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Change in the local geometry of the Fermi surface and the anomaly of the electronic sound absorption coefficient

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A break in a bridge is accompanied by the appearance or disappearance of parabolic-point lines on the Fermi surface. As a result, the change in the connectivity of the Fermi surface leads to an anomaly in the angular dependence of the sound absorption coefficient due to electrons, Γ_e. The closeness of the parabolic-point line to the conical line increases the anomaly. The concept of a phase transition of 2 1/2 order is generalized to include changes in the local geometry of a Fermi surface not accompanied by a change in its connectivity. It is shown that the formation of a "waist" or a "crater" is accompanied by the appearance of angular singularities in Γ_e.

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1. INTRODUCTION

After the prediction of a phase transition of $2\frac{1}{2}$ order[1] due to a change in the connectivity of the Fermi surface, the study of the electron energy spectrum of metals and degenerate semiconductors by means of external action on their Fermi surface has become widespread. Change in the connectivity of the Fermi surface, as is known, takes place because of the rearrangement of the structure of the surface[1)]

$$\varepsilon(\mathbf{p})=\varepsilon_F \tag{1}$$

close to isolated points of p space ($\mathbf{p}=\mathbf{p}_k$), at which either a new cavity of the Fermi surface is created or a bridge is broken.[1,2] The significant effect of such a small change in the spectrum on the thermodynamic characteristics is due to the fact that, because of degeneracy, the latter are determined not by all the electrons, but only by those located on the Fermi surface, and the role of slow electrons is anomalously large, since the density of states at $\varepsilon=\varepsilon_F$ is

$$\nu(\varepsilon)=\frac{2}{(2\pi\hbar)^3}\oint_{(\varepsilon_F)}\frac{dS}{v},\quad \mathbf{v}=\nabla_p\varepsilon. \tag{2}$$

We recall that $\mathbf{v}(\mathbf{p}_k)=0$.

The kinetic properties of a metal are especially sensitive to the structure of the Fermi surface, and, in certain cases, the change in the connectivity of the Fermi surface can lead to radical changes. For example, if, as a result of the breaking of a bridge, the Fermi surface changes from open to closed, then the conductivity in a strong magnetic field along the direction of the openness can increase by factors of tens or even hundreds (see Ref. 2, Sec. 28).

The sound absorption coefficient Γ is especially sensitive to a change in the local structure of the Fermi surface.[3] The fact is that, first, the role of slow electrons in the interaction with the sound is even more important than its role in the thermodynamic characteristics, since the formula that determines the sound absorption coefficient Γ in the form of an integral over the Fermi surface contains v^2 in the denominator [cf. with formula (2)], and second, electrons on the "belt" participate in the absorption of shortwave sound ($kl\gg 1, k=\omega/s$ is the wave vector of the sound wave of frequency ω, s is velocity, l is the mean free path of the electrons)[2)]

$$\mathbf{v}\mathbf{k}=0,\quad \varepsilon(\mathbf{p})=\varepsilon_F, \tag{3}$$

the structure of which can change upon change in the

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Fermi surface.

It was shown previously[4] that an especially important role is played in the anisotropic sound absorption by parabolic points on the Fermi surface, since it is precisely at these that the belt changes its topology upon change in the direction of sound propagation $\mathbf{n}=\mathbf{k}/k$. The break in the bridge is of necessity accompanied by the appearance (or disappearance) of parabolic-point lines[3)] (see Fig. 1).

Parabolic points on the Fermi surface can appear or disappear not only upon change in the connectivity of the Fermi surface. It is shown in Fig. 2 that the appearance of a depression on the Fermi surface is accompanied by the appearance of a parabolic-point line. Two cases are possible here, which are illustrated by the appearance of a "crater" (Fig. 2a) or of a "waist" (Fig. 2b). In the first case, points of the O-type appear, in the second, of the X-type (we use here the terminology of Ref. 4). If it is taken into account that the singularities of Γ investigated in Ref. 4 depend on the local structure of the Fermi surface near the O- and X-type points, then it becomes clear that a small change in the shape of the Fermi surface, one that is quite inconsequential for thermodynamic properties of a metal at $H=0$,[4)] should lead to the formation (or disappearance) of singularities in the dependence of Γ on the direction of the sound. Furthermore, as we shall see, the singular points of Γ in these cases are anomalously large in comparison with the general case treated in Ref. 4, owing to the nearness of the Fermi surface to the critical surface which contains either a cylindrical portion (one of the principal curvatures is equal to zero, Fig. 2b) or a plane one (both principal curvatures are equal to zero, Fig. 2a).

Analysis of the dependence of the sound absorption coefficients on direction of n and on the parameter $z=\varepsilon_F-\varepsilon_k$, which determines the closeness of the Fermi surface to the critical, constitutes the subject of the present paper. At $z=0$, the Fermi surface contains either a conical point or a line of cylindrical points, or

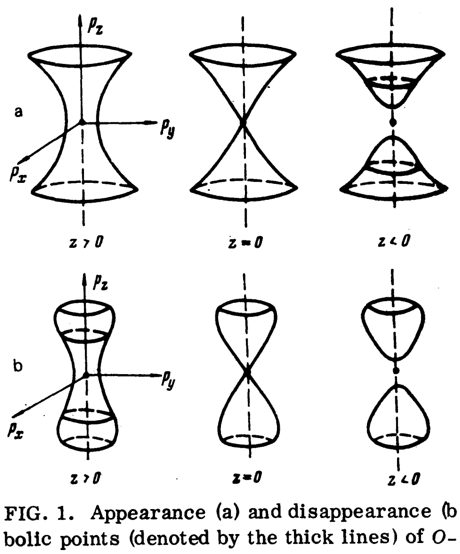

FIG. 1. Appearance (a) and disappearance (b) of lines of parabolic points (denoted by the thick lines) of O- and X-type, respectively, in the breaking of a bridge.

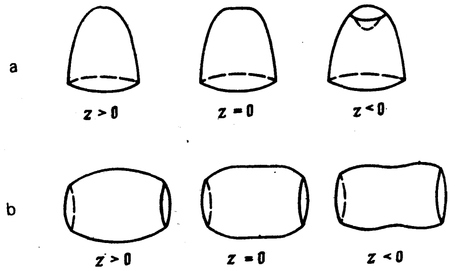

FIG. 2. Formation of a crater (a) and a waist (b).

has a flattening point.[5)]

If the conditions $kl \gg 1$, $T/z \ll 1$, $\hbar\omega/z \ll 1$ and $\omega/kv \to 0$ are satisfied then the sound absorption coefficient can be represented in the form of an integral containing two δ functions:

$$\Gamma=\frac{\pi\omega}{(2\pi\hbar)^3\rho s}\int d^3p|\Lambda|^2\delta(\varepsilon(\mathbf{p})-\varepsilon_F)\delta(\mathbf{vn}), \tag{4}$$

ρ is the density of the metal and Λ is the corresponding term of the renormalized deformation potential.

The problem of the relations among the small parameters $1/kl, T/z, \hbar\omega/z, \omega/kv$ arises, as we shall see, in the consideration of the limiting values of Γ. It must be kept in mind that their action is different (see Refs. 3 and 4). Allowance for the finite sound velocity $(\omega/kv=s/v)$, while not essentially changing the type of the singularities, shifts them somewhat, depriving the results of clarity.[6)]

The quantity $\hbar\omega$ plays a double role. First, account of $\hbar\omega$ somewhat changes the "Cerenkov" condition that selects the electrons taking part in the sound absorption:

$$\cos\theta_e=\frac{s}{v}\left(1-\frac{\hbar\omega}{2ms^2}\right)$$

(θ_e is the angle, measured from the vector $\mathbf{k}$, that determines the direction of the velocity of the electron with mass m). At $\hbar\omega \ll 2ms^2 (ms^2/\hbar \approx 10^{10}\ \mathrm{sec}^{-1})$, account of $\hbar\omega$ is entirely unimportant [especially if it is recalled that we have neglected above the finite sound velocity s under this condition, see (3)]. Second, account of $\hbar\omega$ means the replacement of $\delta(\varepsilon-\varepsilon_F)$ by the difference in the Fermi steps $n_F(\varepsilon)-n_F(\varepsilon+\hbar\omega)$, owing to which the jump in the dependence $\Gamma=\Gamma(z)$ is replaced by two closely located jumps in the derivatives, and the logarithmic singularity is a singularity of the type $x\ln x$. The parameters T/z and $1/kl$ smear out the singularities and an important role, naturally, is played by the larger parameter.

The total sound absorption coefficient is determined by all the cavities of the Fermi surface. However, we shall always be interested only in that cavity which changes its geometry under external action, denoting this by the index z in $\Gamma(\Gamma_z)$. It is necessary to compare the results obtained below with the usual value of $\Gamma \approx \omega(m/M)^{1/2}$, where M is the mass of the metal ion.

2. BREAKING THE BRIDGE

In order to avoid unnecessary complications, we shall assume that the breaking portions of the Fermi surface have axial symmetry and that a symmetry plane passes through the conical point, which we take as the origin of the coordinates ($p_k = 0$), perpendicular to the axis of the bridge. Then the equation of the portion of the Fermi surface of interest to us has the form

$$z = p_\perp^2/2m_\perp - p_z^2/2m_\parallel + \beta p_z^4/4m_\parallel,$$
$$z = \varepsilon_F - \varepsilon_k, \quad p_\perp^2 = p_x^2 + p_y^2, \quad m_\perp, m_\parallel > 0. \tag{5}$$

In order of magnitude, the parameter $|\beta|$ is equal to $a^2/\hbar^2$ in the general case, where a is the linear dimension of the cell of the crystal. The sign of the parameter β determines the type of the appearing (or disappearing) parabolic points: at $\beta > 0$ two symmetrically placed lines of O-type points appear when the bridge breaks (Fig. 1a), and at $\beta > 0$ lines of points of the X-type disappear (Fig. 1b).

Using the axial symmetry of the dispersion law (5), we can represent the sound absorption coefficient Γ_2 in the form of a single integral over p_z:

$$\Gamma_z = \frac{2\pi\omega}{(2\pi\hbar)^3 \rho s \sin\theta} \int dp_z \sum_i \frac{|\Lambda|^2 p_{\perp i}}{v_\perp [1 - (v_z/v_\perp)^2 \operatorname{ctg}^2\theta]^{1/2}}, \tag{6}$$

the angle θ determines the direction of sound propagation ($n_z = \cos\theta$) and

$$v_\perp = (\partial z/\partial p_\perp)|_{p_\perp = p_{\perp i}}, \quad v_z = (\partial z/\partial p_z)|_{p_\perp = p_{\perp i}},$$

$p_{\perp i} = p_{\perp i}(z, p_z)$ is the i-th solution of Eq. (5). The integration is carried out over those values of p_z for which the radicand is positive.

In addition, it should be taken into consideration that the dispersion law (5) is an approximation and that we must restrict the integration over p_z to a value of p_0 that is small in comparison with the dimensions of the cell of the reciprocal lattice $\hbar/a$. The parabolic-point lines whose contribution to Γ_z is especially important (see above) are located, as follows from (5), at distances $|p_z^n| \approx (4m_\parallel |z|/3|\beta|)^{1/4}$ from the origin. Since $p_z^n \to 0$ as $z \to 0$, it follows that p_0 can be chosen to satisfy the inequalities

$$|p_z^n| \ll p_0 \ll \hbar/a, \tag{7}$$

which assure the validity of the applied method of calculation.

Introducing the dimensionless variable $x = p_z|\beta|^{1/2}$, we can write Eq. (6) in the following form:

$$\Gamma_z = \Gamma \left(\frac{1+\mu-\varphi}{\mu}\right)^{1/2} \int_0^{x_0} \frac{\Theta(X)\,dx}{X^{1/2}}. \tag{8}$$

Here

$$X = -(1-\varphi)x^6 + (3/2 - 2\varphi)x^4 + \varphi x^2 + \zeta,$$
$$\Gamma = \pi|\Lambda_0|^2 \omega m_\perp^{3/2} m_\parallel^{1/2}/(2\pi\hbar)^3 \rho s, \quad \Lambda_0 = \Lambda(p=0), \tag{8'}$$

$$x_0 = p_0(|\beta|)^{1/2}, \quad \varphi = 1 - \mu \operatorname{ctg}^2\theta, \quad \mu = m_\perp/m_\parallel, \quad \zeta = 2m_\parallel z\beta. \tag{9}$$

$$\Theta(X) = \begin{cases} 1, & X \geq 0 \\ 0, & X < 0 \end{cases}.$$

The singularities of the absorption coefficient Γ_z arise at those values of φ and ζ ($\varphi = \varphi_c$, $\zeta = \zeta_c$) at which the function $X(x)$ has a multiple root, and are found from the conditions

$$X(x) = 0, \quad dX(x)/dx = 0. \tag{10}$$

Eliminating x from (10), we can find the connection between the critical value of the parameter $\varphi_c = 1 - \mu\cot^2\theta_c$ and the quantity ζ. It is clear, however, that, upon formation of the bridge, when the condition

$$|\zeta| \ll 1 \tag{11}$$

is satisfied, the angle θ_c differs little from $\tilde\theta$ ($\tan^2\tilde\theta = \mu$).

Thus the range of angles of interest to us is very small: $|\varphi| \ll 1$. Using this circumstance, we can simplify the expression in the integrand:

$$X(x) = -x^6 + 3/2\,x^4 + \varphi x^2 + \zeta. \tag{8''}$$

The multiple root $X(x)$ of interest to us tends to zero with ζ. It is easy to prove (see Fig. 3) that it exists at $\varphi = \varphi_c$ only at $\zeta > 0$, and corresponds to the parabolic points of the O-type (at $\beta < 0$) and of the X-type (at $\beta > 0$). According to (8″) and (10) at $|\varphi| \ll 1$ and $|\zeta| \ll 1$ we have

$$\varphi_c \approx -(6\zeta)^{1/2}. \tag{12}$$

Finally, at $\zeta = 0$ and $\varphi = 0$ (i.e., at $\theta = \tilde\theta$) the function (8″) has a fourfold root at zero.

At $\varphi = \varphi_c$, the belt changes its topology. In accord with the general theory[4] the change in the topology takes place at a parabolic point (its coordinate $p_z = p_z^n$, see above); if the parabolic point is of the X type ($\beta > 0$, $z > 0$, Fig. 1b), then Γ_z has a logarithmic singularity as $\varphi \to \varphi_c$:

$$\Gamma_z(\zeta) = \Gamma\left(\frac{27}{2\zeta}\right)^{1/4} \ln\frac{1}{|\delta\varphi|}, \quad |\delta\varphi| = |\varphi - \varphi_c| \ll 1, \tag{13}$$

and in the case of a point of the O-type ($\beta < 0, z < 0$, Fig. 1a) it has a finite discontinuity

$$\Gamma_z(\zeta) = \pi\Gamma(27/2\zeta)^{1/4}. \tag{14}$$

The closeness of the parabolic point line to the conical point $\mathbf{p} = \mathbf{p}_k$ leads to an anomalous increase in the singular part of the sound absorption coefficient ($\Gamma_z \propto \zeta^{-1/4}$). The latter condition makes it difficult to explain how $\Gamma_z(\varphi)$ behaves as $\zeta \to 0$. According to (8) and (8′), the integral that enters into Γ_z, under certain restrictions on the parameter φ ($\varphi > 0$ at $\beta < 0$ and $\varphi < 0$ at $\beta > 0$), di-

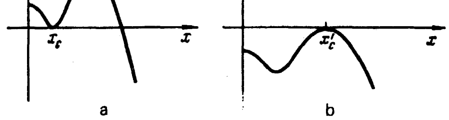

FIG. 3. Dependence of $X(x)$ at a critical value of the parameter φ: a) at $\zeta > 0$, $x_c \approx (-\varphi_c/3)^{1/2} = (2\zeta/3)^{1/4}$; b) at $\zeta < 0$, $x_{c'} \approx 3^{1/2}/2$, $\varphi_c \approx -9/16$.

verges logarithmically as $\zeta \to 0$ from the side of positive values:

$$\Gamma_z(\beta<0, \varphi>0) = \Gamma_z(\beta>0, \varphi<0)$$
$$= \Gamma \left(\frac{1+\mu-\varphi}{\mu|\varphi|} \right)^{1/2} \ln \left| \frac{\varphi}{\zeta} \right| = \Gamma |\sin^2\theta - \mu\cos^2\theta|^{-1/2} \ln \left| \frac{\varphi}{\zeta} \right|. \tag{15}$$

The diverging logarithm is absent from the other side from the critical direction:

$$\Gamma_z(\beta<0, \varphi<0) = \Gamma_z(\beta>0, \varphi>0) = \pi\Gamma \left(\frac{1+\mu-\varphi}{\mu|\varphi|} \right)^{1/2}$$
$$= \pi\Gamma |\sin^2\theta - \mu\cos^2\theta|^{-1/2}. \tag{16}$$

The quantity φ cannot be assumed to be small in these latter formulas, since they are applicable if $|\varphi| \gg \zeta$. We see that, close to the phase transition point of $2\frac{1}{2}$ order ($\zeta \to 0$) and at $\varphi \approx \varphi_c$ the function $\Gamma_z(\varphi, \zeta)$ is very complicated and, in particular, the character of the singularity depends essentially on the ratio of the two small parameters φ and ζ. It is interesting to note that the parameter φ enters in different ways in the factor in front of the logarithm and in the argument of the logarithm itself, thereby additionally complicating the behavior of $\Gamma_z(\varphi, \zeta)$.

The limiting values of Γ_z should be estimated with account taken of the factors that cut off and smear out the singularity ($\hbar\omega, ms^2, 1/kl, T$). For example, if the temperature T turns out to be the smallest parameter, then the maximum value of Γ_z at $\varphi = \varphi_c$ is $\tilde{\Gamma}(\varepsilon_0/T)^{1/4}$, where $\varepsilon_0 \approx (m_{\parallel}\beta)^{-1}$ (we have neglected the logarithmic dependence).

3. FORMATION OF A WAIST (OR CONSTRICTION)

As was mentioned above, the formation of a waist (or constriction) means a change in the sign at one of the principal radii of curvature of the Fermi surface when external action is applied to the metal. If we now mean by z the difference between the Fermi energy and its value when the surface contains a cylindrical-point line, then the equation of the portion of the Fermi surface at small p_z can be written in the form

$$\bar{\varepsilon} + z = vp_\perp + \frac{z}{2m\varepsilon_0} p_z^2 + \frac{\beta}{4m} p_z^4, \quad p_\perp^2 = p_x^2 + p_y^2. \tag{17}$$

We have again assumed for simplicity that the Fermi surface possesses cylindrical symmetry and that the cylindrical-point line ($p_z = 0, z = 0$) lies in the plane of symmetry; v is the value of the velocity at $p_z = 0$, the quantity $\bar{\varepsilon}$ determines the radius of the cylinder ($\bar{p}_\perp = \bar{\varepsilon}/v$); the coefficient of p_z^2, which changes sign upon change of sign of z, is written in the form $z(2m\varepsilon_0)^{-1}$ only for convenience; in the general case ε_0 is of the order of the ordinary electron energy ($\varepsilon_0 \sim \varepsilon_F$) and m is of the order of the mass of the electron. The sign of the coefficient β determines at what sign of z the Fermi surface has a "corrugation" (for definiteness, we henceforth assume $\beta > 0$).

The appearance of a waist at $z < 0$ (see Fig. 2b), as has been pointed out, leads to the development of a parabolic line of points of the X-type.

The angle θ_c reckoned from the axis of the Fermi surface and at which Γ_z has a logarithmic singularity is very small and is defined as

$$\theta_c \approx (4|z|^3/27\beta m^2 v^2 \varepsilon_0^3)^{1/4}, \quad z<0. \tag{18}$$

The coefficient $\Gamma_z(\theta)$ can be calculated from Eq. (6):

$$\Gamma_z(\theta) = \frac{2\cdot 3^{1/2}\pi|\Lambda_0|^2 m\omega\bar{\varepsilon}}{(2\pi\hbar)^3\rho s v^2} \left(\frac{\varepsilon_0}{|z|} \right) \ln\frac{1}{|\delta\theta|},$$
$$|\delta\theta| = |\theta - \theta_c| \ll 1, \quad z<0. \tag{19}$$

If we assume as usual that all the electron energies except z are of the same order ($\sim\varepsilon_F$), then in the given case $\Gamma_z(\theta)$ is $\varepsilon_0/|z|$ larger than usual. The absorbing capability of the electrons then becomes stronger in the presence of cylindrical portions on the Fermi surface.[8] Thus, as $\theta \to 0$,

$$\Gamma_z \approx \frac{2\pi^2|\Lambda_0|^2 m\omega\bar{\varepsilon}}{(2\pi\hbar)^3\rho s v^2} \left(\frac{\varepsilon_0}{|z|} \right). \tag{20}$$

At such a strong singularity ($\sim|z|^{-1}$) the cutoff mechanism is particularly important. If T is the smallest parameter, then the damping coefficient must increase by a factor ε_0/T upon appearance of a cylindrical portion and should exhibit strong anisotropy at a small deviation from the axis of the cylinder.

4. FORMATION OF A CRATER

A simultaneous change of the signs of the two radii of curvature presupposes the existence of axial symmetry of the Fermi surface. Therefore, placing the origin of the coordinates at the flattening point, we can simulate the formation of a crater at $z < 0$ by the following equation (see Fig. 2a):

$$z = vp_z + \frac{z}{2m\varepsilon_0} p_\perp^2 + \frac{\beta}{4m} p_\perp^4, \quad \beta>0. \tag{21}$$

The notation here is clear from the foregoing. The formation of the crater is accompanied by the appearance of a line of parabolic points of the O-type with coordinates

$$p_\perp = p_\perp^n = (|z|/3\beta\varepsilon_0)^{1/2}, \quad p_z = p_z^n = z/v, \quad z<0, \tag{22}$$

due to which, at

$$\theta = \theta_c \approx \pi/2 - (|z|^3/27\beta m^2 v^2 \varepsilon_0^3)^{1/2}, \quad z<0, \tag{23}$$

the absorption coefficient Γ_z undergoes a jump equal to

$$\Gamma_z = \frac{\pi^2|\Lambda_0|^2 m\omega\varepsilon_0^{1/2}}{(2\pi\hbar)^3\rho s v(|z|\beta)^{1/2}\sin\theta_c} \approx \frac{\pi^2|\Lambda_0|^2 m\omega}{(2\pi\hbar)^3\rho s v \beta^{1/2}} \left(\frac{\varepsilon_0}{|z|} \right)^{1/2}, \quad z<0. \tag{24}$$

However, the singularities of the absorption coefficient $\Gamma_z(\theta)$ are not limited to this. Figure 4 shows the belt (3) at $\theta = \pi/2$. The existence of the self-intersection points A_1 and A_2 leads to an additional logarithmic growth of $\Gamma_z(\theta \to \pi/2)$:

$$\Gamma_z(\theta) \approx \frac{\pi|\Lambda_0|^2 m\omega}{(2\pi\hbar)^3\rho s v(\beta)^{1/2}} \left(\frac{\varepsilon_0}{|z|} \right)^{1/2} \ln\frac{1}{\delta\theta}, \quad \delta\theta = \frac{\pi}{2} - \theta, \quad z<0. \tag{25}$$

We emphasize that it is easier to find the singularities (25) and (20) than the others, since in these cases θ_c is identical with selected directions in the crystals.

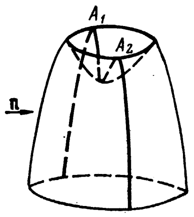

FIG. 4. Shoulder (heavy line) on a Fermi surface containing a crater, at $\theta = \pi/2$. A_1 and A_2 are self-intersection points on the belt.

5. CONCLUDING REMARKS

Analysis of the behavior of the absorption coefficient of short-wavelength sound upon local change in the geometry of the Fermi surface indicates the great sensitivity of Γ_z to subtle characteristics of the electron energy spectrum of the metals. Furthermore, we must also emphasize the following: the appearance of anomalies, which accompany the change (through external action) of sign of the curvature of the Fermi surface, by the same token generalizes the concept of "phase transition of $2\frac{1}{2}$ order." The participation of a small group of electrons in kinetic phenomena makes possible, by comparatively small action on the metal, a significant change in its characteristics, which uncovers additional possibility of investigation of the electron energy spectrum.

In conclusion, we take this opportunity to thank G. T. Avanesyan and I. M. Lifshitz for discussion of the obtained results and stimulating discussion.

[1)] We employ the usual notation (see, for example, Ref. 2).

[2)] In this paper we shall not take into account the fine structure of the singularities (see Ref. 3) due to the finite value of the sound velocity.

[3)] This fact was not taken into account in the work of Davydov and one of the authors.[3] The newly formed cavity is generally an ellipsoid, on which there are no parabolic points. The anomalies in the sound absorption that accompany the formation of the new cavity were investigated in Refs. 3 and 5 (the effect of a comparatively strong magnetic field **H** was considered in Ref. 5).

[4)] At $\mathbf{H} \neq 0$ the formation of depressions on the Fermi surface leads to a change in the density of states and, consequently, to anomalies in the thermodynamic characteristics.

[5)] Experiments in which the appearance or disappearance of a crater would be observed (Fig. 2a) are unknown to us; the appearance of a waist (bridge) (Fig. 2b) has been established in doped tellurium and in bismuth under pressure, in accord with the Shubnikov-de Haas effect (see Refs. 6 and 7).

[6)] The Fermi surface possesses a center of symmetry. For each parabolic point there is an antipode with $v_A = v_{A'}$ (see Fig. 4 in Ref. 4). Therefore, the contribution to the singular part of Γ is made by the vicinities of the two points A and A'. At $s \neq 0$, the singularity is split and the (angle) distance between them is s/v (see Ref. 4).

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