

# Nonlinear selection of optical radiation on reflection from a stimulated Mandel'shtam-Brillouin scattering mirror

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It is shown that in stimulated Mandel'shtam-Brillouin scattering (SMBS) multimode and multifrequency pumping in a light guide it is possible to realize conditions wherein the Stokes radiation consists mainly of the component  $E_s \sim e_{k_0}(\eta) \mathcal{E}_{k_0}^*(\mathbf{r}_1, z)$  which produces only one component, greatly differing in power from  $e_{k_0}(\eta) \mathcal{E}_{k_0}(\mathbf{r}_1, z)$  in the expansion of the pump field  $E_L(\eta, \mathbf{r}_1)$  in a product of the orthogonal (with respect to time and space coordinates) functions  $e_k(\eta)$  and  $\mathcal{E}_k(\mathbf{r}_1, z)$  where  $\eta = t - z/v_L$ . Components with smaller amplitudes are hardly scattered at all, even if their total power at the entrance to the nonlinear medium considerably exceeds the power of the component  $e_{k_0} \mathcal{E}_{k_0}$ . The results yield, in particular, the maximum signal power limited by quantum or thermal noise). Below this value there is no wave front inversion with simultaneous production of the time modulation of the radiation.

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## 1. INTRODUCTION

Interest has increased recently in the study of the physical processes leading to the inversion of a wave front (WFI) of electromagnetic radiation incident on a nonlinear medium.<sup>1-6</sup> Reflection with WFI means that the following relations should hold between the fields of the incident  $E_L$  and reflected  $E_s$  waves in the range of frequencies characteristic or some specific problem:  $E_L = \hat{R} E_s^*$ , where  $\hat{R}$  is the reflection coefficient, which in the general case is a nonlinear operator dependent on the field  $E_L$  or some other fields if there are such in the system.

One of the most important questions connected with the study of WFI processes lies in the finding of the conditions upon whose satisfaction reflection is either total or only partial. Correspondingly,  $R$  is either a constant number or an operator. It is obvious that in the latter case we can speak of selective reflection with WFF while if  $\hat{R}$  does not depend on  $E_L$  or depends only very weakly, then the selection is linear, as in the case, for example, with 3- and 4-photon parametric transformation of the wave  $E_L$  into the wave  $E_s$  in a given pump field (in such a transformation, the reflection coefficient has a resonant dependence on the frequency).<sup>2-4, 6</sup> If the operator  $\hat{R}$  depends on  $E_L$ , then the selection is nonlinear; a characteristic example is a mirror based on stimulated Mandel'shtam-Brillouin scattering (SMBS),<sup>1</sup> reflection from which is possible only at a sufficiently high power level in the incident wave, which is usually higher the broader its spatial or temporal spectrum. The property of nonlinear selection allows appreciable damping of the noise component in radiation reflected with complex-conjugate phase, even in the case in which its power in the arriving radiation is significantly higher than the power of the simple component.<sup>7</sup>

However, the following question remains unclear to date: what is the algorithm of selection by the SMBS mirror of the signals incident on it, i. e., what component of a complicated (in the general case, arbitrary) wave  $E_L$  can be reflected with WFI, and what part can be filtered out. Besides the theoretical interest in the

development of an approach to the solution of the problem of stimulated scattering of an arbitrarily spatially inhomogeneous, complicated wave, this problem also has an important physical aspect, relating to the determination of the maximum achievable (limited by quantum or thermal noise) level of the signal, which can still be reproduced with WFI.

In the present work, it is shown that conditions can be realized in SMBS of multimode and multifrequency pumping in a light guide under which the Stokes radiation will consist basically of only one component  $E_s \sim e_{k_0}(\eta) \mathcal{E}_{k_0}^*(\mathbf{r}_1, z)$ , which produces only a single component  $e_{k_0}(\eta) \mathcal{E}_{k_0}(\mathbf{r}_1, z)$  of distinctly higher power, in the expansion of the field

$$E_L = \sum_{k=0}^N e_k(\eta) \mathcal{E}_k(\mathbf{r}_1, z)$$

in products of the orthogonal (with respect to time and space coordinates) functions  $e_k(\eta)$  and  $\mathcal{E}_k(\mathbf{r}_1, z)$ , where  $\eta = t - z/v_L$ .

Components with smaller amplitudes are hardly scattered, even if their total power at the entrance to the nonlinear medium exceeds the power of the component  $e_{k_0} \mathcal{E}_{k_0}$  considerably. The results enable us to estimate the minimum signal power below which WFF with simultaneous production of the time modulation of the radiation is not achieved.

## 2. INITIAL EQUATIONS

We assume that the pump radiation does not change under the action of the scattered field and, on the boundary of the medium  $z=0$ , consists of a set of several components  $e_k(t) \mathcal{E}_k(\mathbf{r}_1, 0)$ , which are orthogonal in the space coordinates. Neglecting the dispersion of the group velocities of the various pump components, we can write, at  $z > 0$ ,

$$E_L = \sum_{k=0}^N e_k(\eta) \mathcal{E}_k(\mathbf{r}_1, z),$$

where all the  $\mathcal{E}_k$  satisfy the equation

$$(\partial/\partial z + i/2k_L \Delta_L) \mathcal{E}_k = 0$$

and

$$\int \mathcal{E}_i \mathcal{E}_i^* d^2 r_{\perp} = S \delta_{ij}$$

$S$  is the transverse cross section of the beam,  $k_L$  is the wave vector of the pump. We shall find how production of the field  $E_L$  can take place in the Stokes radiation, if  $\mathcal{E}_i$  are independent random functions of  $r_{\perp}$  with small transverse correlation radii, normal distribution laws, and flat envelopes ( $|\mathcal{E}_i^2| = 1$ ). A wave of the form  $E_L$  can be formed in the experiment, for example, if a set of several light beams with small divergences  $\theta_i$  are directed at angles  $\theta_{ij}$  with respect to one another ( $\theta_{ij} > \theta_i, \theta_j$ ) onto a phase plate, which increases the divergence of these beams to a value  $\theta \gg \bar{\theta}_{ij}$ , and then its image is transferred the end of the light guide.<sup>8</sup> It is obvious, in order that the property of orthogonality of the field  $\mathcal{E}_i$  be satisfied, that the number of initial beams  $N$  must be smaller than  $\theta^2/\bar{\theta}_{ij}^2$ , and therefore smaller than  $\theta^2/\bar{\theta}_i^2$ . The radiation thus formed will be multimode at any instant of time and will have a divergence  $\sim \theta$ .

In contrast to Refs. 9, 10, where the expansion of the Stokes field, as applied to the problem of stationary SMBS in a light guide, was carried out in a set of plane waves, in the case considered, for the study of WFF multimode and multifrequency pumping, it is more convenient to make use immediately of an expansion in orthogonal components of the complex-conjugate the laser field itself. In correspondence with this, the field  $E_s$  entering into the system of truncated equations

$$\left( v \frac{\partial}{\partial \eta} + \frac{\partial}{\partial z} + \frac{i}{2k_s} \Delta_{\perp} \right) E_s = i g_s E_L P^*, \quad v = \frac{1}{v_L} + \frac{1}{v_s}; \quad (1)$$

$$\partial P / \partial \eta + \gamma P = i g_s E_s E_s^*, \quad (2)$$

which describe the SMBS, is sought at  $k_s = k_L$  in the form

$$E_s = \sum_{k=0}^N c_k(\eta, z) \mathcal{E}_k^*(r_{\perp}, z) + E_s, \quad (3)$$

where  $\tilde{E}_s$  is a field, orthogonal to all the preferred components  $\mathcal{E}_k^*$ . The function  $\tilde{E}_s$  describes the space-time change of the uncorrelated (with respect to the transverse coordinates) pump waves, and also the change in the so-called noise (in the terminology of three-dimensional holography) waves<sup>9</sup> excited upon scattering of any of the pump components  $e_k \mathcal{E}_k$  by the hypersound lattice created by the other two independent waves  $e_l \mathcal{E}_l$  and  $c_m \mathcal{E}_m^*$  ( $k \neq l, k \neq m$ ).

The component  $\tilde{E}_s$  will not affect the amplification of the components  $c_k \mathcal{E}_k^*$  producing the pump if the total increment

$$M_{\text{corr}} = g z_{\text{corr}} e, \quad g = 2g_s g_z / \gamma, \quad e = \sum_{i=0}^N e_i, \quad e_i = \overline{|c_i|^2}$$

over the longitudinal correlation distance  $z_{\text{corr}} = 1/k_s \theta^2$  is small in comparison with unity (in the nonstationary case  $M_{\text{corr}} = 2(2g_s \gamma \eta z_{\text{corr}} \epsilon)^{1/2}$ ). Then the contribution of the component  $\tilde{E}_s$  to the amplification of the waves  $c_k \mathcal{E}_k^*$  is equivalent to the contribution from an additional delta-correlation source and the intensity of "noise" waves generated by it is small in comparison with the total intensity  $\sigma$  of the Stokes components

$$\sigma = \sum_{i=0}^N \sigma_i, \quad \sigma_i = \overline{|c_i|^2}$$

Nevertheless, at  $M_{\text{corr}} \sim 1$  the integrated contribution from the "noise" waves can be comparable with the contribution from individual weakest Stokes components  $c_k \mathcal{E}_k^*$ , which produce the least intense pump components  $e_k \mathcal{E}_k$ . However, this cannot change the experimental recording of the corresponding Stokes components separately, if their initial divergence  $\theta_i$  is sufficiently small in comparison with  $\theta$ . In fact, the integrated intensity of the noise in Stokes radiation  $\sim M_{\text{corr}} \sigma$  (it is assumed that the intensity of the components uncorrelated with the pump does not exceed  $\sigma$ ) and in a solid angle  $\sim \theta_i^2$  this intensity will be  $\sim M_{\text{corr}} \sigma_i^2 / \theta^2$ .

If this value is small in comparison with the intensity  $\sigma_i$  of the Stokes wave that produces the components  $e_i \mathcal{E}_i$ , i. e.,

$$(\theta_i^2 / \theta^2) M_{\text{corr}} \sigma \ll \sigma_i, \quad (4)$$

then the effect of  $\tilde{E}_s$  on the components  $c_k \mathcal{E}_k^*$  is unimportant. Thus, at  $\theta_i^2 / \theta^2 \ll 1$ , it is not separately required, for the estimate of the intensity of the weak components  $c_k \mathcal{E}_k^*$ , that the parameter  $M_{\text{corr}}$  be small in comparison with the ratio  $\sigma_i / \sigma$  and only the inequality (4) is needed.

As for the amplification of the components entering into  $\tilde{E}_s$  that are uncorrelated with the pump, at  $M_{\text{corr}} \ll 1$  they will be amplified independently, without showing any effect on the pump-producing waves  $c_k \mathcal{E}_k^*$ .<sup>11</sup> These components, if their intensity appreciably exceeds  $\sigma$ , can rise only to the noise level, but even in this case the noise remains small in comparison with the power of the components uncorrelated with the pump. We shall consider the amplification of similar components below in greater detail.

Substituting  $E_L$  and  $E_s$  in Eqs. (1) and (2), and neglecting the contribution of  $\tilde{E}_s$  to the amplification of the waves  $c_k \mathcal{E}_k^*$  at  $M_{\text{corr}} \ll 1$  with account of the orthogonality and the normal distribution law of the fields  $\mathcal{E}_i$ , we obtain

$$v \frac{\partial c_i}{\partial \eta} + \frac{\partial c_i}{\partial z} = -g_s g_z \sum_{k=0}^N e_k(\eta) \int_{-\infty}^{\eta} d\eta' e^{-i(\eta-\eta')} \{ e_i^*(\eta') c_k(z, \eta') + e_k^*(\eta') c_i(z, \eta) \}. \quad (5)$$

The approximations used in the derivation of Eqs. (5) and the procedure of derivation are analogous to the corresponding approximations and procedure customarily employed in the investigation of the nonlinear interaction of waves in the framework of weak turbulence theory.<sup>12</sup> However, the basis functions  $\mathcal{E}_k$  here are expressed in such a way that they all satisfy the free quasi-optical equation and, what is most important, the expansion of the Stokes field is carried out in the problem that we have investigated only over those functions  $\mathcal{E}_k$  which at any instant of time are present in the expansion of  $E_L$  on the entry boundary of the medium with a sufficiently large weight. It should be noted that Eqs. (5) preserve their form even with account of damping of the pump because of SMBS; the coefficients  $e_i$  will then be functions of  $\eta$  and  $z$  and will satisfy an

equation of the same type as (5).

In the next two sections, 3, and 4, we shall consider in succession the amplification of the producing components  $c_i \mathcal{E}_i (i \leq N)$  and those uncorrelated with the pump  $c_j \mathcal{E}_j (j > N)$ .

### 3. AMPLIFICATION OF THE STOKES COMPONENTS PRODUCING THE PUMP

In the quasi-stationary case, the system (5) reduces to

$$dc_i/dz = -g \left( e_i^* \sum_{k=0}^N e_k c_k + c_i \varepsilon \right). \quad (6)$$

At sufficiently large values of the coordinate  $L-z$ , its solution depends weakly on the boundary  $L$  and tends to the values  $c_i = A(z) e_i^*$ , where

$$A(z) = A(L) \exp [2g\varepsilon(L-z)].$$

Since  $c_i \sim e_i^*$ , the reflection coefficients of all the pump components are identical.<sup>9,10</sup>

However, in broadband pumping, the reflection coefficients can be different even under conditions in which the characteristic length of the amplification is less than the time coherence length  $1/\Delta\omega\nu$ , where  $\Delta\omega$  is the width of the pump spectrum. In this case, the mismatch of the group velocities is insignificant and we can set  $\nu = 0$ .<sup>13</sup>

We choose the basis  $\mathcal{E}_k$  such that the functions  $e_k$  are mutually orthogonal:

$$\int_{\tau} e_k(\eta) e_i^*(\eta) d\eta = W_k \delta_{ki}.$$

It was shown in Refs. 14 and 15 how such an expansion can be constructed for a given function  $E_L(r, z, \eta)$ . Since the uniqueness of the chosen expansion is important for the questions considered in the given work, the corresponding proof is given specially in the Appendix.

We first consider the SMBS under conditions in which the intensity of one of the components  $e_0 \mathcal{E}_0 (k_0 = 0)$  significantly exceeds the intensity of each of the remaining components  $e_k \mathcal{E}_k (k = 1, 2, \dots, N)$ , but can be less than the total intensity  $\varepsilon$  of the weaker waves. We assume that the scale  $T$  over which the functions  $e_k(\eta)$  are orthogonal is less than or of the order of the relaxation time  $\tau = 1/\gamma$ . If  $\Delta\omega$  is the characteristic width of the lines, within the limits of which are concentrated the spectra of all the signals  $e_k$ , then the number  $N$  will obviously be bounded from above by the value  $\Delta\omega T$  (it is assumed that  $\Delta\omega T \ll \theta^2/\theta_d^2$ , where  $\theta_d \approx 1/k_s S^{1/2}$  is the diffraction divergence).

As the zeroth approximation in the solution of the system (5), we assume  $c_k^{(0)} = c_0^{(0)} \delta_{k0}$ , where  $c_0^{(0)}$  is given by the equation

$$\partial c_0^{(0)}/\partial z = -2g_1 g_2 e_0(\eta) \int_{-\infty}^{\eta} e_0^*(\eta') c_0^{(0)}(z, \eta') e^{-\tau(\eta-\eta')} d\eta', \quad (7)$$

obtained from (5) upon neglect of all the remaining  $c_k (k = 1, \dots, N)$ . Then, in first approximation for  $c_k^{(1)} (k = 1, \dots, N)$ , we obtain the equation

$$\begin{aligned} \partial c_i^{(1)}/\partial z = & -g_1 g_2 e_0(\eta) \int_{-\infty}^{\eta} [e_i^*(\eta') c_0^{(0)}(z, \eta') \\ & + e_0^*(\eta') c_i^{(1)}(z, \eta')] e^{-\tau(\eta-\eta')} d\eta'. \end{aligned} \quad (8)$$

We assume that the powerful component has a constant mean intensity  $\varepsilon_0 \equiv |e_0(\eta)|^2 = \text{const}$ . In this case, the solutions for  $c_0^{(0)}$  and  $c_i^{(1)}$ , which increase with the largest increment, approximately reproduce the component  $e_k(\eta)$  in time and have the form

$$\begin{aligned} c_0^{(0)}(z, \eta) &= a e_0(\eta) \exp(2g\varepsilon_0(L-z)), \quad (9) \\ c_i^{(1)} &= \frac{\gamma}{2} \frac{c_0^{(0)}(z, \eta)}{\varepsilon_0} \int_{-\infty}^{\eta} d\eta' \exp\left(-\frac{\gamma}{2}(\eta-\eta')\right) e_i^*(\eta') e_0(\eta') \\ & \quad (i=1, 2, \dots, N), \end{aligned} \quad (10)$$

where

$$a = \int_0^T \frac{c_0(L, \eta) e_0^*(\eta)}{W_0} d\eta.$$

Corrections of the second approximation to this solution are equal to

$$\begin{aligned} \delta c_i^{(2)} = c_i^{(2)} - c_i^{(1)} &= \frac{\gamma}{2\varepsilon_0} \sum_{k=1}^N e_k(\eta) \int_{-\infty}^{\eta} d\eta' e^{-\tau(\eta-\eta')} \\ & \times (e_i^*(\eta') c_k^{(1)}(\eta', z) + e_k^*(\eta') c_i^{(1)}(\eta', z)) \quad (i=1, 2, \dots, N). \end{aligned} \quad (11)$$

The relative contribution of these corrections to the power of each of the components  $c_i \mathcal{E}_i^*$  is of the order of  $(\gamma/\Delta\omega)\varepsilon/\varepsilon_0$ , as is not difficult to show; that is, at  $\varepsilon_k \ll \varepsilon_0$ ,  $N \leq \Delta\omega T$ , and  $\gamma \ll \Delta\omega$  we can neglect the corrections  $\delta c_i^{(2)}$ .

Thus, in the considered case,  $c_0$  is determined by Eq. (9), and the values  $c_i = c_i^{(1)} (i = 1, 2, \dots, N)$  by Eq. (10). The contribution of each of the weak components  $e_k \mathcal{E}_k (k = 1, 2, \dots, N)$  to the increment of the Stokes waves  $c_k \mathcal{E}_k^*$  corresponding to them is insignificant (the values of the increment at  $\varepsilon_k \ll \varepsilon_0$  is determined only by the field  $e_0 \mathcal{E}_0$ ) and the effect of the weak components  $e_k \mathcal{E}_k$  on the excitation of the Stokes components  $c_j \mathcal{E}_j^* (j \neq 1)$  that do not produce them is also small. The singularities of the SMBS of compound signals noted here are connected with the fact that the buildup of hypersound with amplitudes  $P_{ki} \sim \mathcal{E}_k \mathcal{E}_i (i, k \neq 0)$  due to the interaction of the waves  $e_k \mathcal{E}_k$  and  $c_i \mathcal{E}_i^*$  takes place less effectively than the buildup of  $P_{00}$  because of the fact that the amplitudes  $e_k$  are orthogonal over the scale  $T \leq \tau$  and are small in absolute value in comparison with  $e_0$ .

The conclusions that we have reached are in accord with the experimental results concerning investigations of SMBS excited by a pump consisting of two components that are orthogonal in space and in time.<sup>8</sup> In the experiments of Ref. 8, the time orthogonality was obtained because of the optical path difference  $\Delta L$  of the two light fields,  $e_0(t) \mathcal{E}_0$  and  $e_1(t) \mathcal{E}_1$  (where  $e_1(t) = e_0(t - \Delta t)$ ,  $\Delta t = \Delta L/c$ ), initially formed from a single broadband pulse of the pump. It turned out that at  $\Delta t \gg 1/\Delta\omega$  the reflection coefficient of the very weak wave  $\mathcal{E}_1 e_1$  falls off sharply in comparison with the case  $\Delta t \leq 1/\Delta\omega$ , which confirms the conclusions obtained above.

It follows from the expression (10) for  $c_i^{(1)}$  that if  $\varepsilon_i$



$$\bar{n}(\omega, T_s) \gg \bar{n}(\omega, T) + \bar{n}(\omega, -T_a) + 1,$$

which is satisfied only for the frequency interval  $\hbar\omega \ll k_B T_s$  under the condition that the temperature of the source  $T_s$  greatly exceeds in value the temperature  $T$  of the medium surrounding the amplifier and the negative temperature  $T_a$  of the quantum amplifier itself.

Satisfaction of the conditions of observation of SMBS naturally does not mean that wave front inversion of the incident radiation can be realized. If, for example, the field  $E_L$  represents part of the thermal radiation with a broad spectrum  $\Delta\omega \gg \gamma$ , then the preferred power components are not contained in the expansion of this field over the time interval  $\sim\tau$  in the orthogonal functions  $e_k$  and  $\mathcal{E}_k$ . In this case, even if the condition of observation of the SMBS is satisfied, total wave front inversion with the reproduction of the time modulation does not occur. However, if  $\Delta\omega\tau \leq 1$ , then on a scale  $\sim\tau$  the thermal radiation will actually consist of a single component  $e_0 \mathcal{E}_0$  which, at  $\hbar\omega \ll k_B T_s$  and  $T_s \gg T$ ,  $|T_a|$  can now be separated from the characteristic noise of the amplifier. Only then can we obtain with the help of SMBS wave front inversion with simultaneous reproduction of the time modulation of the thermal radiation.

Of course, the assertion made above does not exclude the possibility of wave front inversion without production of the time modulation of broadband (with frequencies  $\hbar\omega > k_B T_s$ ) radiation of a sufficiently distant thermal source for which the field at the aperture of the receiver can be represented in the form  $e_0(\eta) \mathcal{E}_0(\mathbf{r}_1)$ . The corresponding procedure of wave front inversion finds application, for example, in the compensation for atmospheric turbulence in optical astronomy where, losing information on the time modulation of the radiation  $e_0(\eta)$ , one succeeds after a sufficiently large time interval (with accumulation of the quanta) in obtaining information of the distortions introduced in the atmosphere and on their transverse structure.<sup>16</sup>

We now discuss the features of SMBS in the case in which a single strongly preferred component is absent in the expansion of  $E_L$  in  $e_k$  and  $\mathcal{E}_k$ . We shall solve the problem under the assumption that the scale of the orthogonality  $T$  is significantly smaller than the relaxation time  $\tau$ . Then the solution of Eq. (5) is conveniently sought in the form

$$c_i = \sum_{k=0}^N \alpha_{ik} c_k,$$

where  $\alpha_{ik}$  are functions of  $z$  that change slowly over the time scale  $T$ .<sup>17</sup> Substituting  $c_i$  in (5) and taking it into account that the functions  $e_k$  are orthogonal in a time scale  $T$  significantly smaller than  $\tau$ , we get, to within terms  $\sim T/\tau$ ,

$$\alpha_{ik}' = -g_i g_z \int_{-\infty}^{\infty} d\eta' \exp(-\gamma(\eta - \eta')) (e_i(\eta') \alpha_{ki}(z, \eta') + e_k(\eta') \alpha_{ik}(z, \eta')), \quad e_i(\eta') = |e_i(\eta')|^2, \quad e_k(\eta') = |e_k(\eta')|^2. \quad (13)$$

If the functions  $e_i$  change slowly with scale  $\tau$ , then Eq. (13) is simplified:

$$\alpha_{ik}' = -g(\varepsilon_i \alpha_{ki} + \varepsilon_k \alpha_{ik}) \quad (14)$$

and its solution

$$\alpha_{ik} = (\varepsilon_k + \varepsilon_i)^{-1} \{ (\alpha_{ik}(L) \varepsilon_k + \alpha_{ki}(L) \varepsilon_i) \exp[g(\varepsilon_i + \varepsilon_k)(L-z)] + (\alpha_{ik}(L) - \alpha_{ki}(L)) \varepsilon_i \}$$

shows that the coefficient  $\alpha_{k_0 k_0}$  of the amplitude of the most intense harmonic  $e_{k_0} \mathcal{E}_0$  increases with maximum increment  $2 |e_{k_0}|^2 g$ .

We find the relative power distribution  $\sigma_k/\sigma_{k_0}$  in the radiation reflected from the SMBS mirror. Assuming all the components  $e_k(\eta)$  and  $\mathcal{E}_k(\mathbf{r}_1, L)$  in the priming noise at the boundary of the medium  $z=L$  are represented approximately with the same weight ( $|\alpha_{ik}(L)|^2 \approx \text{const}$ ) and taking it into account that  $e_k e_j^* = \delta_{kj} \varepsilon_k$ , we find, after uncomplicated transformations,

$$\sigma_k/\sigma_{k_0} = \exp[-M(1 - \varepsilon_k/\varepsilon_{k_0})].$$

Here  $M = g\varepsilon_0 L$  is a parameter equal to one half the value of the total amplitude increment for the component  $e_{k_0} \mathcal{E}_{k_0}$ . If the dependence of  $\varepsilon_k = |e_k|^2$  on  $k$  for the wave  $E_L$  is approximated by the quadratic function

$$\varepsilon_k/\varepsilon_{k_0} = 1 - (k - k_0)^2/\bar{k}^2,$$

where  $\bar{k}$  determines the characteristic number of the most intense terms in the expansion of the field  $E_L$  in  $e_k$  and  $\mathcal{E}_k$  ( $\bar{k} \approx N/2$ ), then

$$\sigma_k/\sigma_{k_0} = \exp[-M(k - k_0)^2/\bar{k}^2].$$

It then follows that  $\sim\bar{k}/M^{1/2}$  orthogonal (in the spatial coordinates) components  $c_k \mathcal{E}_k$  are excited in the Stokes wave. At the startup noise radiation and at  $\bar{k}/M^{1/2} \gg 1$ , these components, as is not difficult to show, are also approximately orthogonal (on a scale  $\sim T$ ), however, their oscillograms  $c_i(\eta)$  do not reproduce the functions  $e_i(\eta)$  and consist of the set of all oscillograms belonging to the preferred power components. If  $\bar{k}/M^{1/2} \sim 1$ , then only a single component  $\sim e_{k_0} \mathcal{E}_{k_0}$  will be limited by saturation effects and under ordinary conditions  $M = M_{th}/4 \approx 7$  ( $M_{th} \approx 25-30$  is the threshold increment of SMBS). However, much higher values of  $M$  can be obtained in principle, by introduction of loss on the return wave as a consequence of resonance absorption, nonreciprocal elements, and so on.<sup>18</sup> Figure 2 shows the relative distribution of the power of the orthogonal components  $\mathcal{E}_k$  before ( $\varepsilon_k/\varepsilon_{k_0}$ , solid line) and after ( $\sigma_k/\sigma_{k_0}$ , dashed lines: a— $M=7$ , b— $M=70$ ) reflection of the wave from the SMBS mirror. It is seen that even  $M=7$  is sufficient to introduce at  $\bar{k}=3$  significant dis-

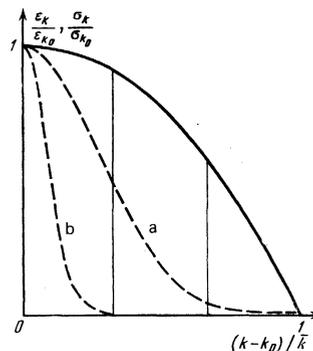


FIG. 2.

crimination in the back-reflected component, singling out the one among them with  $k = k_0$  (see the vertical lines in Fig. 2, the height of which up to the solid curve corresponds to the relative power of the incident and up to the dashed curve, the component reflected from the mirror).

The possibility of distinguishing, among the set of simple components which form the compound pump signal, the single most powerful component can be defined as the property of nonlinear selection. This property is preserved even in the case in which the preferred component has a relatively broad frequency spectrum and its intensity per unit frequency interval is small than that of the other components.

#### 4. AMPLIFICATION OF COMPONENTS UNCORRELATED WITH THE PUMP

We now proceed to the consideration of the possibility of amplification of waves uncorrelated (over the transverse coordinates) with the pump and described by the field  $E_s$ . As has already been noted, at  $M_{th} \ll 1$  these waves are amplified independently of the waves that produce the pump components  $e_k \mathcal{E}_k$ . The equations for the uncorrelated waves are not difficult to obtain by the same method which was used in the derivation of Eqs. (5). It is simplest to proceed in the following fashion. We assume that we are interested in the amplification of the structure of the field  $\mathcal{E}_j (j > N)$  which has no projection on the pump radiation. Nevertheless, we shall assume that the components  $\mathcal{E}_j$  enter into the field  $E_L$  with coefficient  $e_j = 0 (j > N)$ . Then we shall seek the field  $E_s$  in the form of an expansion over all the components entering into  $E_L$ , and also the components  $\mathcal{E}_j$  that are represented in  $E_L$  with zero coefficients.

As a result, we obtain the same equations (5) in which, however, we must set  $e_i = 0 (i > N)$  for the coefficient  $c_i (i > N)$ :

$$v \frac{\partial c_i}{\partial \eta} + \frac{\partial c_i}{\partial t} = -g_i g_2 \int_{-\infty}^{\infty} d\eta' e^{-\tau(\eta-\eta')} c_i(z, \eta') \sum_{k=0}^N e_k^*(\eta') e_k(\eta). \quad (15)$$

In the quasistationary case

$$c_i = c_i(L) \exp \left( g \sum_{k=0}^N |e_k|^2 \right).$$

In the case of a broadband pump we again choose  $\mathcal{E}_i$  as a basis, in order that the functions  $e_j$  be orthogonal. If the scale of orthogonality  $T$  is small in comparison with  $\tau$ , then at  $\nu = 0$  it is convenient to seek the amplitudes  $c_i$ , whose time modulation duplicates the pump components, in the form

$$c_i = \sum_{k=0}^N B_{ik} e_k \quad (i > N),$$

where  $B_{ik}$  is a function of  $z$ , changing slowly in the scale  $T$ . As a result, we get

$$B_{ik}' = -g_i g_2 \int_{-\infty}^{\infty} e^{-\tau(\eta-\eta')} e_k(\eta') B_{ik}(z, \eta') d\eta'. \quad (16)$$

If  $\mathcal{E}_i(\eta)$  is a slowly varying function inside  $\tau$ ,

$$B_{ik} = B_{ik}(L) \exp(g(L-z)e_k).$$

Such a component  $c_i$  whose time modulation reproduces the most intense component in the expansion of the field  $E_L$  in the orthogonal functions  $e_k$  and  $\mathcal{E}_k$  increases with maximum increment. The increment of this component is less than half the increment of the Stokes wave which reproduces not only the time, but also the space (with complex conjugate phase) modulation of the most intense component of the pump.

For estimate of the increment of the components that are uncorrelated with the pump not only in space but also in time coordinates, we must replace

$$\sum_{k=0}^N e_k(\eta) e_k^*(\eta')$$

by

$$\sum_{k=0}^N \overline{e_k(\eta) e_k^*(\eta')} = \Phi_L(\eta - \eta'),$$

where  $\Phi_L = \overline{E_L(\eta, \mathbf{r}_1, z) E_L^*(\eta', \mathbf{r}_1, z)}$  is the correlation function of the pump. The stationary solution of the resultant equation for the Fourier component  $c_i(\omega)$  has the form

$$c_i(\omega) = c_i(\omega, L) \exp \left( g \frac{\Phi_L(\omega)}{1 + i\omega/\gamma} (L - z) \right), \quad (17)$$

where

$$\Phi_L(\omega) = \int_{-\infty}^{\infty} \Phi_L(t) e^{i\omega t} dt$$

is the Fourier spectrum of the pump. If  $\Phi_L(\omega)$  is a smooth function and has a maximum at  $\omega = 0$ , then at  $\Delta\omega \gg \gamma$  we find

$$c_i(t) \sim \exp(g I_L (L - z) / \Delta\omega), \quad \Delta\omega = I_L \left[ \int_{-\infty}^{\infty} \Phi_L(t) dt \right]^{-1},$$

which is identical with the results of Ref. 19, in which the amplification of waves uncorrelated with the pump was considered.

When the mismatch of the group velocities is large and the length of the time coherence is essentially smaller than the characteristic amplification length, the Stokes radiation consists entirely of waves uncorrelated or weakly correlated with the pump, the increment of which is also determined by Eq. (17). It is interesting to note that in the case of waves that are uncorrelated with the pump, the time of the transition process  $t_{tr}$  to the stationary solution (17) is determined, as is easy to see from Eq. (15), by replacing in it

$$\sum_{k=0}^N e_k(\eta) e_k^*(\eta')$$

by  $\Phi_L(\eta - \eta')$ , which is the inverse of the line width  $\Delta\omega$ , while in the amplification of waves that repeat the pump in the time modulation, the value of  $t_{tr}$  is determined by the relaxation time  $\tau$ . Therefore, because of the nonstationarity of the SMBS process, for short broadband pulses of the exciting radiation, the relative discrimination of the increment of the Stokes waves that repeat in time modulation the powerful components of the pump, and the waves that are uncorrelated with the in time, will decrease. For determination of

just how the structure of the Stokes radiation will be formed in this or that specific situation, it is necessary to estimate the relative weight of the components that repeat the pump and do not depend on it in the startup Stokes signal, and to compare the amplified waves with one another, taking into account the difference in their increments. Thus, for example, if the pump consists of a single component  $e_0 \mathcal{E}_0$ , and the startup noise source has the same band  $\Delta\omega$  as the pump, and the same angular spectrum  $\theta$ , then the relative weight of the structure  $e_0 \mathcal{E}_0^*$  in the startup radiation is equal to  $\theta_d^2 \gamma / \theta^2 \Delta\omega$ , so that the amplification factor will have the form

$$\sim \frac{\theta_d^2 \gamma}{\theta^2 \Delta\omega} \exp(2g e_0 L).$$

If in the startup source one selects all components of the form  $e_0(t) \mathcal{E}_L$  or  $\bar{e}_0(t) \mathcal{E}_0^*$ , where  $\mathcal{E}_L(r, z)$  and  $\bar{e}_0(t)$  are components uncorrelated with the pump, then the corresponding factors will be equal to

$$(\gamma/\Delta\omega) \exp(g e_0 L) \text{ or } (\theta_d^2/\theta^2) \exp[2g e_0 L(\gamma/\Delta\omega)].$$

At the same time, the amplification factor is equal to  $\exp[g e_0 L(\gamma/\Delta\omega)]$  for the field  $\bar{E}_s$  uncorrelated with the pump both spatially and in time structure. Comparing these factors, we note that if the first of them significantly exceeds the remainder, then precisely that component  $e_0 \mathcal{E}_0^*$  in the Stokes radiation will be reproduced (see the experiment described in Ref. 20).

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## APPENDIX

We shall prove that in the expansion of the field  $E_L(\eta, \mathbf{r}_\perp)$  only those terms with identical powers  $|e_i|^2$  in the orthogonal functions  $\varphi_i(\eta)$  and  $\mathcal{E}_i(\mathbf{r}_\perp)$ , can be transformed to a new basis  $\varphi'_i(\eta)$  and  $\mathcal{E}'_i(\mathbf{r}_\perp)$ .

We assume that the field  $E_L(\eta, \mathbf{r}_\perp)$  represented in the plane  $z = 0$  in the form (see Refs. 14 and 15):

$$E_L(\eta, \mathbf{r}_\perp) = \sum_{i=0}^N a_i \varphi_i(\eta) \mathcal{E}_i(\mathbf{r}_\perp), \quad \varphi_i(\eta) = e_i(\eta)/a_i, \quad (A.1)$$

$$a_i = \left( \int_{\tau} |e_i|^2 d\eta \right)^{1/2},$$

where  $\varphi_i(\eta)$  and  $\mathcal{E}_i(\mathbf{r}_\perp)$  are complete orthonormal functions over the interval of time  $T$  of the system, admits an expansion in certain orthogonal functions  $\varphi'_i(\eta)$  and  $\mathcal{E}'_i(\mathbf{r}_\perp)$  that differ from  $\varphi_i(\mathbf{r}_\perp)$ :

$$E_L(\eta, \mathbf{r}_\perp) = \sum_{i=0}^N b_i \varphi'_i(\eta) \mathcal{E}'_i(\mathbf{r}_\perp). \quad (A.2)$$

It is obvious that  $\varphi'_i(\eta)$  and  $\mathcal{E}'_i(\mathbf{r}_\perp)$  should not have projection on the functions orthogonal to  $\varphi_i(\eta)$  and  $\mathcal{E}_i(\mathbf{r}_\perp)$ . Then, for any  $\eta$  and  $\mathbf{r}_\perp$ , the following equality is satisfied:

$$\sum_{i=0}^N a_i \varphi_i(\eta) \mathcal{E}_i(\mathbf{r}_\perp) = \sum_{i=0}^N b_i \left( \sum_{k=0}^N c_{ik} \varphi_k(\eta) \right) \left( \sum_{j=0}^N d_{ij} \mathcal{E}_j(\mathbf{r}_\perp) \right). \quad (A.3)$$

Multiplying (A.3) by  $\varphi_n(\eta) \mathcal{E}_m(\mathbf{r}_\perp)$  and integrating over  $\eta$  and  $\mathbf{r}_\perp$ , we obtain

$$a_n \delta_{nm} = \sum_{i=0}^N E_{ni} d_{im}. \quad (A.4)$$

where  $E_{ni} = b_i c_{in}$ .

The equation (A.4) is the condition of orthogonality of the rows of the matrix  $E \equiv [E_{ij}]$  and the columns of the matrix  $D \equiv [d_{ij}]$ . Since the matrix  $D$  is orthogonal according to the condition of orthogonality of the functions  $\mathcal{E}_i(\mathbf{r}_\perp)$ , then the rows of the matrix  $E$  are mutually orthogonal and are proportional to the columns of  $D$ :

$$E_{ni} = \kappa_n d_{in}. \quad (A.5)$$

Since the rows of the matrix  $E$  are mutually orthogonal, while the matrix  $C^T \equiv [c_{ki}]$  is orthogonal by virtue of the orthogonality of the functions  $\varphi'_j(\mathbf{r}_\perp)$ , then it is not difficult to see that the matrix  $C^T$  should be composed of the eigenvectors of the matrix  $B \equiv [b_i \delta_{ij}]$ , i.e.,

$$E_{ni} = c_{in} b_n. \quad (A.6)$$

Using (A.5) and (A.6), and taking into account the normalizability of the matrices  $C$  and  $D$ , we obtain  $c_{in} = d_{in}$ . Taking it into consideration that  $C$  is a matrix composed of the eigenvectors of the matrix  $B$ , in the case of different  $a_i = b_i$ , we obtain  $c_{in} = \delta_{in}$ , i.e., the expansion of  $E_L(\eta, \mathbf{r}_\perp)$  is unique at  $z = 0$ . If there are  $K$  different coefficients among the coefficients  $a_i$  with numbers  $n_j (j = 1, 2, \dots, K)$ , then the admissible systems of orthogonal bases are determined by the equation

$$\varphi'_i(\eta) = \sum_{k=0}^N c_{ik} \varphi_k(\eta), \quad \mathcal{E}'_i(\mathbf{r}_\perp) = \sum_{k=0}^N c_{Lk} \mathcal{E}_k(\mathbf{r}_\perp),$$

where  $c_{ik} = \delta_{ik}$  for  $i \neq n_j (j = 1, 2, \dots, K)$  and

$$c_{ik} = \mu_{ik} \sum_{j=1}^K \delta_{n_j k}$$

for  $i = n_j (j = 1, 2, \dots, K)$ . The values of  $\mu_{ik}$  are determined by the orthogonality condition

$$\sum_{k=0}^N c_{ik} c_{jk} = \delta_{ij},$$

and at  $K > 1$  there exists an infinite set of these values. In this case, there is an infinite set of systems of orthogonal bases  $\varphi'_i, \mathcal{E}'_i$  in terms of which we can expand the desired function  $E_L(\eta, \mathbf{r}_\perp)$ , differing only in the functions  $\varphi'_{n_j}$  and  $\mathcal{E}'_{n_j}$  corresponding to the same value of  $a_{n_j}$ .

At  $z > 0$ , the field  $E_L(\eta, \mathbf{r}_\perp)$  is transformed as the solution of the quasi-optical equation

$$\left( \frac{\partial}{\partial z} + \frac{i}{2k_L} \Delta_\perp \right) E_L = 0$$

to the new field  $E_L(\eta, \mathbf{r}_\perp, z)$ . All the functions  $\xi_i(\mathbf{r}_\perp)$ , scaled to the new plane  $z > 0$  with the help of this quasi-optical equation, remain orthonormal. The functions  $\varphi_i(\eta)$  here obviously do not change. It then follows that if the assertion on the expansion of the field  $E_L(\eta, \mathbf{r}_\perp)$  is valid at  $z = 0$ , then it is preserved for any  $z > 0$ .

<sup>1</sup>The method of solution of Eq. (5) with the help of an expansion in the functions  $e_k$  with a scale of orthogonality  $T$  smaller than the characteristic time of change of the coefficient  $\alpha_{ik}$  is analogous to the well known method of truncated equa-

tions, except that instead of the orthogonal functions  $\exp(i\omega_k t)$  it is convenient to use the functions  $e_k$  in this case (see Ref. 17).

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## Neodymium electron energy deactivation and transfer in highly concentrated phosphate glasses

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The kinetic and spectral luminescence characteristics of neodymium ions in highly concentrated phosphate glasses are measured in the 4.2 to 300 K range. Dipole-dipole interaction between the neodymium and hydroxyl ions is observed and leads to static nonradiative transfer of energy from the neodymium ions to the disordered acceptor  $\text{OH}^-$  group. Migration quenching of luminescence in neodymium by  $\text{OH}^-$  particles is observed at high active-ion concentrations. The values of the microscopic parameters for static quenching interaction between neodymium and hydroxyl ions and for migration donor-donor neodymium-neodymium interaction are determined. Three neodymium-neodymium quenching mechanisms are distinguished and investigated experimentally: 1) static dipole-dipole energy transfer via the intermediate levels  $^4I_{15/2}$  and  $^4I_{13/2}$  with the participation of  $\sim 800 \text{ cm}^{-1}$  phonons; 2) hopping migration of energy along neodymium ions with subsequent quenching by mechanism (1); 3) quenching under ultrarapid migration conditions along neodymium ions. The micro- and macroparameters of the quenching and neodymium-neodymium migration interactions are determined. The causes of the anomalously weak concentration quenching of luminescence in the investigated glasses are explained.

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### 1. INTRODUCTION

The interaction of excited activator centers with one another, with random impurities, and with the base on which they are located determine in final analysis such an important characteristic of a luminescent medium as the quantum yield. This characteristic, in particular, is used to assess the prospects of using a particular matrix as an active laser medium and to determine the optimal concentration of the active ions at which the processes of concentration quenching of the luminescence (CQL) are still quite weak. The search for

media that permit the use of high concentrations of active ions and have a weakly pronounced CQL effect, which is being pursued actively of late, is due to the need for developing miniature effective sources of coherent radiation for integrated optics and for optical-communication systems. In addition, the development of technological materials with increased concentration of active particles make it possible to increase the energy output per unit volume and to miniaturize the existing laser systems. The most considerable successes in the search of such materials were attained among condensed phosphates, both in crystalline form with