nating-sign parasitic signals at the frequency ω_0 and $2\omega_0-\omega$, which were not integrated completely by the synchronous-detector filter and by the substraction circuit.

[1]A. Allen and J. H. Eberly, Optical Resonance and Two-Level Atoms, Wiley, 1975.
[2]S. G. Rautian and I. I. Sobel'man, Zh. Eksp. Teor. Fiz. **41**, 456 (1961); **44**, 934 (1963) [Sov. Phys. JETP **14**, 328 (1962); **17**, 635 (1964)].
[3]V. S. Letokhov and V. P. Chebotaev, Printsipy nelineĭnoĭ lazernoĭ spektroskopii (Principles of Nonlinear Laser Spectroscopy), Nauka, 1975.
[4]Ya. B. Zel'dovich, Usp. Fiz. Nauk **110**, 139 (1973) [Sov. Phys. Usp. **16**, 427 (1973)].
[5]B. R. Mollow, Phys. Rev. **A12**, 1919 (1975).
[6]F. Shuda, C. Stroud, and M. Hercher, J. Phys. **B7**, 198 (1974).
[7]E. B. Aleksandrov, A. M. Bonch-Bruevich, V. A. Khodovoi, and N. A. Chigir', Pis'ma Zh. Eksp. Teor. Fiz. **18**, 102 (1973) [JETP Lett. **18**, 58 (1973)].
[8]F. Y. Wu, S. Ezekiel, M. Ducloy, and B. R. Mollow, Phys. Rev. Lett. **38**, 1077 (1977).
[9]A. M. Bonch-Bruevich, V. A. Khodovoi, and N. A. Chigir', Zh. Eksp. Teor. Fiz. **67**, 2069 (1974) [Sov. Phys. JETP **40**, 1027 (1974)].
[10]A. M. Bonch-Bruevich, S. G. Przhibel'skiĭ, V. A. Khodovoi, and N. A. Chigir', Zh. Eksp. Teor. Fiz. **70**, 445 (1976) [Sov. Phys. JETP **43**, 230 (1976)].
[11]A. M. Bonch-Bruevich, V. A. Khodovoi, and N. A. Chigir', Opt. Spektrosk. **44**, 228 (1978).
[12]A. O. Melikyan, Zh. Eksp. Teor. Fiz. **68**, 1228 (1975) [Sov. Phys. JETP **41**, 610 (1975)].
[13]D. F. Zaretskiĭ and V. P. Krainov, Zh. Eksp. Teor. Fiz. **66**, 537 (1974) [Sov. Phys. JETP **39**, 257 (1974)].
[14]P. A. Apanasevich, Osnovy teorii vzaimodeĭstviya sveta s veshchestvom (Principles of the Theory of Interaction of Light with Matter), Nauka i tekhnika, Minsk, 1977.
[15]V. A. Kovarskiĭ, Mnogokvantovye perekhody (Multiquantum Transitions), Shtiintsa, Kishinev, 1974.
[16]A. M. Bonch-Bruevich, S. G. Przhibel'skii, V. A. Khodovoĭ, and N. A. Chigir', Abstracts, Seventh All-Union Conference on Coherent and Nonlinear Optics (Tashkent), Izd. MGU, 1974, p. 205.
[17]V. A. Yakubovich and V. M. Starzhinskiĭ, Lineĭnye differentsial'nye uravneniya s periodicheskimi koeffitsientami i ikh prilozheniya (Linear Differential Equations with Periodic Coefficients and Their Applications), Nauka, 1972.
[18]W. Wasow, Asymptotic Expansions for Ordinary Differential Equations, Wiley, 1966.
[19]E. B. Aleksandrov and A. P. Sokolov, Opt. Spektrosk. **27**, 896 (1969).

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Dispersion phenomena in the propagation of radiation in media with time-dependent refractive indices

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We consider the question of appearance of total internal reflection from a medium whose dielectric constant passes through zero and becomes negative within a time much longer than the period of the field. When the dielectric constant approaches zero the structure of the field inside the medium is determined mainly by dispersion effects. It is shown that the delay of the long-wave part of the radiation leads to formation of a slowly decreasing power-law tail in the radiation that had entered the medium in earlier instants of time.

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The propagation of radiation in nonstationary media has been extensively discussed recently (see e.g., the reviews[1,2]). Physical situations in which this question arises are quite varied. By way of examples we can cite measurements of the density of a nonstationary plasma by optical and microwave methods, the propagation of optical and infrared radiation in resonant media, passage of radio waves through the ionosphere and their reflection from it, propagation of microwaves in ferrite-filled waveguides, and others. The changes occurring in various parameters, such as the degree of plasma ionization, the resonant frequency of the medium, or the spin-wave frequency in a ferromagnet, cause the dielectric constant and the magnetic permeability of the medium to vary with time. In particular, changes in the refractive index can cause frequency shifts, reflection, changes of pulse waveforms, and others. These effects have found numerous applications in radio engineering and in optical and infrared laser technology.

To describe such phenomena theoretically we can use the fact that a rule the change of the parameters of the medium in one period of the electromagnetic field is small. This makes it possible to construct an adiabatic approximation[1,2] based on the inequality

$$\eta=\max(1/\Delta\omega T,\ 1/\omega T)\ll 1; \tag{1}$$

here T is the characteristic time of variation of the parameters of the medium, and $\Delta\omega$ is the characteristic scale of variation of the dielectric constant of the medium $\varepsilon(\omega)$. However, the adiabatic approximation developed in Refs. 1 and 2 has a rather limited range of validity. This is due to the use of perturbation theory and to expansion of the complex electric field in terms of the parameter η: $E=E^{(0)}+E^{(1)}+E^{(2)}+\dots$, where

$E^{(1)} \sim \eta E^{(0)}, E^{(2)} \sim \eta E^{(1)}, \ldots$[1,2] We note that when a complex field is expanded in a series it is necessary to expand both the amplitude and the phase of the field, and only terms that are small compared with unity can be discarded in the phase. In particular, at $t \ll \omega^2 T$ it suffices to retain in the phase the terms of first order of smallness in η.

We develop in this paper a consistent adiabatic approximation in media whose parameters vary slowly with time. By way of example of the use of the constructed theory, we investigate the reflection of monochromatic radiation from a transparent medium when its dielectric constant passes through zero at a certain instant of time and becomes negative.

The literature (see, e.g., Refs. 3–6) reports also another approach to problems of the propagation of radiation in a nondispersing medium whose refractive index has a prescribed time variation. Various model dependences $\varepsilon(t)$ that lead to an exact solution of the wave equation have been considered (linear,[3,4] quadratic,[4] harmonic,[5] and others). These included[6] the propagation of radiation in a nondispersing medium with arbitrary $\varepsilon(t)$ satisfying the inequality $\omega^{-1}\partial\sqrt{\varepsilon}/\partial t \ll 1$. It should be noted that the model of a nondispersing nonstationary medium has very limited applicability. In fact, in this model the instantaneous field frequency $\tilde{\omega}(t)$ is determined from the dispersion equation $\omega^2\varepsilon(t) = c^2k^2$, i.e., $\varepsilon(\omega, t)$ is considered at a certain fixed frequency ω_0. Comparing the solution of this equation $\tilde{\omega}(t) = ck\varepsilon^{-1/2}(\tilde{\omega}_0, t)c$ with the instantaneous frequency $\omega(t)$ obtained from the exact dispersion equation $\omega^2\varepsilon(\omega, t) = c^2k^2$ we easily obtain an inequality that ensures smallness of the field phase error

$$|\tilde{\omega}-\omega|t \sim \frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial\omega}\left|\left(\frac{\varepsilon(\tilde{\omega}_0, 0)}{\varepsilon(\tilde{\omega}_0, t)}\right)^{1/2}-1\right|\omega^2 t \ll 1. \tag{2}$$

The inequality (2) does not hold at $\omega t \gg 1$ if the dielectric constant changes by even a small amount, and also at all values of the time as $\varepsilon \to 0$.

We recall that a dielectric constant with a parametric time dependence can be introduced into a nonstationary medium only subject to the following inequality[1,2,7] [see (1)]

$$\varepsilon(\omega, t) \gg \eta, \tag{3}$$

which is violated when the dielectric constant ε tends to zero.

The indicated circumstances (neglect of dispersion and introduction of a dielectric constant with a parametric time dependence as $\varepsilon \to 0$) have led in the cited references to unphysical results, such as an electric-field energy that becomes infinite in all of space.

2. The propagation of radiation in nonstationary media is described by the following system of equations for the electric field E and induction D:

$$\partial^2 D/\partial t^2 = c^2\Delta E, \tag{4}$$

$$D = \int_{-\infty}^{t} \varepsilon(t-t', t)E(t')\,dt'; \tag{5}$$

here $\varepsilon(t-t', t)$ is the kernel of the dielectric-constant operator. As first noted by Pitaevskiĭ,[7] in the case of slow variation of the parameters of the medium this kernel has a weak finite-difference variation in terms of the parameter η. The dependence of ε on the time difference $t-t'$ is the same as in the stationary case, i.e., it has a characteristic time scale $\sim 1/\Delta\omega$, while the dependence on the second argument is determined by the character of the change of the parameters of the medium, i.e., it has a time scale $\sim T$. A solution of the system (4), (5) can be sought separately for each spatial Fourier harmonic. When expanding the integral equation (5) in powers of η, it must be remembered that when ε changes by about its own order of magnitude the instantaneous frequency of each spatial harmonic is changed, generally speaking, likewise by about its own order of magnitude, and it is therefore not convenient to seek a solution in a form close to a monochromatic wave. The solution of the system (4), (5) can be found in the form

$$(E, D) \sim (E_k(t), D_k(t))\exp\left[-i\int^{t}\Omega_k\,d\tau + ikx\right]. \tag{6}$$

Here E_k, Dk and Ω_k are slow functions of the time at $\eta \ll 1$.

To establish a connection between E_k and Dk with the aid of (5) we can use the fact that the main contribution to the integral in (5) is made by times t' that differ from t by $\sim 1/\Delta\omega$. The changes of $E_k(t)$ and $\Omega_k(t)$ during these times are small. Expanding $E_k(t)$ and $\Omega_k(t)$ in terms of the parameter η near $t' = t$, we obtain a differential connection between the induction and the field (for the calculations that follow it suffices to retain the terms of first order in η (Refs. 1 and 2):

$$D_k = \varepsilon E_k + i\frac{\partial\varepsilon}{\partial\Omega_k}\frac{\partial E_k}{\partial t} + \frac{i}{2}\frac{\partial\Omega_k}{\partial t}\frac{\partial^2\varepsilon}{\partial\Omega_k^2}E_k + \ldots \tag{7}$$

Expression (7) contains the still unknown running frequency $\Omega_k(t)$ and the spectral form of the dielectric-constant operator $\varepsilon(\Omega_k(t), t)$, which is defined in the standard manner[1,2,7]

$$\varepsilon(\omega, t) = \int_{-\infty}^{t} \varepsilon(t-t', t)\exp[i\omega(t-t')]\,dt'. \tag{8}$$

When solving the system (4), (7) in the adiabatic approximation, we take into account the fact that the dielectric constant of a slowly nonstationary medium $\varepsilon(\omega, t)$ can also be expressed as a series in the adiabatic parameter η:

$$\varepsilon(\omega, t) = \varepsilon^{(0)}(\omega, t) + \varepsilon^{(1)}(\omega, t) + \varepsilon^{(2)}(\omega, t) + \ldots, \tag{9}$$

where $\varepsilon^{(n)} \sim \eta\varepsilon^{(n-1)}$ and $\varepsilon^{(0)}(\omega, t)$ is the quasistationary dielectric constant obtained from the stationary one by substituting in the latter the running values of the parameters of the medium. Thus, $\varepsilon^{(0)}(\omega, t)$ is a universal function that does not depend on the mechanisms of the concrete processes that lead to changes of the parameters of the medium.

Pitaevskiĭ[7] obtained a general expression for the first-order correction

$$\varepsilon^{(1)}(\omega, t) = \frac{i}{2}\frac{\partial^2\varepsilon^{(0)}}{\partial\omega\partial t}. \tag{10}$$

We shall show at the end of the paper that for (10) to be valid it is necessary to satisfy, besides the already mentioned inequality (1) that ensures slowness of the variation of the macroscopic parameters, also some additional conditions. Thus, even the microscopic parameters must be slow enough. In the general case the corrections to the quasistationary dielectric constant must be obtained from the material equations of the medium. Of course, it is possible to construct an adiabatic approximation by starting from the material equations themselves without introducing the dielectric constant.

Substituting (7) and (9) in (4) and equating terms of like order in η, we obtain in the zeroth approximation the usual dispersion equation

$$\Omega_k^2(t)\varepsilon^{(0)}(\Omega_k(t),t)=c^2k^2, \tag{11}$$

which makes it possible to find the phase of the wave in accord with Eq. (6) with sufficient accuracy at $t \ll \omega T^2$. The phase correction obtained from Eq. (11), is of second order in η. From the first-approximation equation we can find the wave amplitude

$$\frac{\partial}{\partial\Omega_k}(\Omega_k^2\varepsilon^{(0)})\frac{\partial E_k}{\partial t}=\left[i\Omega_k^2\varepsilon^{(1)}-\varepsilon^{(0)}\frac{\partial\Omega_k}{\partial t}-2\Omega_k\frac{\partial\Omega_k}{\partial t}\frac{\partial\varepsilon^{(0)}}{\partial\Omega_k}\right.$$
$$\left.-2\Omega_k\left(\frac{\partial\varepsilon^{(0)}}{\partial t}\right)_{\Omega_k}-\frac{\Omega_k^2}{2}\frac{\partial\Omega_k}{\partial t}\frac{\partial^2\varepsilon^{(0)}}{\partial\Omega_k^2}\right]E_k. \tag{12}$$

Equation (11) was used in the literature[1,2] to find the instantaneous frequency of the wave. The amplitude of the wave, on the other hand, was obtained using additional arguments such as the conservation of the photon flux or the weak-dispersion approximation. There is actually no need to resort to additional arguments at all. From (12) we can obtain the field amplitude in a slowly nonstationary medium:

$$E_k=C\exp\left[-i\int^t\Omega_k d\tau+ikx\right]\exp\left\{\int^t d\tau\left[i\Omega_k^2\varepsilon^{(1)}-\varepsilon^{(0)}\frac{\partial\Omega_k}{\partial t}\right.\right. \tag{13}$$
$$\left.\left.-\frac{1}{2}\Omega_k^2\frac{\partial\Omega_k}{\partial t}\frac{\partial^2\varepsilon^{(0)}}{\partial\Omega_k^2}-2\Omega_k\left(\frac{\partial\varepsilon^{(0)}}{\partial t}\right)_{\Omega_k}-2\Omega_k\frac{\partial\Omega_k}{\partial t}\frac{\partial\varepsilon^{(0)}}{\partial\Omega_k}\right]\left(\frac{\partial(\varepsilon^{(0)}\Omega_k^2)}{\partial\Omega_k}\right)^{-1}\right\}.$$

The system (4), (5) has no turning points on the real t axis for any spatial Fourier harmonic, and the adiabatic solution (13) is valid at any instant of time. In problems with sources or with boundary conditions it suffices to choose a superposition of solutions (13) that satisfies the necessary additional requirements.

In the next part of the paper we examine the establishment of total internal reflection, using cold plasma as an example, but without specifying for the time being the nonstationarity mechanism.

3. In the case of a cold plasma

$$\varepsilon^{(0)}(\omega,t)=1-\omega_p^2(t)/\omega^2$$

and the electric field of a fixed spatial harmonic takes the form

$$E_k(t)=C\Omega_k^{-1/2}(t)\exp\left[-i\int^t\Omega_k d\tau+\frac{i}{2}\int^t\Omega_k\varepsilon^{(1)}d\tau+ikx\right], \tag{14}$$

where $\Omega_k^2(t)=c^2k^2+\omega_p^2(t)$ is the solution of the dispersion equation (11).

We consider now the problem of reflection of radiation from the boundary of the plasma, when the plasma dielectric constant goes through zero and becomes negative (Fig. 1). In this case the wave regime of the propagation at $t>0$ give way to cutoff of the light at $t<0$. Total internal reflection from the plasma boundary sets in. In the reflection problem we must satisfy the condition of continuity of the tangential components of the electric and magnetic fields on the plasma boundary. When a monochromatic wave is normally incident on the interface, the electric field inside the plasma and its derivative with respect to the coordinate are connected with the field of the incident wave $A\exp(-i\omega_0 t)$ by the Fresnel formulas (provided that $(1/\omega_0\varepsilon)\times\partial\varepsilon/\partial t \ll 1$):

$$E(t,0)=B(t)e^{-i\omega_0 t},\quad E_x'(t,0)=i\frac{\omega_0}{c}\varepsilon^{1/2}(t)B(t)e^{-i\omega_0 t}; \tag{15}$$

here $B(t)=2A/(1+\varepsilon^{1/2}(t)$, $\varepsilon(t)\equiv\varepsilon^{(0)}(\omega_0,t)$.

As $\varepsilon\to 0$ the condition $(1/\omega_0\varepsilon)\partial\varepsilon/\partial t\ll 1$ is violated. Expressions (15) for the fields $E(t,0)$ and $E_x'(t,0)$ are no longer valid. However, the (x,t)-plane region subject to the influence of the altered boundary conditions is quite small ($x\ll ct/\varepsilon^{1/2}(t)\lesssim\omega_0^{-1}$). To find the solution of Eqs. (4) and (5) in this (x,t) region it suffices to expand $\omega_p^2(t)$ in powers of $t\ll T$ near $t=0$. In the general case this expansion is of the form $\omega_p^2=\omega_0^2+\omega_p^2 t+\ldots$ Retaining only the expansion term linear in time, we can find the exact solution of the wave equation; for lack of space, it will not be written out here. An analysis of this solution shows that retention of $\varepsilon^{1/2}(t)$ in the expression for $B(t)$ is in this case an exaggeration of the accuracy. The correct boundary conditions at $(1/\omega\varepsilon)\partial\varepsilon/\partial t\gtrsim 1$ are

$$E(t,0)=2Ae^{-i\omega_0 t},\quad E_x'(t,0)=0. \tag{16}$$

The regions where expressions (15) and (16) hold overlap.

Using the boundary conditions, we obtain an inhomogeneous equation for each Fourier-Laplace harmonic E_k

$$\frac{\partial^2 D_k}{\partial t^2}+c^2k^2E_k=-ic^2[k+\tilde{k}(t)]B(t)e^{-i\omega_0 t}, \tag{17}$$

where

$$\tilde{k}(t)=\frac{\omega_0}{c}\sqrt{\varepsilon^{(0)}(\omega_0,t)},$$

and $B(t)$ is determined by (15) and (16). We note that since the boundary conditions exert an influence during a short time $t\sim\omega_0^{-1}$, it suffices to use the expression (14) for $B(t)$ in the calculation of the field at $t\gg T$. This approximation results in a negligibly small error in the field amplitude at $\eta\ll 1$.

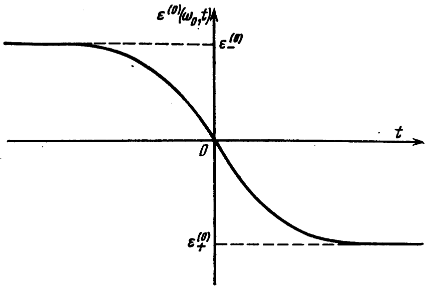

FIG. 1. Time dependence of the dielectric constant.

The system (5), (14) can be solved by variation of the constant in the solution (14) obtained above for the homogeneous system of equations (4) and (5). The final result for the field is

$$E(x,t)=\frac{c^2}{4\pi}\int_{i\sigma-\infty}^{i\sigma+\infty} dk\,\Omega_k^{3/2}(t)\,e^{ikx}$$

$$\times\int_{-\infty}^{t} dt'\,\frac{B(t')[k+\tilde{k}(t')]}{\Omega_k^{5/2}(t')}\exp\left[-i\omega_0 t+i\int_t^{t'}\Omega_k d\tau-\frac{i}{2}\int_t^{t'}\Omega_k\varepsilon^{(1)}d\tau\right], \qquad (18)$$

where $\sigma<0$. The integral with respect to time in (18) diverges formally at the lower limit, since an infinite increment of the phase of the wave accumulates over an infinite time interval. To simplify the exposition, we shall assume that $B(t)\to 0$ as $t\to\pm\infty$ and that the decrease of $B(t)$ takes place over times much longer than all the characteristic times of the problems. To facilitate the investigation of the field at $t\gg T$, we rewrite (18) in the form

$$E(x,t)=\frac{1}{2\pi}\int_{i\sigma-\infty}^{i\sigma+\infty} dk\,F_k(t)\,e^{ikx}+B_+\exp[-i\omega_0 t-|k_+|x]; \qquad (19)$$

the function $F_k(t)$ is defined here by the relation

$$F_k(t)=\frac{c^2(\Omega_k^+)^{3/2}}{2}\int_{-\infty}^{\infty} dt'\,\frac{B(t')[k+\tilde{k}(t')]}{\Omega_k^{5/2}(t')}\exp\left[-i\omega_0 t'\right.$$

$$\left.+i\int_t^{t'}\Omega_k d\tau-\frac{i}{2}\int_t^{t'}\Omega_k\varepsilon^{(1)}d\tau\right], \qquad (20)$$

and the indices ± label here and below quantities taken at the respective instants of time $t=\pm\infty$.

The investigation of the integral in (20) is aided by a simple mechanical analogy: $F_k(t)$ is the amplitude of the natural oscillations of a mathematical pendulum with variable frequency $\Omega_k(t)$, set in motion by a monochromatic force of frequency ω_0. The principal contribution to $F_k(t)$ is made by the resonance regions $(\Omega_k(t)\approx\omega_0)$, inasmuch as at large values of the detuning $\Omega_k(t)-\omega_0$ the integrand oscillates strongly. The function $F_k(t)$ can be calculated in explicit form for those oscillators $\Omega_k(t)$ which pass relatively rapidly through the resonance region, as well as for oscillators whose frequency as $t\to-\infty$ is close to the frequency of the driving force. Using the first case the stationary phase method, we obtain the following expression for $F_k(t)$ at $\delta k=k-k_-<0$ and $|\delta k|\gg 1/cT$

$$F_k(t)=2A\left(\frac{\Omega_k^+}{\omega_0}\right)^{3/2}\frac{c^2k}{\omega_0+ck}\left(\frac{2\pi}{\dot{\Omega}_k(t_k^*)}\right)^{1/2}$$

$$\times\exp\left[-i\omega_0 t_k^*+i\int_t^{t_k^*}\Omega_k\,d\tau-\frac{i}{2}\int_t^{t_k^*}\Omega_k\varepsilon^{(1)}\,d\tau+i\frac{\pi}{4}\right]; \qquad (21)$$

the stationary phase point t_k^* is here a root of the equation $\Omega_k(t_k^*)=\omega_0$.

If $\delta k\gg 1/cT$ but $\delta k>0$, the function $F_k(t)$ is exponentially small. Its value is determined by that singularity of the integrand in (20) which is closest to the real axis.

In the second case, at $|\delta k|\ll 1/cT$, the main contribution to the integral in (20) is made by negative instants of time $|t'|\gg T$ and the nonstationary character of the medium manifests itself only in the phase of the function $F_k(t)$:

$$F_k(t)=\frac{B_-}{i(k-k_-)}\left(\frac{\Omega_{k_-}^+}{\omega_0}\right)^{3/2}\exp\left[-i\Omega_k^- t-i\int_{-\infty}^{t}\Delta_k d\tau+\frac{i}{2}\int_{-\infty}^{t}\Omega_k\varepsilon^{(1)}\,d\tau\right]; \qquad (22)$$

here $\Delta_k(t)=\Omega_k(t)-\Omega_k^-$, $k_-=\tilde{k}(-\infty)$.

We proceed now to calculate the field $E(x,t)$. Different groups of spatial frequencies correspond to definite regions (x,t). The field can be calculated in those regions which correspond to wave packets with $1/cT<k<k_-$ and packets with wave vector near the $F_k(t)$, pole located on the real axis at the point $k=k_-$. The pole of $F_k(t)$ corresponds to wave packets that entered the medium before its dielectric constant changed substantially. Using the stationary-phase method, we can describe the propagation of that part of the momentum which is made up of wave packets with frequencies from $k\gg 1/cT$ to k not too close to k_-, namely $|\delta k|\gg 1/cT$. At

$$V_{k_-}^+ t/(\omega_0 t)^{1/2}\ll x<V_{k_-}^+ t,\quad V_{k_-}^+ t-x\gg V_{k_-}^+ T\quad (V_{k_-}^+=(\partial\Omega_k^+/\partial k)_{k=k_-})$$

the expression for the field (18) takes the form

$$E(x,t)=2A\frac{c^2k_0}{\omega_0+ck_0}\left(\frac{\Omega_{k_0}^+}{\omega_0}\right)^{3/2}\left[\frac{\partial\Omega_{k_0}}{\partial t}(t_{k_0}^*)\left(\frac{\partial}{\partial k}\int_{t_k^*}^{t}V_k\,d\tau\right)_{k=k_0}\right]^{-1/2}$$

$$\times\exp\left[ik_0x-i\omega_0 t_{k_0}^*+i\int_t^{t_{k_0}^*}\Omega_{k_0}\,d\tau-\frac{i}{2}\int_t^{t_{k_0}^*}\Omega_{k_0}\varepsilon^{(1)}d\tau\right]. \qquad (23)$$

The stationary point k_0 is here the root of the equation

$$\int_{t_{k_0}^*}^{t} V_{k_0}d\tau=x,\quad V_{k_0}(\tau)=\left(\frac{\partial\Omega_k}{\partial k}(\tau)\right)_{k=k_0}.$$

Expression (23) describes the change of the field inside the plasma, from values of the order of $\sim A(T/\omega_0 t^2)^{1/2}$ on a distance of the order of $x\sim V_{k_-}^+ t/(\omega_0 t)^{1/2}$ from the boundary to values of the order of A reached atx close to $V_{k_-}^+ t$. Expression (23) becomes much simpler in two intermediate regions. Located at

$$V_{k_-}^+ t/(\omega_0 t)^{1/2}\ll x\ll V_{k_-}^+ t$$

is the long-wave part of the pulse, which is described by the universal expression

$$|E|=2A\left(\frac{\omega_p^+}{\omega_0}\right)^3\frac{\omega_0 x/c}{(\omega_0 t)^{3/2}}\frac{\omega_0}{(\dot{\omega}_p(0))^{1/2}}\exp\left(-\frac{1}{2}\,\mathrm{Im}\int_0^t\Omega_k\varepsilon^{(1)}\,d\tau\right). \qquad (24)$$

It is of interest to note that the fall-off of the field in the tail part of the pulse is a power-law function of both the time and the coordinate. The field fall-off $E\propto x/t^{3/2}$ is the same for different functions $\varepsilon^{(0)}(\omega_0,t)$ on going through zero.

At $x\lesssim V_{k_-}^+ t$ the field increase is faster than linear, and in the region

$$V_{k_-}^+ T\ll V_{k_-}^+ t-x\ll V_{k_-}^+ t$$

it is described by the following asymptotic expression:

$$|E|=\frac{2A}{\omega_0+ck_-}\left(\frac{\Omega_{k_-}^+}{\omega_0}\right)^{3/2}\left(\frac{c^2\omega_0 Tk_-}{V_{k_-}^+ t-x}\right)^{1/2}\exp\left(-\frac{1}{2}\,\mathrm{Im}\int_{-\infty}^{t}\Omega_k\varepsilon^{(1)}\,d\tau\right). \qquad (25)$$

The wave numbers of this part of the momentum are close to k_-, and the field amplitude reaches a value on the order of A.

The radiation that has penetrated into the plasma before its dielectric constant had changed substantially occupies the region

$$x>V_{k_-}^+t,\quad x-V_{k_-}^+t\gg V_{k_-}^+T.$$

The expression for the field in this region is determined by the pole part of $F_k(t)$ and is of the following form

$$E=B_-\left(\frac{\Omega_{k_-}^+}{\omega_0}\right)^{3/2}\exp\left[ik_-x-i\omega_0t-i\int_{-\infty}^{t}\Delta_{k_-}d\tau+\frac{i}{2}\int_{-\infty}^{t}\Omega_{k_-}\varepsilon^{(1)}d\tau\right].\quad(26)$$

Since the regions of applicability of expressions (23) and (26) do not overlap, it is impossible to calculate the field in the intermediate region $(x-V_{k_-}^+t)\sim V_{k_-}^+T$. The field in this region is of the order of A, and its form depends substantially on the character of the variation of the plasma frequency with time. The distribution of the field amplitude inside the plasma at $t\gg T$ is shown in Fig. 2. The qualitative picture of field formation in the medium can be visualized by considering the propagation of wave packets with different spatial frequencies. The wave number of the packet depends on the time of entry into the plasma and changes from $k=k_-$ as $t\to-\infty$ to $k=0$ at $t=0$. At $t<0$ the dielectric constant becomes negative and the radiation no longer enters the plasma. At a given instant $t\gg T$ the spatial frequencies of the wave packets increase with increasing distance from the plasma boundary. The wave packets produced at $|t|\ll T$ have small wave numbers and, according to the dispersion equation (11), propagate slowly into the interior of the plasma. The bulk of the radiation that had entered the plasma at $|t|\gg T$ consists of wave packets with $k\approx k_-$ and propagates with a velocity $\sim V_{k_-}^+$, greatly outstripping the long-wave packets. Thus, the long-wave packets lag in time, by ever increasing distance, the bulk of the radiation, and forms a decreasing power-law tail.

For radiation propagating in a real plasma with $\varepsilon''\neq0$ the frequency $\Omega_k(t)=\Omega_k'(t)+i\Omega_k''(t)$ is a complex quantity. In the case

$$\Omega_k''\ll\Omega_k',\quad(\Omega_k')^{-1}(\Omega_k'')^2t\ll1,\quad\Omega_k''T\lesssim1$$

it is possible to take the energy dissipation into account. To this end it suffices to replace $\Omega_k(t)$ in the phases of expressions (14) and (23)–(26), which describe the field in a nonstationary medium, by $\Omega_k'+i\Omega_k''$.

The expressions derived above enable us to describe the appearance of total internal reflection from an arbitrary transparent medium, particularly from a resonant gas. It suffices for this purpose to substitute in (14) and (23)–(26) the frequency $\Omega_k(t)$ obtained from the dispersion equation (11) with $\varepsilon(\omega_0,t)$, corresponding

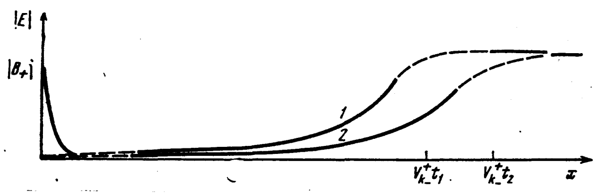

FIG. 2. Dependence of the electric field intensity on the coordinate at $t\gg T$: 1—at the instant of time t_1; 2—at the instant of time $t_2\,(t_2>t_1)$.

to the considered medium. The asymptotic form of the field (24), based on the approximate solution of the dispersion equation at small k, also remains valid, for as $\varepsilon(\omega_0,t)\to0$ the dispersion equation for waves with frequencies $\omega\approx\omega_0$ takes the universal form

$$\omega_0^2\frac{\partial\varepsilon}{\partial\omega}(\omega-\omega_0)=c^2k^2,$$

which coincides with the dispersion equation for a plasma as $\omega\to\omega_p$.

4. To conclude the paper, we consider some of the simplest models of the change of the degree of ionization of a plasma, models that permit calculation of the increment $\varepsilon^{(1)}(\omega,t)$ to the quasistationary dielectric constant. The density of the electronic component of the plasma, which determines the high-frequency dielectric constant, can be altered by ionization, recombination, sticking to neutral particles, plasma motion, etc. These processes change not only the number of electrons, but also their distribution function. The change of the polarization of the medium is connected both with the change of the electron density and with the change of the electron velocity distribution function. It is important here whether the distribution function of the newly produced electrons is capable of adjusting itself to the matched oscillations of the field and of the already present electrons.

In the case of rapid ionization, when the ionization time is much shorter than the period of the field oscillations in the medium, the phase of the oscillations of the newly produced electrons is not equal to the phase of the field. Therefore the initial conditions for the equation of motion of these electrons in the wave field

$$\frac{d^2x}{dt^2}=-\frac{e}{m}Ee^{-i\omega_0t}\quad(27)$$

reduce to zero conditions after averaging over the ensemble of the electrons produced at the instant $t=t_0$:

$$\langle x(t_0)\rangle=\left\langle\frac{dx}{dt}(t_0)\right\rangle=0$$

(x is the coordinate of the electron). The induction D is connected with the electric field E by the relation

$$D=4\pi e\int_{-\infty}^{t}\langle x(t,t_0)\rangle\frac{dn}{dt}(t_0)\,dt_0+E;\quad(28)$$

where dn/dt is the rate of production of electrons at the instant of time t_0. Equation (27) has a solution satisfying the zero boundary conditions, in the form

$$\langle x(t,t_0)\rangle=\frac{eE}{m\omega_0^2}(e^{-i\omega_0t}-e^{-i\omega_0t_0})+\frac{ieE(t-t_0)}{m\omega_0}e^{-i\omega_0t_0}.\quad(29)$$

After substituting (29) in (28) we obtain with the aid of (5) and (8)

$$\varepsilon(\omega_0,t)=\varepsilon^{(0)}(\omega_0,t)+i\frac{\partial^2\varepsilon^{(0)}}{\partial\omega_0\partial t},\quad(30)$$

where $\varepsilon^{(0)}(\omega_0,t)=1-\omega_p^2/\omega_0^2$. The increment to the quasi-stationary dielectric constant

$$\varepsilon^{(1)}=i\partial^2\varepsilon^{(0)}/\partial\omega_0\partial t,\quad(31)$$

is in this case double the increment (10) calculated in Ref. 7.

In the case of slow ionization (the ionization time is much longer than the period of the field), the electrons manage to adjust themselves in the course of their production to the phase of the field in the medium. From the point of view of the contribution to the dielectric constant, this ionization process admits of a model-dependent description in which it is assumed that any parameter in the electron equation of motion (mass, charge, natural frequency) varies smoothly with time. We assume for example, that the electron mass changes, starting with infinity, and reaches a value m near $t=t_0$, and the change of mass per oscillation period of the field is small. It is easy to show that in this case the first-order increment to the dielectric constant coincides exactly with (10).

It is easy to show, using similar models, that in the case of fast recombination

$$\langle x(t,t_0)\rangle=\frac{eE}{m\omega_0^2}[1-\theta(t-t_0)]e^{-i\omega_0 t}$$

(here $\theta(t)=1$ at $t>0$ and $\theta(t)=0$ at $t<0$) and the increment to the quasistationary dielectric constant is zero. In the case of slow recombination $\varepsilon^{(1)}(\omega_0, t)$ also coincides with (10). The difference of $\varepsilon^{(1)}(\omega_0, t)$ from zero in the case of fast recombination was obtained in Refs. 1 and 2 as a result of an error in the differentiation of the current with respect to time.

Thus, depending on the character of the microprocess that leads to the change of the parameters of the medium, the increments to the quasistationary dielectric constant can differ substantially.

It is of interest to examine the following physical situation. Let a stationary degree of ionization be attained in the medium as a result of competition between simultaneous ionization and recombination. Assume that one of these processes is fast and the other slow compared with the period of the field propagating in the medium. In this case the quasistationary dielectric constant does not depend on the time. Nonetheless, since the increments $\varepsilon^{(1)}$ due to various microprocesses do not cancel each other, the field in the macroscopically stationary medium will change as the wave propagates. The cause of this circumstance is the irreversible energy exchange between the field and the medium in the presence of the fast processes. On the other hand if the microprocesses that lead to the nonstationarity of the medium are slow compared with the times of variation of the field in the medium, there is no irreversible energy exchange between the field in the medium. This physical situation corresponds to the analysis of Pitaevskii,[7] and the increment to the dielectric constant can in this case be obtained from the general formula (10). In the case of slow ionization and recombination, an adiabatic invariant is conserved, namely the number of quanta of the propagating radiation $N\propto\alpha(|E_k|^2/\Omega_k)\cdot\partial(\varepsilon^{(0)}\Omega_k^2)/\partial\Omega_k^2$.

In the case of propagation of one spatial harmonic, the amplitude of the electric field varies in the models considered above in the following manner:

$$|E_k|\propto\begin{cases}\Omega_k^{1/2}(t) & \text{—slow ionization and recombination}\\ \Omega_k^{3/2}(t) & \text{—fast recombination}\\ \Omega_k^{-1/2}(t) & \text{—fast ionization.}\end{cases}$$

The expressions obtained here for $\varepsilon^{(1)}(\omega, t)$, together with Eqs. (23)–(26), solve completely the problem of total internal reflection from the plasma.

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[1]Ju. A. Kravtsov, L. A. Ostrovskiy, and N. S. Stepanov, Proc. IEEE 62, 1492 (1974).
[2]L. A. Ostrovskiĭ and N. S. Stepanov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 14, 489 (1974).
[3]G. A. Askar'yan and V. A. Pogosyan, Zh. Eksp. Teor. Fiz. 65, 117 (1973) [Sov. Phys. JETP 38, 58 (1974)].
[4]S. I. Averkov and Yu. T. Khronopulo, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 5, 818 (1960).
[5]V. N. Lugovoy, Opt. Commun. 19, 182 (1976).
[6]S. N. Stolyarov, Kratk. Soobshch. Fiz. (FIAN) 3, 25 (1974).
[7]L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. 39, 1450 (1960) [Sov. Phys. JETP 12, 1008 (1961)].

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