow of the carriers from one part of the Fermi surface to others and this results in mutual modulation of the frequencies of the SdH effect and in nonequidistant positions of the oscillation peaks in terms of the inverse field. This phenomenon was observed by Brandt and Lyubutina.[31] It is also observed in our case for the whiskers of the second type (Fig. 3) in a longitudinal field. It is possible that the abrupt vanishing of the resistance peaks with quantum numbers $n = 3$ and 2 is also connected with the carrier transfer phenomenon. It must then be assumed that the capacity of the last Landau level for the low-frequency component of the SdH oscillations is here such that the next to the last Landau levels for f_4 do not manifest themselves in the resistance. In other words, the rate of change of the Fermi level in the magnetic field is not less than the rate of displacement of the Landau levels for the frequency f_4.

Taking these remarks into account and assuming an average measurement error not larger than 10%, we shall assume that the observed frequencies of the SdH effect are proportional to the extremal areas of the Fermi-surface sections and that the frequency change is always proof of a change in these areas.

1. Dilatation along the direction $\varphi \approx 12°$, $\theta \approx 78°$

We consider the transformation of the hole ellipsoid. The area of its section in a longitudinal field of samples of type I is close to the maximum area of the section passing through the C_3 axis (the angle between these sections is only $\approx 12°$, and the frequencies are respectively 16×10^4 and 19×10^4 Oe). This section increases monotonically and appreciably with elongation of the sample. Thus, at $\xi = 1.2\%$ we have $f_2(\xi)/f_4(0) = 1.8$.

In a transverse field, the observed frequencies correspond to areas close to the minimum section of the hole ellipsoid (the section perpendicular to C_3). These areas decrease slightly up to $\xi \approx 0.5\%$, and then increase (Fig.8). Unfortunately, no large sections were observed in the transverse field, so that one cannot resolve quite definitely the question of the change of the volume of the hole ellipsoid, especially in the region of initial elongations. The only thing not subject to doubt is that the anisotropy of the hole ellipsoid changes strongly. It most likely becomes stretched out along the C_3 axis.

Only two of the three electron ellipsoids were observed. For ellipsoid No. 2 (frequency f_2), it is seen from Figs. 1 and 8 that the areas of all the sections parallel to the tension axis increase monotonically and considerably. The growth rate is the same for all directions. From this one might seemingly conclude that this ellipsoid changes similarly, i.e., its anisotropy does not depend on the tension. However, the results for a longitudinal field (Fig. 5) contradict this. They indicate that the area of the section perpendicular to those considered above remains constant. This picture of variation of the cross sections does not contradict the following behavior of the ellipsoid: its volume increases, it becomes stretched along the long axis, and the angle between this axis and the binary plane becomes smaller than 60°.

The areas of the sections of the ellipsoid with No. 1 (frequency f_1) at all field directions decrease under tension and vanish at $\xi \approx 0.5\%$ (Figs. 5 and 8). The experimental data are not sufficient to understand how the anisotropy of this ellipsoid changes in this case, but there is no doubt that its volume vanishes upon elongation.

We consider now the magnetoresistance. The samples investigated by us had almost quadratic cross sections, so that the anistropy of the resistance should not differ greatly from that of a bulky sample having the same orientation. This anisotropy decreases with increasing tension. Thus, at $\xi = 0$ the ratio of the resistances at the maximum and minimum of the angular dependence (Fig. 1) is approximately equal to 2, and at $\xi = 1.2\%$ it is equal to 4 in a field $H = 10$ kOe—Fig. 9. This may possibly also indicate an increase of the anisotropy of the Fermi surface as a whole.

The considerable decrease of the magnetoresistance (Fig. 9) is explained as being due to the increase of the total volume of the Fermi surface of Bi. In fact, in the case of a strong size effect ($\lambda \gg d$) and high specularity of the electron reflection from the surface (the specularity coefficient for the whiskers is $P \approx 1$, Refs. 30 and 32), the magnetoresistance can be written in the form

$$\rho^d(H) = \rho^\infty(0)\, d/r,$$
$$\rho^\infty(0) = \left[\frac{2c^3\Sigma_F}{3(2\pi\hbar)^3}\lambda\right]^{-1} \psi(\vartheta), \qquad \lambda = v_F\tau, \tag{1}$$

where Σ_F is the total area of the Fermi surface, $\psi(\vartheta)$ is a function that takes into account the anistropy of the Fermi surface, v_F is the velocity on the Fermi surface, and τ is the relaxation time. Since the anisotropy increases and the magnetoresistance decreases at any field direction, this is possible only when the product $(v_F\tau\Sigma_F)^{-1}$ decreases. Various scattering mechanisms at low temperatures do not depend in first-order approximation on the Fermi energy,[33] so that the decrease of the magnetoresistance can be only attributed to an increase of the quantity $\Sigma_F v_F$, which has the same sign of variation as the volume of the Fermi surface.

The increase of the volume of the Fermi surface under tension is confirmed also by measurement of the $R(H)$ dependences (Fig. 9). The change from a quadratic growth of the resistance in a magnetic field to a linear one in bulky samples, and the change in the case of thin samples from a linear growth to saturation, are due to the transition into the ultraquantum region for individual parts of the Fermi surface, when the electron concentration begins to increase linearly with the magnetic field.[32,34] With increasing volume of the Fermi surface, the quantum limit shifts into the region of stronger fields and extends the region of linear growth of the resistance.

The growth of the volume and the change of the anisotropy of the Fermi surface explain also the decrease of the resistance without magnetic field under tension. The resistance of a thin conductor with a section close to quadratic can be written in the form[35,36]

$$\rho^d(\theta, \varphi) = \frac{\rho^\infty\lambda}{d}\left(\frac{1-P}{1+P}\right)\Phi(\theta, \varphi), \qquad \lambda \gg d, \tag{2}$$

where $\Phi(\theta, \varphi)$ is the direction function. The specularity coefficient P decreases with decreasing de Broglie wavelength λ_D.[37] Since $\lambda_D \propto 1/p_F$ (p_F is the Fermi momentum), P decreases with increasing dimensions of the Fermi surface. Thus, the changes of $\rho^\infty\lambda' = -\Sigma_F^1$ and of P are connected in the general case with one another and lead to a mutually weakening influence on the resistance ρ^d. The resistance decreases in tension, so that one can conclude that the factors $\rho^\infty\lambda\Phi(\theta, \varphi)$ in expression (2) exert the dominant influence.

It is impossible to determine the contribution made to the effect by the factors Σ_F^{-1} and $\Phi(\theta, \varphi)$ separately.

If the quantities $\Phi(\theta, \varphi)$ and P were to change little for a given direction of dilatation, then the linear decrease of the resistance would be simply a consequence of the dependence of the total electron Fermi energy on the volume V of the metal. According to I. Lifshitz,[1] when the critical value is approached, the change of the Fermi energy is proportional to the change of the volume:

$$\Delta\varepsilon_F=\varepsilon_F-\varepsilon_\kappa\propto|V-V_\kappa|\propto\xi \quad (3)$$

($\xi\equiv\Delta l/l_0$). Next,

$$\rho^d(\xi)\propto\Sigma_F^{-1}(\xi)=[\Sigma_F(0)+\Delta\Sigma_F(\xi)]^{-1}\approx\Sigma_F^{-1}(0)[1-\Delta\Sigma_F(\xi)/\Sigma_F(0)]. \quad (4)$$

Since $\Sigma_F\propto\varepsilon_F$ and $\Delta\Sigma_F\propto\Delta\varepsilon_F$, it follows that

$$\rho^d(\xi)=\rho^d(0)-\beta\xi, \quad (5)$$

where β is a coefficient.

When one electron ellipsoid vanishes, a jump should occur in the rate of change of the quantities Σ_F, $\Phi(\theta,\varphi)$ and P, and this should lead to a singularity in the function $\rho^d(\xi)$. Depending on the ratio of the new rates of change of these three quantities, there are three possible versions of the change in the rate of decrease of the resistance. This rate can increase, decrease, or remain unchanged. It was observed in experiment that the decrease of the resistance slows down once an elongation $\xi\approx0.5\%$ is reached. Inasmuch as at this elongation value one of the electron ellipsoids vanishes there are grounds to assume these two effects to be causally connected. This would confirm the theoretical conclusion[1] that anomalies occur in the physical quantities when a topological transition takes place in the Fermi surface.

At room temperature there are no grounds for assuming the size effect to be strong and the resistivity should not differ from that of the bulky sample ρ^∞. Therefore the decrease of the resistance at room temperature under tension undoubtedly confirms the conclusion that the volume of the Fermi surface of Bi is increased.

2. Dilatation along the direction $\varphi\approx3°$, $\theta\approx55°$

Judging from the results in a transverse field, the volume of the hole ellipsoid first increases (approximately by 50% up to $\xi\approx0.7\%$), and then begins to decrease. The conclusion that the volume decreases is more definite, for in a longitudinal field the section of the hole ellipsoid decreases at large elongations.

The results for the second and third electron ellipsoids indicate that they vanish under tension not before an elongation of 2% is reached. Up to these elongations, no topological transition takes place in the Fermi surface of bismuth.

For the electron ellipsoid with No. 1 (frequency f_1), the initial stage of tension is characterized by an increase of the areas of the sections in both a longitudinal field and in a wide range of directions of the transverse field. An uncertainty remains only for a narrow region near the maximum sections (Fig. 2). The growth of the areas of the ellipsoid sections then slows down and starting with $\xi>1\%$ they decrease weakly (Figs. 12 and 14). We can therefore assume, with greater certainty than for the hole ellipsoid, that the volume of this electron ellipsoid first increases, reaches a maximum, and then decreases. From the anisotropy of the frequency f_1 under zero tension and at $\xi=1.2\%$ (shown in Fig. 2), we can qualitatively deduce that the rate of change of the areas of the Fermi-surface sections decreases as large sections are approached, i.e., the ellipsoid does not change similarly, and its anisotropy decreases corresponding to the weak change of the form and volume of the Fermi surface is in this case also the weak change in anisotropy and magnitude of the magnetoresistance (Fig. 2).

The behavior of the resistance in the absence of a magnetic field under tension, for samples of type II, is even more anomalous than for samples of type I: it decreases, reaches a minimum at $\xi\approx0.5-0.7\%$, and then begins to increase sharply (Fig. 12). Unlike samples of type I, however, this unusual dependence cannot be connected with a toplogical transition in the Fermi surface, and can be attributed only to the reversal of the signs of the change of the volumes of the Fermi surface.

In conclusion, it is of interest to compare our results with those of Brandt and co-workers,[17,18,38] who subjected samples of bulky bismuth to complicated inhomogeneous deformations close to pure shear in the central part of the samples, in which nonequivalent changes of Fermi-surface volumes of equal sign were observed for the first time. The comparison can be only qualitative, since neither the type of deformation nor the orientation of the samples in our study agree with those in the cited studies. A comparison is possible only for strong anisotropic deformations. First, our results confirm the nonequivalent change (increase or deccrease) of the electron ellipsoids, which depends substantially on the tension direction. Second, just as in our study, in Refs. 18 and 38 they observed a topological transition of the Fermi surface of bismuth at approximately the same tension strain 0.6% along the bisector axis (the orientation of the samples of the first type is close to this direction).

There are also differences. Thus, in Refs. 17 and 38 it was established that all the Fermi-surface section areas for deformations of arbitrary sign in the basal plane change linearly with the deformation, and the shape of Fermi-surface ellipsoids remains unchanged (similar variation), while the total volume of the Fermi surface always increases. Nor do the directions of the ellipsoid axes change. None of this is observed in our case, probably because of the difference in the orientations, types, and magnitudes of the deformations. A more detailed comparison will be made possible only by development of a theory for the change of the energy spectrum of the bismuth electrons under strong anisotropic deformations.

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