Anomalous absorption of microwave radiation by a plasma in the electron cyclotron band

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We study the anomalous absorption of radiation which is connected with the parametric excitation of high-frequency upper-hybrid and low-frequency ion-sound or lower-hybrid oscillations and which occurs when we pump with a slow extraordinary electromagnetic wave which propagates across a magnetic field in a plasma. We show that when the wavelength of the pump field is less than some minimum value in the region near the threshold "red" and "blue" pump wave satellites are excited under weak coupling conditions. When the pump field frequency lies between the electron cyclotron frequency and twice its value, the upper hybrid oscillations excited in a rarefied plasma have an anomalous dispersion which leads to an energy transfer to excitations with a shorter wavelength in the case of a cascade mechanism for the saturation of the parametric instability. In that case the intensity of the blue satellite in the high-frequency turbulence spectrum excited under resonance conditions turns out to be of the same order of magnitude as the intensity of the Stokes line; this agrees qualitatively with the experimentally observed results.

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The anomalous absorption occurring during microwave heating of a plasma in a magnetic field may be connected with the parametric excitation of strong plasma oscillations. For instance, when we pump with a slow extraordinary (SE) electromagnetic wave propagating at right angles to the external magnetic field we observe the excitation of high-frequency upper-hybrid (HU) and low-frequency (LF) ion-sound or lower hybrid (LH) oscillations.[1-4] The spectrum of the high-frequency (HF) perturbations then consists of a set of peaks which are spaced at the frequency of the LF wave. One can explain the existence of such a spectrum by a cascade decay process of HU waves into a HU wave with a smaller frequency and LF perturbations, similar to what happens to Langmuir oscillations in an isotropic plasma.[5-7] However, in contrast to the results of Refs. 5–7 it turned out that in the region near the threshold there may appear in the HU perturbations spectrum a "blue" satellite as well as the "red" peaks (i.e., waves with frequencies lower than the pumping frequency). Its existence is connected with the finite value of the wavelength of the pump field. The HU oscillations excited by the SE wave have an anomalous dispersion and this also leads to an effect which differs from the characteristics for Langmuir waves with which we compare it[5-7]: energy transfer in the direction of shorter wavelengths.

The paper is divided into three sections. In section 1 we consider the linear stage of the parametric instability in the field of the SE wave. We show that when we take the finite wavelength of the pump field into account there is realized a situation different from the ones studied earlier and it is possible to satisfy simultaneously resonance conditions for red and blue satellites. We find the growth rates and threshold values of the fields for the excitation of the Stokes and anti-Stokes satellites of the pump wave in the case of ion-sound and lower-hybrid LF perturbations.

In section 2 we study for the determination of the amplitudes of the electrical fields of the satellites the non-linear stage of the parametric instability in the case of a cascade mechanism for the saturation of the plasma turbulence. We find the stationary values of the amplitudes of the satellites in the case of collisional absorption of the HF waves in their dependence on the field strength. We discuss also the effect of electron cyclotron absorption in a strong magnetic field.

In section 3 we give estimates of the magnitude of the power absorbed in the plasma and of the effective relaxation length of the pump wave. We discuss the agreement of the theory developed here with the experimentally observed results.

1. CONDITIONS FOR THE EXCITATION OF PLASMA OSCILLATIONS WITH AN ANOMALOUS DISPERSION

We consider a situation which is often encountered in experiments when vhf radiation in the form of an SE wave (a pump wave of frequency ω_0 and wavevector $\mathbf{k}_0$) propagates in the plasma at right angles to the external magnetic field $\mathbf{B}_0$ ($\mathbf{k}_0 \perp \mathbf{B}_0$). The SE wave is then polarized in such a way that its electrical field vector $\mathbf{E}_0$ lies in the plane perpendicular to the external magnetic field:

$$\mathbf{E}_0(\omega,k)=\frac{1}{2}E_{0x}\left\{\left(-i\frac{\Omega_e}{\omega_0}\frac{\omega_{Le}^2}{\omega_0^2-\omega_H^2}\varkappa_0+[\varkappa_0\mathbf{h}]\right)\delta(\omega-\omega_0)\delta(\mathbf{k}-\mathbf{k}_0)+\left(i\frac{\Omega_e}{\omega_0}\frac{\omega_{Le}^2}{\omega_0^2-\omega_H^2}\varkappa_0+[\varkappa_0\mathbf{h}]\right)\delta(\omega+\omega_0)\delta(\mathbf{k}+\mathbf{k}_0)\right\},\quad \varkappa_0=\mathbf{k}_0/k_0,\quad \mathbf{h}=\mathbf{B}_0/B_0. \tag{1}$$

Here ω_{Le} and Ω_e are the electron Langmuir and gyrofrequencies and $\omega_H=(\omega_{Le}^2+\Omega_e^2)^{1/2}$ is the upper hybrid resonance frequency.

For an elucidation of the conditions for the excitation of LF potential plasma oscillations with frequency ω and wavevector $\mathbf{k}$ by the pump wave (1) we turn to the dispersion relation obtained in the paper by Pustovalov and one of the present authors[8] from which we find

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using standard procedures

$$\frac{1}{1+\delta\varepsilon_e(\omega+i\gamma,\mathbf{k})}+\frac{1}{\delta\varepsilon_i(\omega+i\gamma,\mathbf{k})}+\left(\frac{keE_{0x}}{2m(\omega_0^2-\Omega_e^2)}\right)^2$$
$$\times\left\{\left[a^2\frac{((\mathbf{k}-\mathbf{k}_0)\mathbf{e}_{0x})^2}{|\mathbf{k}-\mathbf{k}_0|^2}+b^2\frac{((\mathbf{k}-\mathbf{k}_0)\mathbf{e}_{0y})^2}{|\mathbf{k}-\mathbf{k}_0|^2}\right]\frac{1}{\varepsilon(\omega-\omega_0+i\gamma,\mathbf{k}-\mathbf{k}_0)}\right.$$
$$\left.+\left[a^2\frac{((\mathbf{k}+\mathbf{k}_0)\mathbf{e}_{0x})^2}{|\mathbf{k}+\mathbf{k}_0|^2}+b^2\frac{((\mathbf{k}+\mathbf{k}_0)\mathbf{e}_{0y})^2}{|\mathbf{k}+\mathbf{k}_0|^2}\right]\frac{1}{\varepsilon(\omega+\omega_0+i\gamma,\mathbf{k}+\mathbf{k}_0)}\right\}=0. \quad (2)$$

In Eq. (2), e and m are the electron charge and mass, $\delta\varepsilon_{e,i}$ are the partial electron and ion plasma dielectric permittivities, $\varepsilon=1+\delta\varepsilon_e+\delta\varepsilon_i$, the polarization factors a and b are defined by the equations

$$a=1+\frac{\Omega_e^2}{\omega_0^2}\frac{\omega_{Le}^2}{\omega_0^2-\omega_H^2},\quad b=\frac{\Omega_e}{\omega_0}\frac{\omega_0^2-\Omega_e^2}{\omega_0^2-\omega_H^2},$$

γ is the growth rate of the parametric instability. The external magnetic field $\mathbf{B}_0$ is directed along the Z-axis and the pump wavevector $\mathbf{k}_0$ along the Y-axis; $\mathbf{e}_{ox}$ and $\mathbf{e}_{oy}$ are unit vectors along the X- and Y-axes of the system of coordinates used here.

In the limit $\mathbf{k}_0=0$ the dispersion relation (2) is the same as the equation for potential perturbations of a plasma in the field of a linearly polarized ($a=1$, $b=\Omega_e\omega_0^{-1}$) wave (see Refs. 9, 10).

It turns out to be important for the effect discussed below to take into account the fact that the pump wave has a large, but finite wavelength, $k_0\ll k$. This allows us to write the resonance denominators in Eq. (2) in the form

$$\varepsilon(\omega+\omega_0+i\gamma,\mathbf{k}+\mathbf{k}_0)=\frac{\partial\varepsilon'(\omega_0,\mathbf{k})}{\partial\omega_0}[\Delta\omega_0+\Omega+i(\gamma+\tilde{\gamma})],$$
$$\varepsilon(\omega-\omega_0+i\gamma,\mathbf{k}-\mathbf{k}_0)=\frac{\partial\varepsilon'(\omega_0,\mathbf{k})}{\partial\omega_0}[\Delta\omega_0-\Omega-i(\gamma+\tilde{\gamma})], \quad (3)$$

where the mismatch $\Delta\omega_0$ is equal to

$$\Delta\omega_0=\varepsilon'(\omega_0,\mathbf{k})\left(\frac{\partial\varepsilon'(\omega_0,\mathbf{k})}{\partial\omega_0}\right)^{-1}\equiv\varepsilon_0'\bigg/\frac{\partial\varepsilon_0'}{\partial\omega_0}, \quad (4)$$

and $\tilde{\gamma}=\varepsilon''(\omega_0,\mathbf{k})(\partial\varepsilon_0'/\partial\omega_0)^{-1}$ is the damping rate of the HF wave.

It is important that in the resonance denominators in (3) in the case of excitation by a finite wavelength pump wave there occurs the shifted frequency of the LF field

$$\Omega=\omega+\mathbf{k}_0\frac{\partial\varepsilon'(\omega_0,\mathbf{k})}{\partial\mathbf{k}}\left(\frac{\partial\varepsilon'(\omega_0,\mathbf{k})}{\partial\omega_0}\right)^{-1}. \quad (5)$$

Substituting the expansion (3) into the dispersion Eq. (2) and separating the real and imaginary parts we get the equations determining the frequency $\omega(\mathbf{k})$:

$$1+\delta\varepsilon_e'(\omega,\mathbf{k})+\delta\varepsilon_i'(\omega,\mathbf{k})+2(1+\delta\varepsilon_e'(\omega,\mathbf{k}))\delta\varepsilon_i'(\omega,\mathbf{k})\left(\frac{\partial\varepsilon_0'}{\partial\omega_0}\right)^{-1}$$
$$\times\left(\frac{keE_{0x}}{2m(\omega_0^2-\Omega_e^2)}\right)^2\{[(\Delta\omega_0)^2-\Omega^2+(\gamma+\tilde{\gamma})^2]^2+4\Omega^2(\gamma+\tilde{\gamma})^2\}^{-1}$$
$$\times\left\{[(\Delta\omega_0)^2-\Omega^2+(\gamma+\tilde{\gamma})^2]\left[(b^2(\varkappa\varkappa_0)^2+a^2(1-(\varkappa\varkappa_0)^2))\Delta\omega_0\right.\right.$$
$$\left.+2(\varkappa\varkappa_0)\frac{k_0}{k}(a^2-b^2)(1-(\varkappa\varkappa_0)^2)\Omega\right]$$
$$\left.-4(\gamma+\tilde{\gamma})^2\Omega(\varkappa\varkappa_0)\frac{k_0}{k}(a^2-b^2)(1-(\varkappa\varkappa_0)^2)\right\}=0, \quad (6)$$

and the growth rate $\gamma(\mathbf{k})$ for the excitation of the plasma oscillations:

$$\gamma+\gamma_s+(1+\delta\varepsilon_e'(\omega,\mathbf{k}))\delta\varepsilon_i(\omega,\mathbf{k})\left(\frac{\partial\varepsilon_0'}{\partial\omega_0}\frac{\partial\varepsilon'(\omega,\mathbf{k})}{\partial\omega}\right)^{-1}$$
$$\times\left(\frac{keE_{0x}}{m(\omega_0^2-\Omega_e^2)}\right)^2\{[(\Delta\omega_0)^2-\Omega^2+(\gamma+\tilde{\gamma})^2]^2+4\Omega^2(\gamma+\tilde{\gamma})^2\}^{-1}$$
$$\times(\gamma+\tilde{\gamma})\left[\Delta\omega_0\Omega(b^2(\varkappa\varkappa_0)^2+a^2(1-(\varkappa\varkappa_0)^2))\right.$$
$$\left.+[(\Delta\omega_0)^2+\Omega^2+(\gamma+\tilde{\gamma})^2](\varkappa\varkappa_0)\frac{k_0}{k}(a^2-b^2)(1-(\varkappa\varkappa_0)^2)\right]=0. \quad (7)$$

Here $\gamma_s=\varepsilon''(\omega,\mathbf{k})(\partial\varepsilon'(\omega,\mathbf{k})/\partial\omega)^{-1}$ is the damping rate of the LF wave.

It follows from (7) that the pump wave (1) excites plasma oscillations in the following range of mismatches $\Delta\omega_0$ and frequencies Ω:

$$\Delta\omega_0\Omega[b^2(\varkappa\varkappa_0)^2+a^2(1-(\varkappa\varkappa_0)^2)]+[(\Delta\omega_0)^2+\Omega^2+(\gamma+\tilde{\gamma})^2]$$
$$\times(\varkappa\varkappa_0)\frac{k_0}{k}(a^2-b^2)(1-(\varkappa\varkappa_0)^2)>0. \quad (8)$$

The maximum growth rate $\gamma(\mathbf{k})$ is reached in the case of decay conditions $\Delta\omega_0=\Omega$ when the real part of the plasma dielectric permittivity vanishes at high frequencies. The minimum threshold for pumping $E_{0\min}$ occurs then for plasma waves propagating along the Y-axis: $(\mathbf{n}\cdot\mathbf{n}_0)^2=1$. The high-frequency decay products of the pumping–potential HU oscillations–in a rarefied plasma, $\omega_{Le}^2<3\Omega_e^2$, have an anomalous dispersion:

$$\omega_{\rm HF}=\omega_H-\frac{\omega_{Le}^2\Omega_e^2}{2\omega_H^3}(\Delta\vartheta)^2+\frac{3k^2v_{Te}^2\omega_{Le}^2}{2\omega_H(\omega_{Le}^2-3\Omega_e^2)},$$

and thanks to that the decay of the pump wave (1) considered by us turns out to be possible; the mismatch (4) is then equal to

$$\Delta\omega_0=\frac{\omega_0^2-\Omega_e^2}{2\omega_0\omega_{Le}^2}\left\{\omega_0^2-\omega_H^2-3\frac{k^2v_{Te}^2\omega_{Le}^2}{\omega_0^2-4\Omega_e^2}\right.$$
$$+(\Delta\vartheta)^2\left[\frac{\Omega_e^2}{\omega_0^2}(\omega_0^2-\Omega_e^2)\left(1+\frac{\omega_0^2-\omega_H^2}{\omega_0^2}\right)-6\frac{k^2v_{Te}^2\omega_{Le}^2}{\omega_0^4}\right.$$
$$\left.\left.\times\left(\frac{(\omega_0^2-\Omega_e^2/2)\Omega_e^2}{\omega_0^2-4\Omega_e^2}+\frac{\omega_0^4}{\omega_0^2-4\Omega_e^2}-\frac{1}{6}\frac{6\omega_0^6-3\omega_0^4\Omega_e^2+\Omega_e^4\omega_0^2}{(\omega_0^2-\Omega_e^2)^2}\right)\right]\right\}, \quad (9)$$

and the frequency (5) has the form

$$\Omega=\omega-\frac{\omega_0^2-\Omega_e^2}{\omega_0}\frac{\mathbf{k}_0\mathbf{k}}{k^2}\left[3\frac{k^2v_{Te}^2}{\omega_0^2-4\Omega_e^2}\right.$$
$$\left.+(\Delta\vartheta)^2\left(\frac{\Omega_e^2}{\omega_0^2}+3\frac{k^2v_{Te}^2\Omega_e^2(2\omega_0^2-\Omega_e^2)}{\omega_0^4(\omega_0^2-4\Omega_e^2)}\right)\right]. \quad (10)$$

The angle ϑ between the wavevector $\mathbf{k}$ and the magnetic field $\mathbf{B}_0$ is here close to $\frac{1}{2}\pi$ ($\vartheta=\frac{1}{2}\pi-\Delta\vartheta$, $\Delta\vartheta\ll\vartheta$), and v_{Te} is the electron thermal velocity.

In experiments ion-sound oscillations with frequency[4] $\omega_s=kv_s$ (v_s: sound velocity) or LH oscillations[3] with frequency

$$\omega_L=\omega_{Li}(1+\Omega_i^2\omega_{Li}^{-2})^{1/2}(1+\omega_{Le}^2\Omega_e^{-2})^{-1/2}$$

(ω_{Li} and Ω_i are the ion Langmuir and gyro-frequencies) are registered as LF decay products. In what follows we consider ion-sound LF oscillations; at the end of the section we shall say something about the change in the obtained results when we apply them to LH oscillations.

According to (7) the minimum threshold field (for $\gamma=0$) for which ion-sound oscillations must be excited with a frequency less than the ion Langmuir frequency is determined by the relation[1)] (n_e is the electron density in the plasma)

$$E_{0\,min}^2=64\pi n_e \varkappa_B T_e \frac{\tilde{\gamma}\gamma_s}{\omega_0\omega_s}\frac{\omega_0^2}{\omega_{Le}^2}\left[\left(\frac{\omega_{Le}^2}{\omega_0^2-\Omega_e^2}\right)^2 + \frac{\omega_0^2}{\Omega_e^2}\left(\frac{\omega_0^2-\omega_H^2}{\omega_0^2-\Omega_e^2}\right)^2\right]\left(1+\left(\frac{\tilde{\gamma}}{2\Omega}\right)^2\right). \tag{11}$$

In Eq. (11) T_e is the temperature of the plasma electrons and $\varkappa_B$ the Boltzmann constant. The contribution from the ion-ion collisions to the ion-sound damping rate turns out to be negligibly small as compared to the Landau damping. For a comparison we note that ion-ion collisions make a considerable contribution to the damping of slow magneto-acoustic waves with a spectrum $\omega = kv_s|\cos\vartheta|$. The corresponding threshold fields for the excitation of such oscillations are appreciably higher than the threshold field (11) and the slow magneto-acoustic waves are therefore not observed experimentally.[4]

The damping rate of the HF waves for small, but finite values of the component $k_z (k_z \ll k_\perp)$ of the wavevector **k** along the direction of the magnetic field $\mathbf{B}_0$

$$\frac{\tilde{\gamma}}{\omega_0} = \nu_{ei}\frac{\omega_{Le}^2(\omega_0^2+\Omega_e^2)}{2\omega_0^5} + \left(\frac{\pi}{8}\right)^{1/2}\frac{(\omega_0^2-\Omega_e^2)^2}{\omega_0(kv_{Te})^3|\cos\vartheta|} \times\left\{\exp\left(-\frac{\omega_0^2}{2(kv_{Te})^2\cos^2\vartheta}\right) + \frac{k^2v_{Te}^2}{2\Omega_e^2}\exp\left(-\frac{(\omega_0-|\Omega_e|)^2}{2(kv_{Te})^2\cos^2\vartheta}\right) + \frac{k^4v_{Te}^4}{8\Omega_e^4}\exp\left(-\frac{(\omega_0-2|\Omega_e|)^2}{2(kv_{Te})^2\cos^2\vartheta}\right)\right\} \tag{12}$$

is (for a given value of $|\cos\vartheta|$) determined by the electron-ion collisions with frequency ν_{ei} in the wavelength region $k < k_{st}$ (k_{st} is defined by the contributions of the collisional and the collisionless absorption to Eq. (12) being equal). For shorter wavelengths $k > k_{st}$ the collisionless absorption makes the main contribution to the damping (12).

In strong magnetic fields $\Omega_e^2 \gg \omega_{Le}^2$ (which is, for instance, realized in toroidal apparatus[11]) collisionless absorption of HF waves occurs due to electron cyclotron absorption which is important when $k_z > k_{z\,st}$ where (r_{De} is the electron Debye radius)

$$k_{z\,st}=\frac{1}{r_{De}}\frac{\omega_0-|\Omega_e|}{2^{1/2}\omega_{Le}}\left\{\ln\frac{\pi^{1/2}\omega_0^2(\omega_0-|\Omega_e|)(\omega_0+|\Omega_e|)^2}{2\nu_{ei}\omega_{Le}^2(\omega_0^2+\Omega_e^2)}\right\}^{-1/2}, \tag{13}$$

while the HF waves excited at the threshold of the parametric instability have $k_z = k_{z\,min} < k_{z\,st}$, where

$$k_{z\,min}=\frac{1}{r_{De}}\frac{\omega_0-|\Omega_e|}{2^{1/2}\omega_{Le}}\left\{\ln\frac{\pi^{1/2}\omega_0^2(\omega_0-|\Omega_e|)(\omega_0+|\Omega_e|)^2}{\nu_{ei}\omega_{Le}^2(\omega_0^2+\Omega_e^2)}\right\}^{-1/2}. \tag{14}$$

The condition for the build-up of plasma oscillations $\Delta\omega_0=\Omega$ (decay condition) determines, depending on the sign of the scalar product $\mathbf{k}\cdot\mathbf{k}_0$ for an arbitrary angle ϑ, two values, $\mathbf{k}_1$ and $\mathbf{k}_2$, of the wavevectors of the plasma waves. For instance, when ion-sound oscillations with $(\varkappa\cdot\varkappa_0)^2$ propagate at almost right angles to the magnetic field $(\Delta\vartheta)^2 \ll 3(kv_{Te})^2\omega_0^2\Omega_e^{-2}|\omega_0^2-4\Omega_e^2|^{-1}$) we have

$$k_{1,2}=\mp k_0-\frac{\omega_0 v_s}{3v_{Te}^2}\frac{\omega_0^2-4\Omega_e^2}{\omega_0^2-\Omega_e^2}+\left\{\left(\mp k_0-\frac{v_s\omega_0}{3v_{Te}^2}\frac{\omega_0^2-4\Omega_e^2}{\omega_0^2-\Omega_e^2}\right)^2 + \frac{(\omega_0^2-\omega_H^2)(\omega_0^2-4\Omega_e^2)}{3v_{Te}^2\omega_{Le}^2}\right\}^{1/2}. \tag{15}$$

The plasma oscillations with wavevector $\mathbf{k}=\mathbf{k}_2$ are characterized by the minimum threshold field $E_{0\,min}$ obtained from (11) in which the ratio $\tilde{\gamma}/2\Omega(\mathbf{k}_2)$ is negligibly small. In that case the condition $\Delta\omega_0=\Omega \gtrsim \omega(\mathbf{k}_2)$ is the usual condition for the decay of the pump wave into a red satellite and ion sound with frequency $\omega(\mathbf{k}_2)$. A new circumstance for the theory of parametric excitation of plasma waves is the possibility of the excitation of a blue satellite in the region near threshold. It turns out that for oscillations with wavevector $\mathbf{k}=\mathbf{k}_1$ and when we take the spatial dispersion of a pump wave of frequency (10) into account, $\Omega(\mathbf{k}_1)$ is lowered and becomes comparable to the HF damping rate $\tilde{\gamma}$ (i.e., $\tilde{\gamma}/2\Omega(\mathbf{k}_1)\sim 1$, while the weak coupling condition $\omega(\mathbf{k}_1) \gg \tilde{\gamma}, \gamma$ is not violated) when

$$k_{0\,min}\approx\frac{v_s-\tilde{\gamma}/2k_1}{3v_{Te}^2}\frac{4\Omega_e^2-\omega_0^2}{\omega_0^2-\Omega_e^2}\omega_0 .$$

According to the first of Eqs. (3) the condition $|\Delta\omega_0| \sim |\Omega| \sim \tilde{\gamma}$ corresponds to the resonance condition for the excitation of the blue satellite. Its frequency differs from the frequency ω_0 of the pump wave by the frequency $\omega(\mathbf{k}_1)$ of the ion sound. Both the blue satellite and the accompanying ion sound with $\mathbf{k}=\mathbf{k}_1$ propagate in the direction opposite to that of the pump wave, similar to what takes place for induced Raman scattering of laser radiation in a plasma.[12] The threshold field (11) necessary for the appearance of a blue satellite in the plasma turbulence spectrum is larger by a factor $\sqrt{2}$ (for $\tilde{\gamma}=2\Omega(\mathbf{k}_1)$ than the minimum threshold field $E_{0\,min}(\mathbf{k}_2)$; such a field value can easily be reached and even be exceeded experimentally (see, e.g., Ref. 4).

In the growth rate for plasma oscillations with wavenumbers (15), written down for the near-threshold region:

$$\gamma=-\frac{1}{2}(\tilde{\gamma}+\gamma_s)+\left[\frac{1}{4}(\tilde{\gamma}-\gamma_s)^2+\frac{\omega_s\omega_{Le}^2}{\omega_0}\left(\frac{\Omega_e}{\omega_0}\frac{\omega_0^2-\Omega_e^2}{\omega_0^2-\omega_H^2}\right)^2 \times\left(1+\frac{\tilde{\gamma}^2}{4\Omega^2}\right)^{-1}\frac{E_{0x}^2}{64\pi n_e\varkappa_B T_e}\right]^{1/2}, \tag{16}$$

we have taken into account the pump polarization and the possibility for the occurrence of an anti-Stokes line. For oscillations with a wavevector $\mathbf{k}=\mathbf{k}_1$ Eq. (16) is valid in weak pump fields

$$\frac{E_{0x}^2}{16\pi n_e\varkappa_B T_e}\leqslant 16\frac{\omega_0\tilde{\gamma}(\tilde{\gamma}+\gamma_s)}{\omega_s\omega_{Le}^2}\left(\frac{\omega_0}{\Omega_e}\frac{\omega_0^2-\omega_H^2}{\omega_0^2-\Omega_e^2}\right)^2\left(1+\frac{\tilde{\gamma}^2}{4\Omega^2}\right)^{-1}.$$

If the pump is strong so that

$$\frac{E_{0x}^2}{16\pi n_e\varkappa_B T_e}\gtrsim\frac{\omega_0^2}{\Omega_e^3\omega_s\omega_{Le}^2}\left(\frac{\omega_0^2-\omega_H^2}{\omega_0^2-\Omega_e^2}\right)^2\max\{\gamma_s^2(\gamma_s^2+4\Omega^2),\tilde{\gamma}^2(\tilde{\gamma}^2+4\Omega^2)\},$$

the growth rate $\gamma(\mathbf{k}_1)$ is determined by the formula

$$\gamma(k_1)=\sqrt{2}\left\{-\Omega^2+\Omega\left[\Omega^2+\frac{\omega_s\omega_{Le}^2}{\omega_0}\left(\frac{\Omega_e}{\omega_0}\frac{\omega_0^2-\Omega_e^2}{\omega_0^2-\omega_H^2}\right)^2\frac{E_{0x}^2}{64\pi n_e\varkappa_B T_e}\right]^{1/2}\right\}^{1/2}. \tag{17}$$

The growth rates of plasma oscillations propagating at an arbitrary angle $\sphericalangle(\varkappa,\varkappa_0)$ to the pump wave are obtained from (16) and (17) by replacing E_{0x}^2 by

$$E_{0x}^2[(\varkappa\varkappa_0)^2+(1-(\varkappa\varkappa_0)^2)(a/b)^2]$$

in the field terms. The decay values of the wavevectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are in that case determined by Eq. (15) where we use $|(\mathbf{k}_0\cdot\varkappa)|$ instead of the quantity k_0 while the resonance condition $|\Omega(\mathbf{k}_1)| \approx \tilde{\gamma}/2$ for the excitation of the blue satellite takes the form

$$|\mathbf{k}_0\varkappa_1| = -(v_s \pm \tilde{\gamma}/2k_1)(\omega_0^2-4\Omega_e^2)(3v_{Te}^2(\omega_0^2-\Omega_e^2))^{-1}\omega_0.$$

The minimum threshold field for the excitation of the parametric instability corresponds as before to a LF perturbation with wavevector $\mathbf{k}_2$ along the Y-axis $[(\varkappa_2\cdot\varkappa_0)^2=1]$. The threshold field for the excitation of oscillations with wavevector $\mathbf{k}=\mathbf{k}_1$ is a minimum when $\mathbf{k}_1 \parallel \mathbf{k}_0$ for negative mismatches $\Delta\omega_0(\mathbf{k}_1)<0$ corresponding to oscillations with

$$|\mathbf{k}_0\varkappa_1| > v_s\omega_0(4\Omega_e^2-\omega_0^2)[3v_{Te}^2(\omega_0^2-\Omega_e^2)]^{-1}.$$

In the opposite case of positive mismatches $\Delta\omega_0(\mathbf{k}_1)>0$ the threshold field $E_{\text{thr}}(\mathbf{k}_1)$ reaches its smallest value for an angle $\measuredangle(\varkappa_0,\varkappa_1)$ given by the relation

$$(b^2-a^2)\left(1+\frac{\tilde{\gamma}^2}{4\Omega^2(k_1)}\right)=\frac{\omega-\Omega(k_1)}{\Omega(k_1)}\left[b^2-a^2+\frac{a^2}{(\varkappa_1\varkappa_0)^2}\right]\frac{\tilde{\gamma}^2}{4\Omega^2(k_1)}. \quad (18)$$

We show now how Eqs. (11) to (17) change in the case when we consider not ion-sound waves but LF lower hybrid oscillations propagating across the external magnetic field $(\Delta\vartheta)^2 \ll \omega_{Li}^2\omega_{Le}^{-2}$. The threshold field for decay wavenumbers

$$k_{1,2}=\pm|\mathbf{k}_0\varkappa|+\left[(\mathbf{k}_0\varkappa)^2+\frac{(\omega_0^2-\omega_H^2)(\omega_0^2-4\Omega_e^2)}{3v_{Te}^2\omega_{Le}^2}-\frac{2\omega_L\omega_0}{3v_{Te}^2}\frac{\omega_0^2-4\Omega_e^2}{\omega_0^2-\Omega_e^2}\right]^{1/2}$$

differs by a factor $\gamma_L\omega_L k_s^2(\gamma_s\omega_s k_L^2)^{-1}$ (see Ref. 13) from $E^2_{0\,\min}$ given by Eq. (11). In the damping rate γ_L of the LH oscillations we have then taken into account not only the collisional absorption but also Landau damping by ions $(\omega_{Li}\gg\Omega_i)$:

$$\gamma_L=\nu_{ei}\frac{\omega_{Le}^2}{\Omega_e^2}\left(1+\frac{\omega_{Le}^2}{\Omega_e^2}\right)^{-1}\left\{1+\left(\frac{\pi}{8}\right)^{1/2}\frac{\Omega_e^2}{\omega_{Le}^2}\frac{\omega_{Li}}{\nu_{ei}}\left(1+\frac{\omega_{Le}^2}{\Omega_e^2}\right)^{-1}\times\frac{1}{(kr_{Di})^3}\exp\left(-\frac{(1+\omega_{Le}^2\Omega_e^{-2})^{-1}}{2(kr_{Di})^2}\right)\right\}.$$

The contribution from the latter is important for wavenumbers $k>k_{st\,i}$ (r_{Di} is the ion Debye radius) where

$$k_{st\,i}=\frac{1}{2^{1/2}r_{Di}}\left(1+\frac{\omega_{Le}^2}{\Omega_e^2}\right)^{-1/2}\left\{\ln\frac{\pi^{1/2}\omega_{Li}\Omega_e^2}{\nu_{ei}\omega_{Le}^2}\left(1+\frac{\omega_{Le}^2}{\Omega_e^2}\right)^{1/2}\right\}^{-1/2},$$

which are larger than the decay wavenumbers $k_{1,2}$ for a not too small ratio r_{De}^2/r_{Di}^2:

$$\frac{r_{De}^2}{r_{Di}^2}>\frac{2(4\Omega_e^2-\omega_0^2)}{3\omega_{Le}^4}\left[\omega_H^2-\omega_0^2+\frac{2\omega_0\omega_{Le}^2\omega_L}{\omega_0^2-\Omega_e^2}\right]\times\ln\left(\frac{\pi^{1/2}\omega_{Li}\Omega_e^2}{\nu_{ei}\omega_{Le}^2}\left(1+\frac{\omega_{Le}^2}{\Omega_e^2}\right)^{1/2}\right).$$

The resonance condition $2\Omega(\mathbf{k}_1)\sim\tilde{\gamma}$ for the excitation of a blue satellite is realized for the minimum value of the magnitude of the pump field wavenumber

$$k_{0\,\min}=(\omega_L-{}^1/_2\tilde{\gamma}(\mathbf{k}_1))\omega_0(4\Omega_e^2-\omega_0^2)[3k_1v_{Te}^2(\omega_0^2-\Omega_e^2)]^{-1},$$

and Eqs. (16) and (17) are valid as before for the growth rates of the parametric instability in the near-threshold region; in these equations we must take into account the factor $(k_Lv_s)^2(\omega_s\omega_L)^{-1}$ in the field terms and replace γ_s by γ_L.

2. THE SPECTRUM OF THE TURBULENT PLASMA WAVES

In the preceding section we showed that when the field of the SE wave exceeds the threshold value (11) HF oscillations (of the Stokes and the anti-Stokes line) are built up in the plasma in the frequency range of the order of the electron gyro- and ion-sound frequencies. Energy is transferred from the SE wave to the HF oscillations and to ion sound until saturation sets in. The available experimental data allow us to establish that in a wide range of frequencies and wavenumbers saturation of the instability occurs due to cascade processes. Experiments show[2-4] that the stationary spectral distribution of the HF plasma oscillations has a clearly distinguishable peak structure while in the spectrum of the turbulent plasma waves there are not only red peaks but also a blue satellite.

This framework agrees with the non-linear theory developed in what follows as a starting point of which we have taken the equations for the non-linear interaction between the HF waves

$$M_{ij}(\omega_l,\mathbf{k}_l)E_j(\omega_l,\mathbf{k}_l)+\frac{1}{2}\int d\omega'\,d\omega''\,d\mathbf{k}'\,d\mathbf{k}''\,S_{ijp}(\omega_l,\mathbf{k}_l;\omega'',\mathbf{k}'')\times E_j(\omega',\mathbf{k}')E_p(\omega'',\mathbf{k}'')\delta(\omega_l-\omega'-\omega'')\delta(\mathbf{k}_l-\mathbf{k}'-\mathbf{k}'')=0 \quad (19)$$

and the LF waves in the plasma

$$M_{ij}(\omega_s,\mathbf{k}_s)E_j(\omega_s,\mathbf{k}_s)+\frac{1}{2}\int d\omega'\,d\omega''\,d\mathbf{k}'\,d\mathbf{k}''\,S_{ijp}(\omega_s,\mathbf{k}_s;\omega'',\mathbf{k}'')\times E_j(\omega_l',\mathbf{k}_l')E_p(\omega_l'',\mathbf{k}_l'')\delta(\omega_s-\omega'-\omega'')\delta(\mathbf{k}_s-\mathbf{k}'-\mathbf{k}'')=0. \quad (20)$$

In Eqs. (19) and (20) M_{ij} is the Maxwell tensor, S_{ijp} the nonlinear dielectric permittivity tensor, and the indexes l and s refer to the HF and LF waves. Under the conditions considered by us the main contribution to saturation comes from the HF waves for the amplitudes of which we get, using (19) and (20) (in what follows we have dropped the indexes l of the frequencies)

$$M_{ij}(\omega,\mathbf{k})E_j(\omega,\mathbf{k})+\frac{1}{2}\int d\omega'\,d\omega''\,d\omega'''\,d\mathbf{k}'\,d\mathbf{k}''\,d\mathbf{k}'''\times S_{ijp}(\omega,\omega''+\omega''')A_{pq}(\omega''+\omega''')S_{qmr}(\omega''+\omega''',\omega''')E_j(\omega',\mathbf{k}')\times E_m(\omega'',\mathbf{k}'')E_r(\omega''',\mathbf{k}''')\delta(\omega-\omega'-\omega''-\omega''')\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}''-\mathbf{k}''')=0,$$
$$A_{ki}M_{ij}=\delta_{kj},\quad S_{ijp}(\omega,\mathbf{k};\omega'',\mathbf{k}'')\equiv S_{ijp}(\omega,\omega''), \quad (21)$$

Combinations of the high frequencies ω'' and ω''' give in this equation a low frequency: $|\omega''+\omega'''|\approx|\omega_s|$. In the case of a weak coupling between the pump and the plasma waves $[\gamma<\max(\tilde{\gamma},\gamma_s)]$ we can write the HF fields in (21) in the form of a set of n monochromatic lines:

$$E_j(\omega,\mathbf{k})=\frac{1}{2}\sum_n[E_j(\mathbf{k}_n)\delta(\omega-\omega(\mathbf{k}_n))+E_j^*(\mathbf{k}_n)\delta(\omega+\omega(\mathbf{k}_n))],$$

where the $\mathbf{k}_n$ determined by the energy and momentum conservation laws for the interacting waves. There arises then from (21) a system for the Stokes (red satellite) lines of second and higher order $(n\geq 2)$:

$$M_{ij}(\omega_n,\mathbf{k}_n)E_{jn}+{}^1/_4S_{ijp}(\omega_n,\omega^{(m)}+\omega^{(r)})A_{pq}(\omega^{(m)}+\omega^{(r)})S_{qmr}(\omega^{(m)}+\omega^{(r)},\omega^{(r)})\times[E_{j\,n-1}(E_{rn}E^*_{m\,n-1}+E_{mn}E^*_{r\,n-1})+E_{j\,n+1}(E_{mn}E^*_{r\,n+1}+E^*_{m\,n+1}E_{rn})]=0,$$
$$\omega_n\equiv\omega(\mathbf{k}_n),\quad E_{jn}\equiv E_j(\omega_n,\mathbf{k}_n). \quad (22)$$

The interaction between the HF and the pumping fields leads to the necessity to take into account (apart from the terms on the left-hand side of (20) for $n=\pm1$) in the equation for the first-order Stokes line also the term

$$\frac{1}{4}S_{ijp}(\omega_1,\omega_1-\omega_0)\frac{(\mathbf{k}_1-\mathbf{k}_0)_p(\mathbf{k}_1-\mathbf{k}_0)_q}{\varepsilon(\omega_1-\omega_0,\mathbf{k}_1-\mathbf{k}_0)}S_{qmr}(\omega_1-\omega_0,\omega_1)\frac{E_{0j}E_{0m}^*E_r(\omega_1)}{|\mathbf{k}_1-\mathbf{k}_0|^2}$$

and the term

$$\frac{1}{4}S_{ijp}(\omega_1, \omega_1-\omega_0)\frac{(\mathbf{k}_1-\mathbf{k}_0)_p(\mathbf{k}_1-\mathbf{k}_0)_q}{\varepsilon(\omega_1-\omega_0, \mathbf{k}_1-\mathbf{k}_0)} \times S_{qmr}(\omega_1-\omega_0, \omega_1-2\omega_0)\frac{E_{0j}E_{0m}E_r(\omega_1-2\omega_0)}{|\mathbf{k}_1-\mathbf{k}_0|^2},$$

and also the analogous terms in the equation for the blue satellite in which $\omega_1, \mathbf{k}_1$ are replaced by $\omega_{-1}, \mathbf{k}_{-1}$.

These equations can be considerably simplified if we take into account the fact that we are dealing with potential waves and use the dispersion laws of the interacting plasma oscillations. For instance, for potential plasma waves with wavevectors (15) along the Y-axis we find that the rate for the systematic change of the amplitude $A_{ln} = E_{ln}(\omega_{ln}^2 - \Omega_e^2)^{-1/2}$ of the n-th order Stokes line is determined by the damping ξ_{ln}^{-1} of this line and the balance between a flow of energy from the n-1st and a sink into the $n+1$st line ($n \geq 2$):

$$\frac{1}{\tilde{\gamma}_{ln}}\frac{\partial A_{ln}}{\partial t} = -\frac{1}{\xi_{ln}} + \xi_{sn}|A_{l\,n-1}|^2 - \xi_{s\,n+1}|A_{l\,n+1}|^2. \tag{23}$$

Here

$$\xi_{ln}^{-1} = \tilde{\gamma}_{ln}\frac{\partial\varepsilon'(\omega_{ln}, \mathbf{k}_{ln})}{\partial\omega_{ln}}\left(\frac{e}{2mr_{De}}\frac{\omega_0}{\omega_0^2-\Omega_e^2}\right)^{-1}, \quad \tilde{\gamma}_{ln} = \tilde{\gamma}(\mathbf{k}_{ln}),$$

and

$$\xi_{sn}^{-1} = \gamma_{sn}\frac{\partial\varepsilon'(\omega_{sn}, \mathbf{k}_{sn})}{\partial\omega_{sn}}\left(\frac{e}{2mr_{De}\omega_0}\frac{1}{k_{sn}^2 r_{De}^2}\right)^{-1}$$
$$\gamma_{sn} = \gamma_s(\mathbf{k}_{sn})$$

are the renormalized damping rates of the HF and ion-sound waves.

The cascade (23) in the turbulence spectrum is continued downwards along the frequencies ω_{ln} down to the line $n = N$, the intensity of which is less than the threshold value; then

$$\frac{1}{\tilde{\gamma}_{lN}}\frac{\partial A_{lN}}{\partial t} = -\frac{1}{\xi_{lN}} + \xi_{sN}|A_{lN-1}|^2. \tag{24}$$

The change in the amplitude A_{l1} of the first red satellite is connected with the influx of energy from the pump wave $A_0 = bE_{0x}(\omega_0^2 - \Omega_e^2)^{-1/2}$ and from the blue satellite, and also with the expenditure of energy on the excitation of the second-order Stokes line (23):

$$\frac{1}{\tilde{\gamma}_{l1}}\frac{\partial A_{l1}}{\partial t} + \left[\frac{1}{\xi_{l1}} - \xi_{s1}|A_{l-1}|^2 + \xi_{s2}|A_{l2}|^2 - \xi_{s1/2}|A_0|^2\right]A_{l1} + \xi_{s1/2}A_0^2 A_{l-1}^* = 0. \tag{25}$$

Similarly we get for the amplitude A_{l-1} of the blue satellite

$$\frac{1}{f\tilde{\gamma}_{l-1}}\frac{\partial A_{l-1}}{\partial t} + \left[\frac{1}{f\xi_{l-1}} + \xi_{s1}|A_{l1}|^2 + \xi_{s1/2}|A_0|^2\right]A_{l-1}$$
$$-\xi_{s1/2}A_0^2 A_{l1}^* + 2\Omega\tilde{\gamma}_{l-1}^{-1}\xi_{s0}A_{l-1}\sum_r |A_{lr}|^2 = 0. \tag{26}$$

In Eqs. (23) to (26)

$$f = [1 + 4\Omega^2\tilde{\gamma}_{l-1}^{-2}]^{-1}, \quad \xi_{s0} = \frac{e}{2mr_{De}\omega_0}\frac{r_{De}^2}{r_{De}^2 + r_{Di}^2};$$

the indexes sn and ln indicate, respectively, LF and HF waves in the cascade processes

$$\omega_0 \mp \omega_{l1} + \omega_{s1/2}, \quad \omega_0 + \omega_{s1/2} \mp \omega_{l-1}, \quad \omega_{l-1} \mp \omega_{l1} + \omega_{s1},$$
$$\omega_{l1} \mp \omega_{l2} + \omega_{s2}, \ldots, \omega_{ln} \mp \omega_{l\,n+1} + \omega_{s\,n+1}.$$

In Eq. (26) for the blue satellite there is, apart from the decay terms corresponding to the interaction of HF and ion-sound waves, a term (the last one) which takes into account the interaction of HF waves through oscillations with a frequency well below the ion-sound frequency. The contribution from this last term in (26) is the smaller the better the resonance conditions for the excitation of the blue satellite are satisfied. One can neglect this term when $\omega\Omega(k_z v_{Te}\gamma)^{-1} \ll 1$ which is, for instance, satisfied under the conditions $k_0 \approx k_{0\,\min}$.

In that stage of the excitation of plasma oscillations when the amplitudes of the blue and the red satellites are linearly coupled through the pumping one can determine from Eqs. (25) and (26) the growth rates and threshold fields which were considered in detail in the preceding section.

In the saturation stage a stationary ($dA_{ln}/dt = 0$) plasma turbulence state is established. Equations (23) and (24) are then simply recurrence relations for the real stationary amplitudes A_{ln} of the system of second- and higher-order red peaks. Equations (25) and (26) determine the amplitudes of the red satellite:

$$A_{l1}^2 = \xi_{s1/2}\xi_{s1}^{-1}\left(A_0^2 + \frac{1}{\xi_{s1/2}\xi_{l-1}f}\right)$$
$$\times\left[-1 + A_0^2\left(A_0^2 - \frac{\xi_{s2}}{\xi_{s1/2}}A_{l2}^2 - \frac{1}{\xi_{s1/2}\xi_{l1}}\right)^{-1/2}\left(A_0^2 + \frac{1}{\xi_{s1/2}\xi_{l-1}f}\right)^{-1/2}\right] \tag{27}$$

and of the blue satellite:

$$A_{l-1}^2 = \xi_{s1/2}\xi_{s1}^{-1}\left(A_0^2 - A_{l2}^2\frac{\xi_{s2}}{\xi_{s1/2}} - \frac{1}{\xi_{s1/2}\xi_{l1}}\right)$$
$$\times\left[-1 + A_0^2\left(A_0^2 - \frac{\xi_{s2}}{\xi_{s1/2}}A_{l2}^2 - \frac{1}{\xi_{s1/2}\xi_{l1}}\right)^{-1/2}\left(A_0^2 + \frac{1}{\xi_{s1/2}\xi_{l-1}f}\right)^{-1/2}\right], \tag{28}$$

which are nonlinearly coupled through the amplitude A_0 of the pump wave.

In the set of red peaks the intensities of the even modes are (for an odd number of N HF and LF wave pairs) given by the relations (see Ref. 6)

$$A_{l\,N-(2k+1)}^2 = \sum_{m=0}^{k}(\xi_{l\,N-2m}\xi_{s\,N-2m})^{-1}\prod_{p=0}^{k-m}\frac{\xi_{s\,N-(2p+1)}}{\xi_{s\,N-2(p+1)}}. \tag{29}$$

In particular, the intensity of the second-order Stokes line equals

$$A_{l2}^2 = \sum_{m=0}^{(N-3)/2}(\xi_{l\,N-2m}\xi_{s\,N-2m})^{-1}\prod_{p=0}^{(N-3)/2-m}\frac{\xi_{s\,N-(2p+1)}}{\xi_{s\,N-2(p+1)}}. \tag{30}$$

Knowing the intensity (30) of this line we can use Eqs. (27) and (28) to find the amplitudes of the red A_{l1} and of the blue A_{l-1} satellites and afterwards by using the stationary recurrence relation (23) the amplitudes of the odd-order modes:

$$A_{l\,2k+1}^2 = -\sum_{m=0}^{k-1}(\xi_{l\,2k-2m}\xi_{s\,2k-2m+1})^{-1}\prod_{p=0}^{m}\frac{\xi_{s\,2k-2p}}{\xi_{s\,2k-2p+1}} + A_{l1}^2\prod_{p=0}^{k-1}\frac{\xi_{s\,2k-2p}}{\xi_{s\,2k-2p+1}}. \tag{31}$$

Equations (26) for an even number of N peaks in the spectrum determine the amplitudes of the odd modes and, hence, the intensity of the first-order Stokes line:

$$A_{l1}^2 = \sum_{m=0}^{(N-2)/2}(\xi_{l\,N-2m}\xi_{s\,N-2m})^{-1}\prod_{p=0}^{(N-2)/2-m}\frac{\xi_{s\,N-(2p+1)}}{\xi_{s\,N-2(p+1)}}. \tag{32}$$

We can in this case use Eqs. (27), (28), and (32) to find the amplitude of the blue satellite:

$$A_{l-1}=A_{l1}A_0^2\xi_{s1/2}\xi_{l-1}(f^{-1}+A_0^2\xi_{s1/2}\xi_{l-1}+A_{l1}^2\xi_{s1/2}\xi_{l-1})^{-1}, \tag{33}$$

and also the intensities of the second- and higher-order Stokes lines which are connected through the recurrence relations.

In a laboratory plasma conditions are often realized when the damping of HF waves is caused by electron-ion collisions $\tilde{\gamma}\sim\nu_{ei}$, i.e., $\xi_{ln}=\xi_l$, and the dissipation of ion sound by Landau absorption: $\xi_{sn}=\xi_s$. In that case, the relations given above which determine the spectrum of the plasma oscillations can be simplified. For instance, in a spectrum consisting of an odd number of N peaks we have for the intensities of the first two red satellites

$$A_{l1}^2=\left(A_0^2+\frac{1}{\xi_s\xi_l f}\right)\left[-1+A_0^2\xi_s\xi_l\left(A_0^2\xi_s\xi_l-\frac{N-1}{2}\right)^{-1/2}(A_0^2\xi_s\xi_l+f^{-1})^{-1/2}\right],$$
$$A_{l2}^2={}^1/_2(N-3)(\xi_s\xi_l)^{-1} \tag{34}$$

and for the intensity of the anti-Stokes line:

$$A_{l-1}^2=\left(A_0^2-\frac{N-1}{2}\frac{1}{\xi_s\xi_l}\right)$$
$$\times\left[-1+A_0^2\xi_s\xi_l\left(A_0^2\xi_s\xi_l-\frac{N-1}{2}\right)^{-1/2}(A_0^2\xi_s\xi_l+f^{-1})^{-1/2}\right]. \tag{35}$$

In a spectrum of an even number of red peaks the intensities of the satellites are not given by (34) and (35) but by the relations

$$A_{l1}^2=\frac{N}{2}(\xi_s\xi_l)^{-1},$$
$$A_{l2}^2=A_0^2-(\xi_s\xi_l)^{-1}-A_0^4\xi_s\xi_l\left(A_0^2\xi_s\xi_l+\frac{N}{2}+f^{-1}\right)^{-2}(A_0^2\xi_s\xi_l+f^{-1}), \tag{36}$$
$$A_{l-1}^2=A_0^4\xi_s\xi_l\frac{N}{2}\left(A_0^2\xi_s\xi_l+\frac{N}{2}+f^{-1}\right)^{-2}.$$

Estimates show that the difference between the series of peaks with an even or an odd number $N\gg1$ of modes which is connected with the choice of the boundary value A_{ln} is insignificant. The intensity of the higher-order red peaks decreases linearly with increasing number n (i.e., in the direction of decreasing frequency ω_l) similarly to what happens in an isotropic plasma[5,7] for Langmuir waves with normal dispersion. When one is well above threshold, $p^2=E_0^2E_{0\,\min}^{-2}(\mathbf{k}_2)\gg f^{-1}$ the total number of red peaks $N\approx1.23\,p^2$; it then follows from Eqs. (34) to (36) that the intensity of the red satellite $E_{l1}^2\approx0.613\,b^2E_{0x}^2$ is roughly 2.62 times larger than the intensity of the blue peak.

Equations (24) to (36) are also valid for lower-hybrid LF perturbations provided we replace r_{De} by k_L^{-1} in the quantity ξ_{ln} and use instead of the quantity ξ_{sn} the quantity ξ_{Ln}:

$$\xi_{Ln}^{-1}=\gamma_{Ln}\frac{\partial\varepsilon'(\omega_{Ln},k_{Ln})}{\partial\omega_{Ln}}\left(\frac{ek_L}{2m\omega_0}\right)^{-1}(1+\omega_{Le}^2\Omega_e^{-2})^{-2}.$$

The HF waves excited in secondary instabilities also have an anomalous dispersion. Hence the pumping energy goes during the relay transfer over into plasma oscillations with shorter wavelengths (i.e., in more fine-scaled perturbations) where collisionless absorption of HF waves becomes important. In a strong magnetic field this manifests itself in that the components of the wavevectors along the direction of the magnetic field k_{zn} of the plasma waves excited in the secondary instability are given by Eqs (14) in which one must replace ω_0 by ω_{ln-1}. When the frequency ω_{ln} decreases (i.e., when n increases) the quantities k_{zn} change little and the inequality $k_{zn}<k_{zst}(n)$ remains valid [$k_{zst}(n)$ is given by Eq. (19) with ω_0 replaced by ω_{ln-1}]. The way the intensities A_{ln}^2 change is in that case determined by Eq. (23) which for a small change in the wavenumber k_n of number n,

$$k_n\approx k_1+(|n|-1)\Delta,$$
$$\Delta=\frac{v_e}{3v_{Te}r_{De}}\frac{\omega_H(3\Omega_e^2-\omega_{Le}^2)}{\omega_{Le}^3}\ll k_1, \tag{37}$$

can be written in differential form

$$\frac{dA_{ln}^2}{dn}=-\frac{1}{2\xi_s\xi_l}\frac{k_nk_{z\,\min}}{k_1k_{zn}}[1+g(k_{zn})],\quad n\geqslant2,$$
$$g(k_{zn})=\left(\frac{\pi}{8}\right)^{1/2}\frac{\omega_0^4(\omega_0^2-\Omega_e^2)^2}{\omega_{Le}^2\Omega_e^2\nu_{ei}(\omega_0^2+\Omega_e^2)}\frac{1}{k_{zn}v_{Te}}\exp\left(-\frac{(\omega_0-|\Omega_e|)^2}{2k_{zn}^2v_{Te}^2}\right). \tag{38}$$

Neglecting the difference between k_{zn} and $k_{z\,\min}$ we find from Eq. (38) that a violation of the linear law of change of the intensities occurs only for large $n\sim k_1\Delta^{-1}$:

$$A_{ln}^2=A_{l2}^2-\frac{n-2}{2}(1+g(k_{z\,\min}))\frac{1}{\xi_s\xi_l}\left[1+\frac{(n-1)\Delta}{2k_1}\right]. \tag{39}$$

If the frequency of the external field differs from the HF frequency by an amount $\delta\omega_0=\omega_H-\omega_0$ comparable to

$$\delta\omega_0\omega_0^{-1}=\frac{v_{Te}^2}{8v_s^2}\frac{\omega_{Le}^6}{\Omega_e^6}\ln^{-1}\left(\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_{Le}^3}{8\Omega_e^2\nu_{ei}}\right),$$

the long-wavelength ion-sound oscillations with $k_z\approx k_{z\,\min}$ are strongly damped. For such a detuning weakly damped ion sound is excited for smaller values of the angles ϑ and the threshold values of the fields exceed in that case the quantity $E_{0\,\min}(k_{z\,\min})$ corresponding to Eq. (11).

For a plasma oscillations excited when the mismatch is negative, $\Delta\omega_0(\mathbf{k}_1)<0$, in the non-linear stage of the development of the parametric instability one must take into account the non-linear frequency shift

$$\delta\omega_l=-\frac{E_l^2}{32\pi n_e\varkappa_B(T_e+T_i)}\frac{\omega_{Le}^2}{\omega_0}, \tag{40}$$

which leads to a decrease in the magnitude of the detuning $|\Delta\omega_0(\mathbf{k}_1)|$ when the energy of the plasma noise increases and, hence, to an increase in the threshold value of the field $E_{\text{thr}}^2(\mathbf{k}_1)$ and a stabilization of the instability. The stationary value of the energy of the plasma oscillations

$$\frac{E_l^2}{32\pi n_e\varkappa_B(T_e+T_i)}=\frac{\omega_0}{\omega_{Le}^2}\left\{|\Delta\omega_0(\mathbf{k}_1)|-\frac{\tilde{\gamma}}{2}(E_0^2E_{0\,\min}^{-2}(\mathbf{k}_2)-1)^{-1/2}\right\}, \tag{41}$$

which is in that case determined from the condition $E_0^2=E_{\text{thr}}^2(|\Delta\omega_0(\mathbf{k}_1)|+\delta\omega_l)$ is bounded from above at the level

$$E_{l\,\max}^2(32\pi n_e\varkappa_B(T_e+T_i))^{-1}\sim\omega_0|\Delta\omega_0|\omega_{Le}^{-2}$$

the maximum noise level, which is reached in cascade processes when we are well above threshold $N\gg1$. Thus, for a pump wavenumber

$$k_0=\frac{v_s+\tilde{\gamma}/2k_1}{3v_{Te}^2}\frac{|\omega_0^2-4\Omega_e^2|}{\omega_0^2-\Omega_e^2}\omega_0,$$

which corresponds to $\Delta\omega_0(\mathbf{k}_1)=-\frac{1}{2}\tilde{\gamma}$ the maximum number N of peaks for oscillations with wavevectors $\mathbf{k}=\mathbf{k}_1$ along the Y-axis does not exceed the magnitude $N_{\max} \sim\omega_s^{1/2}\gamma_s^{-1/2}$.

In concluding this section we make a few remarks about the integrated intensity of the blue satellite. We showed above that in directions close to the direction of propagation of the pump wave, when we are when well above threshold, $p^2\sim N\gg f^{-1}$, the first red and blue lines are of the same order in the spectrum of the plasma turbulence. However, if we are well above threshold the red satellite will be excited in the whole range of angles between $\mathbf{k}_0$ and $\mathbf{E}_0$ and the phase volume necessary for the blue satellite is for $\mathbf{k}_0\approx\mathbf{k}_{0\min}$ determined by the interval of small angles near the direction of $\mathbf{k}_0$ and is smaller by a factor $(\tilde{\gamma}/\omega_s)^{1/2}$ then the phase volume for the Stokes line. When k_0 increases the phase volume for the blue satellite increases and this enhances the possibility to observe it.

3. DISCUSSION OF THE RESULTS AND COMPARISON WITH EXPERIMENTS

We showed in the preceding sections that an SE wave with a field strength E_0 above the minimum $E_{0\min}$ propagating across the magnetic field in a plasma leads to a parametric instability. The anomalous absorption of the pump wave connected with it is characterized by an effective collision frequency $\nu_{\text{eff}}=\tilde{\gamma}E_1^2E_0^{-2}$ which enables us to estimate the magnitude of the power absorbed per unit volume in the plasma $Q=(4\pi)^{-1}\nu_{\text{eff}}E_0^2$. When we are just above threshold, when $\gamma<\max(\gamma_s,\tilde{\gamma})$, it follows from Eqs. (34) to (36) that

$$Q=\frac{1}{512\pi^2n_e\varkappa_BT_e}\frac{\omega_0\omega_s}{\gamma_s}E_0^4\left[\left(\frac{\omega_{Le}^2}{\omega_0^2-\Omega_e^2}\right)^2+\frac{\omega_0^2}{\Omega_e^2}\left(\frac{\omega_0^2-\omega_H^2}{\omega_0^2-\Omega_e^2}\right)^2\right]^{-2}\left[k'^2+\frac{3}{2}(1+k'^4)\right].$$

In the opposite limit of being well above threshold, i.e., when $\gamma\gg(\tilde{\gamma},\gamma_s)$, we get by using the results of the paper by Bychenkov *et al.*,[7] applied to our situation, the following estimate for the quantity Q:

$$Q=\frac{\omega_{Le}}{24\pi^2\omega_0}\left(\frac{\omega_0\omega_s}{\pi n_e\varkappa_BT_e}\right)^{1/2}E_0^3\left[\left(\frac{\omega_{Le}^2}{\omega_0^2-\Omega_e^2}\right)^2+\frac{\omega_0^2}{\Omega_e^2}\left(\frac{\omega_0^2-\omega_H^2}{\omega_0^2-\Omega_e^2}\right)^2\right]^{-3/2}[2(1+k'^2)\mathrm{E}(k)-k'^2\mathrm{K}(k)],$$

($E(k)$ and $K(k)$ are complete elliptical integrals, $k'^2=a^2b^{-2}, k^2=1-k'^2$). The effective relaxation length of the pump wave

$$l_{\text{eff}}=c(2\omega_H\delta\omega_0)^{3/2}(\omega_0\omega_{Le}\Omega_e\nu_{\text{eff}})^{-1}$$

(c is the velocity of light in vacuo) then shortens with decreasing mismatch $\delta\omega_0=\omega_H-\omega_0$, i.e., when we approach the point of the upper hybrid resonance.

At the limit of the applicability of the weak coupling conditions $\nu_{\text{eff}}\sim 2\omega_s\sim 2\gamma$ for ion-sound LF perturbations with wavenumbers (15) we get for $\delta\omega_0\omega_0^{-1}\gg v_s^2\Omega_e^2/v_{Te}^2\omega_{Le}^2$ the simple estimate

$$l_{\text{eff}}\sim(3/2)^{1/2}cv_{Te}\delta\omega_0(v_s\Omega_e)^{-1}(3\Omega_e^2-\omega_{Le}^2)^{-1/2}.$$

Under the conditions of thermonuclear reactors where the instability considered by us will develop in the plasma volume included between the cyclotron absorption and the linear SE wave transformation surfaces, l_{eff} is for typical plasma parameters ($\Omega_e^2\omega_{Le}^{-2}\approx 4$, $\delta\omega_0\omega_0^{-1}\approx 0.1$, $\omega_0\approx\Omega_e\approx\omega_H\approx 10^{12}$ sec^{-1}) three orders of magnitude smaller than the characteristic size of the plasma inhomogeneity for a radius $d\approx 10^2$ cm which indicates the high efficiency of the parametric absorption of vhf radiation.

In a laboratory plasma a situation close to the one considered in the present paper has been observed by Albers *et al.*[4] Ion sound with a frequency $\omega_s=3.45\times 10^7$ sec^{-1} and oscillations with a frequency close to that of the upper hybrid wave were in that experiment excited in the plasma as a result of the action of an SE pump wave of frequency $\omega_0=1.45\times 10^{10}$ sec^{-1}. For a magnetic field $B_0=520$ gauss found in the experiment the pump wave frequency ω_0 was lying in the interval between the electron gyro-frequency $\Omega_e=0.92\times 10^{10}$ sec^{-1} and twice the gryo-frequency $2\Omega_e$. The wavenumber of the pump field determined from the dispersion law $n^2(\omega_0)$, $k_0\approx 0.23$ to 0.5 cm^{-1} differs by more than two orders from the measured value $k_\perp=96$ cm^{-1} of the component of the wavevector $\mathbf{k}$ at right angles of the magnetic field $\mathbf{B}_0$ while a measurement of the longitudinal component of the wavevector $k_\parallel=6$ cm^{-1} shows that the phase velocity of the LF waves along the magnetic field is less than the electron thermal velocity.

For typical plasma parameters in the experiment ($n_e\approx 7\times 10^{10}$ cm^{-3}, $T_e\approx 3$ eV) the collisionless absorption of the HF modes is unimportant; an estimate of the minimum threshold field (11) gives a magnitude $E_{0\min}\approx 10$ V/cm while changes in the geometric dimensions of the plasma within wide limits did not affect the plasma turbulence parameters. The spectrum of the HF oscillations in the stationary state consisted of a series of red peaks and a blue satellite and already for $p^2\gtrsim 3$ the ratio E_{11}^2/E_0^2 remained unchanged with increasing E_0^2 which corresponds to our estimate. The value of the mismatch $\Delta\omega_0(\mathbf{k}_1)$, evaluated under the experimental conditions, turns out to be of the order of the HF damping rate $\tilde{\gamma}=10^5$ sec^{-1}, i.e., the excitation of the blue satellite occurred in the work by Albers *et al.*[4] under resonance conditions.

[1)]Equation (11) differs from the results of Porkolab's paper[1] by the last two factors which take into account, respectively, the polarization of the pump (1) and the presence of a blue satellite in the turbulence spectrum.

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Thermal runaway and convective heat transport by fast electrons in a plasma

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The distribution of the electron velocities in an inhomogeneously heated fully ionized plasma is investigated. It is shown that the fast growth of the mean free path with increasing energy gives rise to thermal runaway of the electrons—an abrupt growth of the number of fast electrons in the region of the cold plasma. The electron distribution function has a double Maxwellian character, i.e., it is characterized by two electron temperatures. The higher temperature is possessed by the fast electrons. Critical discontinuities appear, namely unusual types of discontinuities of the distribution function. The structure of the kinetic discontinuity is investigated. It is shown that besides the usual heat flux there appears a convective energy flux carried by the fast electrons. At not too small temperature gradients, the convective transport plays the principal part.

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1. INTRODUCTION

The mean free path of electrons in a plasma increases rapidly with increasing electron energy. Consequently the coupling between the energetic electrons and the plasma is very weak and even small forces cause a considerable deviation of their distribution from equilibrium. For example, even in a weak constant electric field the electron distribution function becomes strongly distorted in the region of high velocities, and a flux of runaway electrons is produced.[1]

Similar distortions of the distribution functions can arise also in the presence of electron-temperature gradients. Indeed, assume that in a given direction x there is present in the plasma a rather weak electron-temperature gradient

$$\gamma=\frac{l_T}{T_e}\left|\frac{dT_e}{dx}\right|\ll 1. \tag{1}$$

Here l_T is the mean free path of the thermal electrons. The temperature gradient (1) produces, naturally, only a small perturbation of the equilibrium distribution of the electrons in the principal (thermal) velocity region. For fast particles, however, the situation is substantially different. Their mean free path increases: $l_\varepsilon = l_T(\varepsilon/T_e)^2$, so that electrons with sufficiently high energy

$$\varepsilon\geqslant\varepsilon_0=T_e/\gamma^{1/4} \tag{2}$$

move freely between the regions with substantially different temperatures. This leads to a strong distortion of the distribution function. In particular, the number of high-energy electrons in the region of the cold plasma increases sharply. This phenomenon can naturally be called thermal runaway of the electrons. The present paper is devoted to its investigation.

It is important that electrons with energy $\varepsilon\gtrsim\varepsilon_0$ carry heat by convection. Thus, so to speak, two heat fluxes are produced. One is by usual thermal conductivity and is due mainly to the electrons with low energies $\varepsilon \lesssim 10T_e$. The second flux is convective and due to fast electrons $\varepsilon\gtrsim\varepsilon_0$. At a sufficiently small electron-temperature gradient $\gamma<\gamma_k\approx 10^{-2}$ the principal role is played by the thermal conductivity. At $\gamma>\gamma_k$, on the contrary, the convective transport is more important. In this case the deformation of the distribution function exerts a decisive influence on the heat transport in the plasma, which proceeds mainly via convection by the fast electrons. It has a kinetic character and cannot be described within the framework of ordinary transport theory.

In the present paper we confine ourselves to plasma electrons; effects of similar type are typical, however, also of ions. We note also that, as shown in Ref. 2, perfectly analogous phenomena arise in transverse transport of supertrapped electrons and ions in toroidal magnetic traps.

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