

$\times 10^{-11}$ sec, which corresponds to $E_0 \sim 50$ V/cm. At helium temperatures, scattering by ionized impurities predominates in the passive region, and at $n \sim 2.5 \times 10^{14}$ cm $^{-3}$ the value of q_D/p_0 is of the order of 0.25, so that good conditions are obtained for the observation of NDC in a frequency region slightly exceeding the fly-through frequency. The fly-through frequency is $\nu_E \sim 30$ GHz at $E_0 \sim 74$ V/cm.

We note in conclusion that despite the strong disequilibrium in the electron system in this case, the spectral density of the current fluctuations, defined in Ref. 4, is closely connected with $\sigma(\omega)$. Both quantities differ substantially from zero only in resonant regions, where both have Lorentzian shapes. Therefore in the vicinity of each resonance the Callen-Welton relation (1.2) can be suitably defining the proportionality coefficient T , which is interpreted as the noise temperature. In the present situation T is large and is proportional to either $(\tau_E/\tau^+)^{2/3}$ or τ^-/τ_E ; this indicates that the electron system is strongly heated. In addition, T must be chosen complex and written under the Re sign. The phase of T defined in this manner is equal to the phase of σ^c and is directly connected with the onset of the NDC or, in other words, with the possibility of development of unstable fluctuations. The electron system for which a complex T follows from (1.22) is a unique intermediate case between a stable system with real positive T and unstable fully inverted system with negative T for which NDC exists in the entire resonance region.

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Current fluctuations in a strong electric field under conditions of frequent interelectron collisions that ensure a Maxwellian distribution with drift

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A quantitative description is presented of noise in strong electric fields at high carrier densities when the form of the electron distribution both in energy and in momentum is governed by the collisions between the atoms. It is shown that under these conditions, in the general case, neither the longitudinal nor the transverse noise temperatures are equal to the electron temperature which fluctuates in a Maxwellian distribution with drift. The difference is due to an additional correlation of the occupation numbers of the electronic states which results from (and only from) the collisions between the electrons. The corresponding expressions are obtained for the spectral density of the current fluctuations in a wide temperature interval.

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1. INTRODUCTION

In nonequilibrium systems the current fluctuations and the carrier diffusion coefficients are no longer connected with the conductivity by the Nyquist and Einstein relations. These quantities contain new information, and their investigations provides a method for the diagnostics of the nonequilibrium electron gas in a semiconductor.

Price¹ was apparently the first to call attention to the important circumstance that although neither the Ny-

quist theorem nor the Einstein relation holds in the nonequilibrium state, there nevertheless remains in force a simple relation between the diffusion coefficient and the spectral density of the current fluctuations. Price's "nonequilibrium fluctuation-diffusion relation" is widely used, since it makes it possible to extract, from measurements of noise in a spatially homogeneous nonequilibrium system, the information on the response of a nonequilibrium system to a spatial gradient produced in it, and vice versa.²⁻⁴

The foregoing, however, is incontrovertibly true only

so long as the carrier density is low enough to be able to neglect the collisions between the electrons. It has been shown⁵⁻⁷ (see also Ref. 8 and the review⁹) that in a nonequilibrium electron gas with pair collisions the latter lead to an additional correlation between the electrons in the nonequilibrium state, as a result of which the nonequilibrium fluctuation-diffusion relation is violated.⁷ In particular, a detailed study was made of the case when the collisions between the electrons govern the electron distribution in energy but not in momentum:

$$\tau_p \ll \tau_{ee} \ll \tau_e. \quad (1)$$

Here τ_p and τ_e are the electron momentum and energy relaxation times following their collisions with the lattice (i. e., with impurities, phonons, etc.), and τ_{ee}^{-1} is the frequency of the collisions between the electrons. In this "case of effective electron temperature T ," analytic expressions were obtained both for the spectral density of the current fluctuations,^{10,11} and for the diffusion-coefficient tensor.¹²⁻¹⁴ A comparison of these expressions demonstrates the violation of the fluctuation-diffusion relation in a concrete example.

The purpose of the present study was to investigate the current fluctuations in the case of electron-electron collisions that are so frequent that they control the electron distribution both in energy and in momentum:

$$\tau_{ee} \ll \{\tau_p, \tau_e\}. \quad (2)$$

We were prompted to investigate this case, on the one hand, by the intrinsic logic of the theory development, and on the other by the progress made in the creation and investigation of nonequilibrium states in those semiconductors in which one can expect the inequality (2) to be realized. It was just in recent years that experimental studies were made of kinetic phenomena in strong electric fields in lead chalcogenides.¹⁵⁻¹⁷ It is known¹⁸ that in semiconductors of this class, owing to the lattice polarization, the screening of the static Coulomb potential of the immobile scatterers is more effective than the "dynamic" screening of the electron-electron interaction. The case (2) is therefore a physical reality in uncompensated lead-chalcogenide samples at large carrier densities and low temperatures. Thus, the theory developed in the present paper lays the groundwork for a quantitative description of noise in strong electric fields in lead chalcogenides at large carrier densities.

It should be noted that a thorough experimental investigation of the noise of hot electrons in germanium, silicon, gallium arsenide, and other classical semiconductors, carried out in the last 10-15 years, yielded valuable information on the details of the scattering mechanisms, on the degree of heating of the carriers by the electric field, and on the relation times of the energy and of the intervalley transition.^{2-4,19-21} Obviously similar experiments on lead chalcogenides are highly desirable. When the theory developed in the present paper is applied to concrete compounds, it will be necessary to take their band structure into account. (We note that in the case of rapid energy and momentum exchange between electrons of different equivalent valleys, the individuality of the latter does not come into play.)

We were interested in the present study in only the most general problems of the kinetics of current fluctuations in case (2)—in the case when the collisions between electrons govern completely the form of the averaged (over the ensemble, or—in the stationary case—over the time) carrier distribution ("Maxwellian with drift"). We wished to ascertain the influence exerted in this case by the electron-electron collisions on the current fluctuations; in particular, the role played by aforementioned additional correlation between the occupation numbers of the electronic states, which arises in a nonequilibrium system, and the nature and the cause of the anisotropy of the noise temperature. The last question is closely linked with preceding one and seems most interesting to us. In fact, it follows from the very definitions that in the case of a Maxwellian distribution with drift the diffusion-coefficient tensor is proportional to the differential-conductivity tensor (i. e., the "Wannier conjecture" is true^{22,23}). This means in turn that the anisotropy of the noise temperatures (i. e., the fact that the current-fluctuation spectral-density tensor is not proportional to differential-conductivity tensor) is possible only to the extent that the fluctuation-diffusion relation is violated. This distinguishes the case of pure electron-electron collisions (2) from the case of the electron temperature (1), in which the anisotropy of the noise temperatures is due both to the additional correlation (i. e., to the violation of the fluctuation-diffusion relation) and to the absence of a connection between the diffusion and the differential conductivity.^{13,14,9}

Our principal results are the following. As expected [in analogy with the results obtained by Shul'man¹¹ for case (1)], the influence of the additional correlation on the current fluctuations in the case (2) turned out to be substantial. Under the condition $\tau_p \sim \tau_e$ the additional correlation affects not only the longitudinal (along the stationary current) but also the transverse current fluctuations. By the same token, the Nyquist relation and the fluctuation-diffusion relation are violated also in the transverse direction. Another remarkable circumstance is that the influence of the additional correlation and the ensuing violation of the Nyquist relation as well as the anisotropy of the noise temperature take place in a wide range of frequencies up to $\omega \leq \tau_{ee}^{-1}$.

If the energy relaxes more slowly than the momentum:

$$\tau_{ee} \ll \tau_p \ll \tau_e, \quad (3)$$

the situation becomes simpler. The additional correlation affects then only the longitudinal fluctuations of the current, and only at frequencies $\omega \leq \tau_e^{-1}$. In weakly inelastic scattering, the expression for the longitudinal noise temperature (n terms of the electron temperature, of its relaxation time, and of the ration of the static longitudinal differential and specific conductivities) coincides with the one previously obtained by Kogan and Shul'man^{10,11} for the case (1). It is interesting to note that the contribution of the additional correlation to the longitudinal current fluctuations is different in cases (1) and (2); this difference, however, is exactly cancelled out by the difference between the longitudinal dif-

fusion coefficients.

We recall that in the case of the electron temperature.²⁴ (1) the contribution of the additional correlation vanishes if the Maxwellian energy distribution is ensured by the very dependences of the times τ_p and τ_e on the energy (namely, if $\tau_p \tau_e = \text{const}$).^{11,13,14,9} Analogously, the additional correlation vanishes also in our case (2) if the mechanisms of the interaction with the lattice ensure by themselves the applicability of a Maxwellian distribution with drift. In this special case, both fluctuation-diffusion relation and the connection between the current fluctuations and the differential conductivity are re-established, so that the noise temperature becomes equal to the electron temperature.²⁴

2. KINETIC DESCRIPTION OF THE CURRENT FLUCTUATIONS

We use as our basis the kinetic theory of fluctuations in a nonequilibrium electron gas, a detailed description of which can be found in Ref. 9. We recall briefly its results.

It is assumed that as a result of a constant electric field E and of collisions with the thermostat (phonons, impurities, etc.) the electron gas goes over into a stable stationary nonequilibrium state, so that the mean distribution (over the ensemble or time) of the electrons in the momenta (F_p) is determined by the kinetic equation¹

$$eE\partial_p F_p + I_p^{th} F_p + I_p^{ee} \{F, F\} = 0, \quad (4)$$

where I_p^{th} is a linear operator that describes the collisions of the electrons with the thermostat, $I_p^{ee} \{F, F\}$ is the integral of the interelectron pair collisions. The density of the stationary current is given by

$$j = \frac{e}{v_0} \sum_p v F_p = en_0 V, \quad (5)$$

here $n_0 = N/v_0$ is the electron density, v_0 is the volume. The distribution function is normalized to the total number of the electrons $N = \sum_p F_p$, which will be assumed to be non-fluctuating. Next, $v \equiv 2_p c_p$ is the electron velocity, V is the drift velocity of the electron system, and p and ε_p are the momentum and energy of the electrons.

The spectral density of the fluctuations of the distribution function is given by the expression [see Eq. (14) of Ref. 8 or (1.44) of Ref. 9]:

$$(\delta F_p \delta F_{p'})_{\omega} = (-i\omega + I_p)^{-1} (i\omega + I_{p'})^{-1} [(I_p + I_{p'}) F_p \delta_{pp'} - I_{pp'}^{ee} \{F, F\}]. \quad (6)$$

Here

$$I_p = I_p^{th} + eE\partial_p + I_p^{ee}(F) \quad (7)$$

is the linearized operator of the kinetic equation (4); $I_p^{ee}(F)$ is a linear operator obtained by linearization of the pair collisions. The "source of the additional correlation" $I_{pp'}^{ee} \{F, F\}$ differs from the pair-collision integral in the absence of one summation:

$$\sum_{pp'} I_{pp'}^{ee} \{F, F\} = I_p^{ee} \{F, F\}. \quad (8)$$

For the spectral density of the current fluctuations

$$(\delta j_{\alpha} \delta j_{\beta})_{\omega} = \left(\frac{e}{v_0}\right)^2 \sum_{pp'} v_{\alpha} v_{\beta} (\delta F_p \delta F_{p'})_{\omega}, \quad (9)$$

using (6), we obtain the expression [see Eq. (1.68) in Ref. 9]:

$$\begin{aligned} (\delta j_{\alpha} \delta j_{\beta})_{\omega} &= \frac{e^2 n_0}{v_0} \left[\sum_p v_{\alpha} (-i\omega + I_p)^{-1} (v_{\beta} - V_{\beta}') \partial_p F_p + \text{H.c.} \right] \\ &+ \left(\frac{e}{v_0}\right)^2 \sum_{pp'} v_{\alpha} v_{\beta} (-i\omega + I_p)^{-1} (i\omega + I_{p'})^{-1} [(I_p + I_{p'}) \\ &\quad \times (F_p - N \partial_p F_p) \delta_{pp'} - I_{pp'}^{ee} \{F, F\}], \end{aligned} \quad (10)$$

where

$$V' = \sum_p v \partial_p F_p, \quad (11)$$

is the differential (with respect to the change of the concentration) drift velocity. Expression (10) is the starting point for the subsequent analysis.

3. THE RESPONSE PROBLEM

Our problem is to obtain the concrete form of the formal expression (10) in the case (2). To this end we must learn how to invert the response operator $(-i\omega + I_p)$, i. e., solve the equation for the response,

$$(-i\omega + I_p) g_p = X_p, \quad \sum_p X_p = 0, \quad (12)$$

in the case

$$\tau_e \ll \{\tau_p, \tau_e\}, \quad \omega \ll \tau_e^{-1}. \quad (13)$$

In the kinetic equation (4), under the condition (2), the electron-electron collision term shapes the distribution function and makes it close to a Maxwellian distribution with drift²⁵

$$F_p = F_p^M + \Delta F_p, \quad (14)$$

$$F_p^M = A \exp \{-(\varepsilon_p - pV)/T\}, \quad (15)$$

$$\Delta F_p / F_p^M \sim \tau_{ee} / \tau_p. \quad (16)$$

In exactly the same way, the form of the response g_p under the condition (13) is determined mainly by the linearized operator $I_p^{ee}(F)$ that enters in I_p . The operator $I_p^{ee} \{F, F\}$ is made to vanish by the function (15) with arbitrary independent parameters—the electron temperature T and their drift velocity V . Therefore

$$\partial_T I_p^{ee} \{F_p^M, F_p^M\} = I_p^{ee} (F_p^M) \partial_T F_p^M = 0,$$

$$\partial_V I_p^{ee} \{F_p^M, F_p^M\} = I_p^{ee} (F_p^M) \partial_V F_p^M = 0,$$

so that the zeros of the linearized operator $I_p^{ee}(F)$ are the functions $\partial_T F_p^M$ and $\partial_V F_p^M$. Moreover, they have the property

$$\sum_p \partial_T F_p^M = 0, \quad \sum_p \partial_V F_p^M = 0 \quad (17)$$

(actually $\sum_p \partial_T F_p^M = \partial_T N = 0$ and the same holds for $\partial_V F_p^M$), so that

$$g_p = B \partial_T F_p^M + C \partial_V F_p^M + o(\tau_{ee} / \tau_p, \omega \tau_{ee}). \quad (18)$$

The stationary values of the temperature and of the drift velocity of the electron gas are obtained from the energy and momentum balance equations that follow from (4). After multiplying (4) by ε_p or p and summing over the momenta, the integral of the electron-electron collisions vanish identity by virtue of the electron

and momentum conservation in electron-electron collisions. Substituting the principal part of the function (14) in the remaining terms of (4), we get

$$eE\nu - \dot{\varepsilon}^{th}(T, V; T_0) = 0, \quad (19)$$

$$eE - \dot{p}^{th}(T, V; T_0) = 0, \quad (20)$$

where $\dot{\varepsilon}^{th}$ and \dot{p}^{th} are the rates of energy and momentum losses by the electron system in the collisions with the thermostat:

$$\dot{\varepsilon}^{th}(T, V; T_0) = (\partial_t \varepsilon)_{coll} = \frac{1}{N} \sum_p \varepsilon_p I_p^{th} F_p^M, \quad (21)$$

$$\dot{p}^{th}(T, V; T_0) = (\partial_t p)_{coll} = \frac{1}{N} \sum_p p I_p^{th} F_p^M, \quad (22)$$

T_0 is the thermostat temperature.

The constants B and C in the expression (18) for the response can be obtained by multiplying (12) by ε_p or p and summing over the momenta. The terms with the electron-electron collisions then drop out identically, so that it suffices to substitute in the remaining terms the principal part of (18). Assuming that the medium is isotropic and that the electron dispersion law is parabolic ($\varepsilon_p = p^2/2m$), we get

$$B \left(-\frac{3}{2} i\omega + \partial_\tau \dot{\varepsilon}^{th} \right) + C_\alpha (-i\omega m V_\alpha + \partial_{V_\alpha} \dot{\varepsilon}^{th} - eE_\alpha) = \frac{1}{N} \sum_p \varepsilon_p X_p, \quad (23)$$

$$B \partial_\tau \dot{p}^{th} + C_\alpha (-i\omega m \delta_{\alpha\beta} + \partial_{V_\alpha} \dot{p}_\beta^{th}) = \frac{1}{N} \sum_p p_\beta X_p,$$

(summation over the dummy indices is implied). In the calculation of (10) we need not g_p itself, but only the "response current"

$$\sum_p v_\alpha g_p = \sum_p v_\alpha (-i\omega + I_p)^{-1} X_p = \sum_p v_\alpha (B \partial_\tau F_p^M + C_p \partial_{V_p} F_p^M) = NC_\alpha. \quad (24)$$

Choosing the z axis along the E direction, we obtain from (23)

$$\sum_p v_x g_p = \sum_p v_x (-i\omega + I_p)^{-1} X_p = \sum_p p_x X_p / [-i\omega m + (\partial_{V_p} \dot{\varepsilon}^{th})_x], \quad (25)$$

$$\sum_p v_z g_p = \sum_p v_z (-i\omega + I_p)^{-1} X_p = \sum_p (p_z + K_z(\omega) \varepsilon_p) X_p / m\Omega(\omega), \quad (26)$$

where

$$K_z(\omega) = -\partial_\tau \dot{p}_z^{th} / v(\omega), \quad v(\omega) = -\frac{3}{2} i\omega + \partial_\tau \dot{\varepsilon}^{th}, \quad (27)$$

$$\Omega(\omega) = -i\omega + \frac{1}{m} (\partial_{V_p} \dot{\varepsilon}^{th})_z + \left(-i\omega V_z + \frac{1}{m} d_{V_z} \dot{\varepsilon}^{th} - \frac{eE_z}{m} \right) K_z(\omega). \quad (28)$$

It remains to apply formulas (25) and (26) to (10).

4. SPECTRAL DENSITY OF CURRENT FLUCTUATIONS

The only nontrivial operation is the calculation of the double sum

$$\sum_{pp'} (p_\alpha + K_\alpha(\omega) \varepsilon_p) (p_{\beta'} + K_{\beta'}(\omega) \varepsilon_{p'}) [(I_p + I_{p'}) (F_p - N \partial_{V_p} F_p) \delta_{pp'} - I_{pp'}^{**} \{F, F\}]. \quad (29)$$

The linearized interelectron-collision operators that enter in the operator $(I_p + I_{p'})$ vanish identically after multiplication by the momentum or energy and summation. Consequently, it suffices to substitute in the first term in the square brackets of (29) the principal part of the function F_p , after which this term vanishes. We transform the second term using the energy and mo-

mentum conservation laws (cf. Ref. 11):

$$\Gamma = \sum_{pp'} (p_\alpha + K_\alpha(\omega) \varepsilon_p) (p_{\beta'} + K_{\beta'}(\omega) \varepsilon_{p'}) I_{pp'}^{**} \{F, F\} \quad (30)$$

$$= - \sum_p (p_\alpha + K_\alpha(\omega) \varepsilon_p) (p_\beta + K_\beta(\omega) \varepsilon_p) I_p^{**} \{F, F\},$$

followed by the use of the kinetic equation (4)

$$\Gamma = \sum_p (p_\alpha + K_\alpha(\omega) \varepsilon_p) (p_\beta + K_\beta(\omega) \varepsilon_p) [eE \partial_p + I_p^{**}] F_p. \quad (31)$$

In the last expression it suffices to substitute the principal part of F_p .

As a result we obtain for the transverse current fluctuations

$$(\delta j_\alpha^z)^2 = \frac{2e^2 n_0 T}{v_0 m^2} \frac{(\partial_{V_p} \dot{\varepsilon}^{th})_{xx} - (\dot{p}_x^z)^{th} / 2T}{\omega^2 + (\partial_{V_p} \dot{\varepsilon}^{th})_{xx}^2 / m^2}, \quad (32)$$

where

$$(\dot{p}_x^z)^{th} = \frac{1}{N} \sum_p p_x^2 I_p^{th} F_p^M. \quad (33)$$

We call attention to the fact that the term (33) was obtained (32) from expression (30), i. e., it is the result of the additional correlation.

The expression for the longitudinal current fluctuations are more cumbersome:

$$(\delta j_z^z)^2 = \frac{2e^2 n_0}{v_0 m} \left[T \operatorname{Re} \frac{1 + V_z K_z(\omega)}{\Omega(\omega)} + \frac{eE_z V_z - (\dot{p}_z^z)^{th} / 2m + L/2m |v(\omega)|^2}{|\Omega(\omega)|^2} \right], \quad (34)$$

where

$$L = \{ [eE_z V_z (5T + mV_z^2) - (\dot{\varepsilon}^z)^{th}] \partial_{V_z} \dot{p}_z^{th} - [eE_z (5T + 3mV_z^2) - 2(e p_z)^{th}] \partial_{V_z} \dot{\varepsilon}^{th} \} \partial_{V_z} \dot{p}_z^{th}, \quad (35)$$

$$(\dot{\varepsilon}^z)^{th} = \frac{1}{N} \sum_p \varepsilon_p^2 I_p^{th} F_p^M, \quad (e p_\alpha)^{th} = \frac{1}{N} \sum_p \varepsilon_p p_\alpha I_p^{th} F_p^M. \quad (36)$$

The two terms in (34) correspond to the two terms in (10)—the second term is due to the additional correlation.

5. EFFECT OF ADDITIONAL CORRELATION ON THE LONGITUDINAL AND TRANSVERSE NOISE TEMPERATURES

We compare now the expressions obtained for the spectral density of the current fluctuations with the corresponding responses of a nonequilibrium electron gas to a weak external alternating electric field, i. e., with the differential conductivity at the given frequency. From the general expression for the differential-conductivity tensor

$$\delta_{\alpha\beta}(\omega) = -\frac{e^2}{v_0} \sum_p v_\alpha (-i\omega + I_p)^{-1} \partial_p F_p, \quad (37)$$

[see, e. g., Ref. 9, Eq. (1.24)] we get in case (2), using (25) and (26)

$$\delta_{xx}(\omega) = \frac{e^2 n_0}{-i\omega m + (\partial_{V_p} \dot{\varepsilon}^{th})_{xx}}, \quad (38)$$

$$\delta_{zz}(\omega) = \frac{e^2 n_0}{m} \frac{1 + V_z K_z(\omega)}{\Omega(\omega)}. \quad (39)$$

Comparison of (34) with (39) and of (32) with (38) shows that neither the longitudinal noise temperature

$$T_{||}^{(n)}(\omega) = v_0 (\delta j_{||}^z)^2 / \omega^2 \operatorname{Re} \delta_{||}(\omega), \quad (40)$$

nor even the transverse noise temperature

$$T_{\perp}^{(n)}(\omega) = v_0 (\delta j_{\perp}^2)_{\omega} / 2 \operatorname{Re} \sigma_{\perp}(\omega) \quad (41)$$

of the nonequilibrium electron gas interacting with the thermostat in the case (2) (the electron-electron collisions control the form of the distribution function) in the entire frequency interval $\omega \tau_{ee} \ll 1$, generally speaking, is equal to the electron-gas temperature T . The difference is due to (and only to) the additional correlation.

We begin the investigation with the "high-frequency" case

$$\{\tau_p^{-1}, \tau_e^{-1}\} \ll \omega \ll \tau_{ee}^{-1}. \quad (42)$$

Retaining the first nonvanishing terms in $1/\omega\tau$, we have

$$(\delta j_x^2)_{\omega} = \frac{2e^2 n_0 T}{v_0 m^2 \omega^2} \left[(\partial_v \dot{p}^x)_{xx} - \frac{(\dot{p}_x^2)_{xx}}{2T} \right] \quad (43)$$

$$(\delta j_z^2)_{\omega} = \frac{2e^2 n_0 T}{v_0 m^2 \omega^2} \left[(\partial_v \dot{p}^z)_{zz} - \frac{(\dot{p}_z^2)_{zz}}{2T} + \frac{eE_z V_x m}{T} \right], \quad (44)$$

whereas

$$\operatorname{Re} \sigma_{\alpha\alpha}(\omega) = \frac{e^2 n_0}{m^2 \omega^2} (\partial_v \dot{p}^{\alpha})_{\alpha\alpha}, \quad (45)$$

so that the high-frequency noise temperatures are

$$T_{\perp}^{(n)}(\omega) = T [1 - (\dot{p}_x^2)_{xx} / 2T (\partial_v \dot{p}^x)_{xx}], \quad (46)$$

$$T_{\parallel}^{(n)}(\omega) = T [1 - (\dot{p}_z^2)_{zz} - 2eE_z V_x m] / 2T (\partial_v \dot{p}^z)_{zz}].$$

Before we consider (in the sections that follow) the lower frequencies, we make one general remark. There is a possible (albeit very special) case when the Maxwell distribution with drift is established in the electron gas not as a result of collisions between electrons, but because of the special form of the operator I_p^{th} , i.e., it is due to the interaction with the thermostat:

$$(eE\partial_p + I_p^{\text{th}}) F_p^{\text{M}} = 0. \quad (47)$$

In this case the stationary distribution of the electrons is Maxwellian with drift, independently of the frequency of the interelectron collisions. This means that in the case of frequent interelectron collisions [in case (2)] the correction ΔF_p is missing from (14), and consequently $I_{pp1}^{ee} \{F, F\} = 0$, i.e., in this special case the interelectron collisions produce no additional correlation in the stationary state. (That the second terms of (32) and (34) vanish under the condition (47) can be easily verified also directly.) The noise temperatures (40) and (41) are then equal to the electron temperatures: $T_{\perp}^{(n)} = T_{\parallel}^{(n)} = T$. The equality of the noise and electron temperatures in the case of a Maxwellian distribution with drift, brought about by relaxation mechanisms, was noted by Nougier and Rolland.²⁴

6. LOW FREQUENCY CURRENT FLUCTUATIONS. COMPARISON WITH THE DIFFUSION COEFFICIENT

We compare now the low-frequency fluctuations with the diffusion coefficient of a nonequilibrium electron gas. The diffusion current, as the response to a smooth density gradient of nonequilibrium carriers, was determined by Wannier.²² He expressed the diffusion coefficient in terms of the solution of the corresponding spatially homogeneous kinetic equation. The generalization to the case of a nonlinear kinetic equa-

tion was given in Ref. 7:

$$D_{\alpha\beta} = \sum_p v_{\alpha} I_p^{-1} (v_{\beta} - V_{\beta}') \partial_{\pi} F_p, \quad (48)$$

[V' is defined in (11)]. It was noted there that the low-frequency spectral density of the current fluctuations is generally speaking, not expressed in terms of $D_{\alpha\beta}$ [cf. (48) and (10)]:

$$(\delta j_{\alpha} \delta j_{\beta})_{\omega} = \frac{e^2 n_0}{v_0} (D_{\alpha\beta} + D_{\beta\alpha} - \Delta_{\alpha\beta}), \quad \omega \ll \{\tau_p^{-1}, \tau_e^{-1}\}, \quad (49)$$

where $\Delta_{\alpha\beta}$ is determined by the second term of (19). In our case (2) we have according to (32) and (34)

$$D_{xx} = T / (\partial_v \dot{p}^x)_{xx} = T \bar{\sigma}_{\perp} / e^2 n_0, \quad (50)$$

$$\Delta_{xx} = (\dot{p}_x^2)_{xx} / (\partial_v \dot{p}^x)_{xx}^2, \quad (51)$$

whereas for the longitudinal components

$$D_{zz} = T \frac{\partial_x \dot{p}^z - V_x \partial_x \dot{p}_z^z}{(\partial_v \dot{p}^z)_{zz} \partial_x \dot{p}^z - (\partial_v \dot{p}_z^z - eE_z) \partial_x \dot{p}_z^z} = \frac{T \bar{\sigma}_{\parallel}}{e^2 n_0}, \quad (52)$$

$$\Delta_{zz} = \frac{[(\dot{p}_z^2)_{zz} - eE_z V_x m] (\partial_x \dot{p}^z)^2 - L}{[(\partial_v \dot{p}^z)_{zz} \partial_x \dot{p}^z - (\partial_v \dot{p}_z^z - eE_z) \partial_x \dot{p}_z^z]^2}, \quad (53)$$

[L is defined by (35)]. We use $\bar{\sigma}_{\perp}$ and $\bar{\sigma}_{\parallel}$ to denote the corresponding components of the differential conductivity tensors at low frequencies $\omega \ll \{\tau_p^{-1}, \tau_e^{-1}\}$; the expressions for them follow from (38) and (39).

The connection between the corresponding components of the diffusion-coefficient and low-frequency-differential-conductivity tensors, as stated by Eqs. (50) and (52):

$$D_{\perp} = T \bar{\sigma}_{\perp} / e^2 n_0, \quad D_{\parallel} = T \bar{\sigma}_{\parallel} / e^2 n_0,$$

is evidence of the applicability of the so-called "Wannier conjecture"^{22,23} to the case (2). This connection in case (2) is by far no accident and is in no way connected with the assumptions that the medium is isotropic and the electron dispersion is parabolic and isotropic, under which expression (50) and (52) were derived. As follows from the general expressions (48) and (37), these tensors are proportional to each other to the extent that the stationary distribution of the electrons is Maxwellian with drift:

$$D_{\alpha\beta} = T \bar{\sigma}_{\alpha\beta} / e^2 n_0 + o(\Delta F_p / F_p^{\text{M}}). \quad (54)$$

Expressions (49)–(52) mean that in our case (2), owing to the essential role of the additional correlation, the Price fluctuation-diffusion relation

$$(\delta j_{\alpha} \delta j_{\beta})_{\omega \ll 1} = \frac{e^2 n_0}{v_0} (D_{\alpha\beta} + D_{\beta\alpha})$$

is violated, and not only in the longitudinal but also in the transverse direction.

7. CURRENT FLUCTUATIONS AT $\tau_{ee} \ll \tau_p \ll \tau_e$

We proceed to the case when the electron system loses energy much more slowly than momentum. Discarding terms of order τ_p / τ_e we obtain from (32) and (34)

$$(\delta j_x^2)_{\omega} = \frac{2e^2 n_0 T}{v_0 m^2} \frac{(\partial_v \dot{p}^x)_{xx}}{\omega^2 + (\partial_v \dot{p}^x)_{xx}^2 / m^2} = \frac{2T}{v_0} \bar{\sigma}_{\perp}(\omega), \quad (55)$$

$$(\delta j_z^2)_{\omega} = \frac{2e^2 n_0 T}{v_0 m^2} \frac{\Theta_{\perp}(\omega) + 2L_{\perp} / 9T}{\Lambda_{\perp}(\omega)}, \quad (56)$$

whereas

$$\delta_{xx}(\omega) = \frac{e^2 n_0}{m^2} \frac{\Theta_{<}(\omega)}{\Lambda_{<}(\omega)}, \quad (57)$$

where we have put

$$\Lambda_{<}(\omega) = \omega^4 + \omega^2 (\partial_{\nu} \dot{p}^{ih})_{xx}^2 / m^2 + \omega^2 R_{<}^2 / m^2, \quad (58)$$

$$\Theta_{<}(\omega) = \omega^2 (\partial_{\nu} \dot{p}^{ih})_{xx} + \omega^2 (\partial_{\tau} \dot{e}^{ih} - V_x \partial_{\tau} \dot{p}_x^{ih}) R_{<} \quad (59)$$

$$R_{<} = (\partial_{\nu} \dot{p}^{ih})_{xx} \partial_{\tau} \dot{e}^{ih} + e E_x \partial_{\tau} \dot{p}_x^{ih}, \quad (60)$$

whereas

$$L_{<} = \{ [5eE_x V_x T - (\dot{e}^i)^{ih}] \partial_{\tau} \dot{p}_x^{ih} + 2T^2 (\partial_{\tau} \dot{p}_x^{ih}) \partial_{\tau} \dot{e}^{ih} \} \partial_{\tau} \dot{p}_x^{ih} \quad (61)$$

is the same as (35), in which we used the smallness of τ_p / τ_e and the balance equation (19). In particular, in this case

$$2(e p_x)^{ih} = 2T^2 \partial_{\tau} \dot{p}_x^{ih} + 5T \dot{p}_x^{ih}. \quad (62)$$

In the same approximation

$$(\partial_{\nu} \dot{p}^{ih})_{xx} = (\partial_{\nu} \dot{p}^{ih})_{yy} = (\partial_{\nu} \dot{p}^{ih})_{zz} = \dot{p}_x^{ih} / V_x. \quad (63)$$

Equations (55)–(57) cover a wide range of frequencies $\omega \ll \tau_{ee}^{-1}$. We note that because of the inequality $\tau_p \ll \tau_e$, a simple relation between $(\delta j_x^2)_{\omega}$ and $\bar{\sigma}_{\perp}(\omega)$ was established in this entire temperature interval [see (55)]. In the case $\omega \tau_p \ll 1$ it is necessary to discard in (58) the term ω^4 . Expressions (56) and (57) then take the more illustrative form

$$\text{Re} (\delta_{xx}(\omega) / \bar{\sigma}_{\perp} - 1) = (\bar{\sigma}_{\parallel} / \bar{\sigma}_{\perp} - 1) (1 + \omega^2 \tau_r^2)^{-1}, \quad (64)$$

where

$$\tau_r = \frac{1}{2} (\partial_{\tau} \dot{e}^{ih} + V_x \partial_{\tau} \dot{p}_x^{ih})^{-1} = \frac{1}{2} n_0 (n_0 \partial_{\tau} \dot{e}^{ih} - E_x^2 \partial_{\tau} \bar{\sigma}_{\perp})^{-1} \quad (65)$$

is the electron-temperature relaxation time;

$$(\delta j_x^2)_{\omega} = \frac{2T}{v_0} \bar{\sigma}_{\perp} \left[1 + (1 + \omega^2 \tau_r^2)^{-1} \left(\frac{\bar{\sigma}_{\parallel}}{\bar{\sigma}_{\perp}} - 1 + \frac{2\tau_r^2 \bar{\sigma}_{\perp} L_{<}}{9n_0 e^2 T} \right) \right]. \quad (66)$$

In the derivation of (64) and (66) we used the balance equations (19) and (20), $\bar{\sigma}_{\parallel}$ and $\bar{\sigma}_{\perp}$ are the longitudinal and transverse components of the differential conductivity as $\omega \rightarrow 0$:

$$\bar{\sigma}_{\parallel} = \bar{\sigma}_{\perp} \left[1 + \frac{4E_x^2}{3n_0} \partial_{\tau} \bar{\sigma}_{\perp} \right], \quad \bar{\sigma}_{\perp} = \frac{e^2 n_0}{(\partial_{\nu} \dot{p}^{ih})_{xx}} = \frac{en_0 V_x}{E_x}. \quad (67)$$

In the "high-frequency" case $\tau_p^{-1} \ll \omega \ll \tau_{ee}^{-1}$, on the other hand, we get

$$(\delta j_x^2)_{\omega} = \frac{2e^2 n_0 T}{v_0 m^2} \frac{(\partial_{\nu} \dot{p}^{ih})_{xx}}{\omega^2} = \frac{2T}{v_0} \bar{\sigma}_{\perp}(\omega), \quad (68)$$

$$(\delta j_x^2)_{\omega} = \frac{2e^2 n_0 T}{v_0 m^2} \frac{(\partial_{\nu} \dot{p}^{ih})_{zz}}{\omega^2} = \frac{2T}{v_0} \bar{\sigma}_{\parallel}(\omega), \quad (69)$$

and also [see (64)]

$$\bar{\sigma}_{\parallel}(\omega) = \bar{\sigma}_{\perp}(\omega), \quad (70)$$

i. e., a simple relation between the spectral density of the current fluctuations and the coefficient of the differential conductivity was established also in a direction longitudinal relative to the field.

The expression (6) for $(\delta j_x^2)_{\omega}$ at $\omega \tau_p \ll 1$ becomes even simpler if we assume in addition to the smallness of the parameter also a weak inelasticity of the scattering by the thermostat ($\Delta \varepsilon \ll \bar{\varepsilon}$, where $\Delta \varepsilon$ is the characteristic change of electron energy upon collision). Then

$$(\dot{e}^i)^{ih} = 2T^2 \partial_{\tau} \dot{e}^{ih} + 3T \dot{e}^{ih} - 2 \frac{TT_0}{T - T_0} \dot{e}^{ih} \quad (71)$$

[see Ref. 11; in Ref. 9 this is Eq. (3.133)]. Now

$$L_{<} = \frac{9n_0 e^2}{8\tau_r^2 \bar{\sigma}_{\perp}} \frac{T^2}{T - T_0} \left(\frac{\bar{\sigma}_{\parallel}}{\bar{\sigma}_{\perp}} - 1 \right)^2 \quad (72)$$

and (66) takes the form

$$(\delta j_x^2)_{\omega} = \frac{2T \bar{\sigma}_{\perp}}{v_0} \left[1 + \left(\frac{\bar{\sigma}_{\parallel}}{\bar{\sigma}_{\perp}} - 1 \right) (1 + \omega^2 \tau_r^2)^{-1} \left(1 + \frac{T}{4(T - T_0)} \left(\frac{\bar{\sigma}_{\parallel}}{\bar{\sigma}_{\perp}} - 1 \right) \right) \right]. \quad (73)$$

Thus, in our case in the quasielastic approximation the spectral density of the current fluctuations is expressed in terms of the static differential conductivities, the electron temperature T , and its relaxation time just as in case (1) (see Refs. 11 and 9). It must be borne in mind, however, that the contribution of the additional correlation between the occupation numbers of the single-electron states, due to the interelectron collisions, to $(\delta j_x^2)_{\omega}$ is different in cases (1) and (2). In our case it equals

$$\frac{T \bar{\sigma}_{\perp}}{2v_0} (1 + \omega^2 \tau_r^2)^{-1} \left(\frac{\bar{\sigma}_{\parallel}}{\bar{\sigma}_{\perp}} - 1 \right)^2 \frac{T}{T - T_0}, \quad (74)$$

whereas in case (1) it was

$$\frac{T \sigma}{2v_0} (1 + \omega^2 \tau_r^2)^{-1} \left(\frac{\sigma_{\parallel}}{\sigma} - 1 \right) \left[\frac{T}{T - T_0} \left(\frac{\sigma_{\parallel}}{\sigma} - 1 \right) - \frac{2T}{\sigma} \partial_{\tau} \sigma \right] \quad (75)$$

(see Ref. 11 or (1.136) in Ref. 9).

8. CONCLUSION

Thus, in a strong electric field in an isotropic medium, in the case of frequent collisions between the electrons, the quantity

$$(\delta j_x \delta j_y)_{\omega \tau_{ee} \ll 1}$$

ceases to be proportional to $\bar{\sigma}_{\alpha\beta}$, but the proportionality to $D_{\alpha\beta}$ is preserved.

In this case $(\delta j_x \delta j_y)_{\omega}$ contains two characteristic terms: a term proportional to $\bar{\sigma}_{\alpha\beta}$ (or $D_{\alpha\beta}$) and an additional correlation. This distinguishes the case of a Maxwellian distribution with drift, due to the frequent electron collisions ($\tau_{ee} \ll \tau$) from the case of the effective electron temperature $\tau_p \ll \tau_{ee} \ll \tau_e$, in which the quantities

$$(\delta j_x \delta j_y)_{\omega \tau_{ee} \ll 1}, \bar{\sigma}_{\alpha\beta}, D_{\alpha\beta}$$

are not proportional to one another (see Refs. 10, 11, 13, 14, and 9) and $(\delta j_x^2)_{\omega}$ consists of three characteristic terms: proportional to $\bar{\sigma}_{\parallel}$, "transforming" $\bar{\sigma}_{\parallel}$ into D_{\parallel} , and the additional correlation.

Therefore the experimental separation of an interesting physical effect—the contribution made to the current fluctuations by the additional correlation due to collisions between the electrons—may turn out to be relatively simple in the case of frequent collisions between the electrons $\tau_{ee} \ll \tau$. If it is established independently that the case $\tau_{ee} \ll \tau$ is realized in experiment, then to investigate the additional correlation it suffices to compare (δj^2) with $\text{Re} \bar{\sigma}(\omega)$ (i. e., it is not mandatory, as in the case $\tau_p \ll \tau_{ee} \ll \tau_e$, to have independently measured δj^2 and D). This makes an experimental investigation of the current fluctuations in the case of frequent interelectron collisions $\tau_{ee} \ll \tau$ quite enticing.

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