

the electron is in a σ state).

There are two ranges of values for the distance R between the nuclei of the interacting systems in which the exchange-interaction potential $\Delta(R)$ is represented by different functions of R . Thus, when $R \gg 2z$, Eq. (20) for the exchange interaction reduces to the limiting expression (22), which can be derived within the framework of the known asymptotic theory.²¹ When $4z^{1/2} < R < 2z$, the asymptotic theory is inapplicable, owing to the fact that when $z > 4$ the asymptotic expression for the wave function for the electron centered on the ion cannot be used at these distances. At the same time, the calculations show that the quasiclassical method worked out here correctly gives the interaction both in the intermediate range $4z^{1/2} < R < 2z$ and in the asymptotic limit $R > 2z$. In addition, the quasiclassical approach allows the results to be easily generalized to the case in which the electron's orbital angular momentum l ceases to be zero when the electron makes a transition from the atom to the MCI.

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Stimulated photoassociation in the field of an intense electromagnetic wave

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The quantum-mechanical problem of the stimulated transition of a system in the field of an intense electromagnetic wave from the continuous spectrum to a bound state having a finite lifetime is considered. The formulas derived are used to calculate the stimulated production of mesic atoms and the mesic molecule $dd\mu$. It is shown that the probability for the production of such systems may be considerably enhanced in the presence of an external electromagnetic field.

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1. Transitions from the continuous spectrum to a bound state with spontaneous emission of a photon are possible in particle collisions. As a rule, however, the probability for such transitions is small. The probab-

ility for such transitions in atomic collisions may be considerably enhanced in the field of an intense electromagnetic wave, however, on account of stimulated photon emission.¹⁻³

The stimulated transitions that arise in collisions of ordinary atoms or ions were considered in the papers cited above. However, induced transitions in the field of an intense electromagnetic wave may also arise in the production of mesic atoms, muonium, positronium,⁴ antiprotonium, mesic molecules, etc. In these cases the stimulated photoassociation process has a number of specific features that must be taken into account in calculating the corresponding probabilities.

In particular, in the production of the "exotic" atoms and molecules mentioned above at thermal energies, the wavelength of the colliding particles considerably exceeds the size of the system in the bound state. Moreover, electronic terms may be excited in collisions of ordinary atoms, but internal degrees of freedom are not excited in the production of exotic atoms (or molecules).

In this paper we consider the quantum-mechanical problem of the stimulated transition of a system from the continuous spectrum to a bound state that has a finite lifetime against spontaneous transitions to lower states (gamma-ray emission, the Auger effect, etc.). The resulting formulas will be used to calculate probabilities for stimulated production of mesic atoms in states with large principal quantum numbers and for the production of mesic molecule $dd\mu$ in collisions of mesic deuterium with deuterium in the field of an intense electromagnetic wave. The second process is possible in principle since the $dd\mu$ system has several bound states, including one with a binding energy of ~ 2 eV.⁵

2. Let the energy of the particles before collision be $E_1 = \varepsilon_p + \varepsilon_1 + \varepsilon_2$, where ε_p is the kinetic energy of the relative motion of the particles, and ε_1 and ε_2 are the internal energies of the colliding particles. We shall write the potential for the interaction of the system of particles with the external electromagnetic field in the dipole approximation:

$$V(t) = \hat{d} \mathbf{E}_0 \cos \omega t, \quad (1)$$

where ω and \mathbf{E}_0 are the frequency and electric field strength, respectively, of the external electromagnetic field and \hat{d} is the operator for the transition dipole moment.

We shall seek the wave function of the system in the external electromagnetic field in the form ($\hbar = c = 1$)

$$\Psi(t) = A(t) \Phi_{E_1} + B_{p_0 n}(t) \Phi_{E_n p_0}^* + \sum_{p'} D_{p'}(t) \Phi_{E'} + \sum_{\lambda} C_{\lambda}(t) \Phi_{\lambda}, \quad (2)$$

where Φ_{E_1} is the wave function of the system with energy E_1 in the continuous spectrum (the initial state), $\Phi_{E_n p_0}^*$ is the wave function of the bound quasistationary state of energy E_n produced from the initial state of momentum p_0 , $\Phi_{E'}$ is the continuum wave function of the system of energy E' that results from the breakup of the bound quasistationary state $\Phi_{E_n p_0}^*$, and Φ_{λ} is the wave function for the bound state to which the spontaneous transition from the state $\Phi_{E_n p_0}^*$ goes. We shall use Heitler's method⁶ to solve the time dependent Schrödinger equation with interaction (1). We subject the amplitudes $A(t)$, $B_{p_0 n}(t)$, $D_{p'}(t)$, and $C_{\lambda}(t)$ to Fourier transformations:

$$A(t) = -\frac{1}{2\pi i} \int dE A(E) \exp\{i(E_1 - E)t\},$$

$$B_{p_0 n}(t) = -\frac{1}{2\pi i} \int dE B_{p_0 n}(E) \exp\{i(E_{p_0 n} - E)t\} \quad (3)$$

etc. After substituting (3) into the time dependent Schrödinger equation we obtain the following set of equations for the amplitudes ($\hbar = c = 1$):

$$(E - E_1 + i\varepsilon) A(E) = 1 + \sum_{p_0} V_{p_0 n} B_{p_0 n}(E),$$

$$(E - E_{p_0 n} - \omega + i\varepsilon) B_{p_0 n}(E) = V_{p_0 n}^* A(E) + \sum_p V_{p_0 p} D_p(E) + \sum_{\lambda} H_{\lambda} C_{\lambda}(E), \quad (4)$$

$$(E - E_{\lambda} - \omega + i\varepsilon) C_{\lambda}(E) = H_{\lambda}^* B_{p_0 n}(E),$$

$$(E - \varepsilon_p + \varepsilon_{p_0} + i\varepsilon) D_p(E) = V_{p_0 p}^* B_{p_0 n}(E),$$

$$V_{p_0 n} = \int \Phi_{E_n}^* d(\mathbf{R}) \mathbf{E}_0 \Phi_{E_1} d\mathbf{R},$$

where H_{λ} is the matrix element for the transition from the bound state $\Phi_{E_n}^*$ to another bound state Φ_{λ} of the system with the emission, for example, of a photon, and λ is the set of quantum numbers specifying the new bound state of the system and the emitted photon (\mathbf{k} and μ are the wave vector and polarization of the emitted photon). Equations (4) were derived in the resonance approximation, i.e. under the assumption that $\omega \gg \varepsilon_{p_0} + |I_0| - \omega$, where I_0 is the energy of the bound state reckoned from the energy of the atoms at infinite separation ($R \rightarrow \infty$). The solution of Eqs. (4) for the amplitude $C_{\lambda}(E)$ can be expressed in the form

$$C_{\lambda}(E) = H_{\lambda} V_{p_0 n} [(E - E_{\lambda} - \omega + i\varepsilon) (E - E_{p_0 n} - \omega + i\Gamma/2) (E - E_1 + i\Gamma_0/2)]^{-1}, \quad (5)$$

$$\Gamma_0 = 2 \sum_{p_0} \frac{|V_{p_0 n}|^2}{E - E_{p_0 n} - \omega + i\Gamma/2}, \quad \gamma = 2 \sum_{\lambda} |H_{\lambda}|^2 \zeta(E - E_{\lambda} + \omega),$$

$$\Gamma_{np} = 2 \sum_p |V_{p_0 p}|^2 \zeta(E - \varepsilon_p + \varepsilon_{p_0}), \quad \Gamma = \Gamma_{np} + \gamma,$$

where γ is the width of the bound level due to the spontaneous decay with photon emission and Γ_{np} is the width associated with the transition of the system from the bound state to the continuum under the action of the external electromagnetic field.

3. As is shown in Heitler's book,⁶ the transition probability per unit time to the state Φ_{λ} can be obtained with the aid of Eq. (5):

$$W = \frac{2\pi}{\hbar} \sum_{\lambda} \frac{|H_{\lambda}|^2 |V_{p_0 n}|^2 \delta(E_1 - E_{\lambda} - \omega)}{(\varepsilon_{p_0} + |I_0| - \omega)^2 + \Gamma^2/4}. \quad (6)$$

On summing (5) over λ and dividing (6) by the particle flux, we obtain the cross section for the process at a fixed value of energy ε_{p_0} :

$$\sigma = g\pi \frac{\hbar^2}{p_0^2} \frac{\Gamma_{np}\gamma}{(\varepsilon_{p_0} + |I_0| - \omega)^2 + \Gamma^2/4}, \quad (7)$$

where g is the statistical weight and p_0 is the momentum of the particles. We shall obtain expressions for Γ_{np} for two cases: a) systems with Coulomb interaction (mesic atoms, muonium, positronium, antiprotonium, etc.) and b) the $dd\mu$ system. For the systems listed under case a), resonant transitions in the optical range are possible only to excited levels with $n \gg 1$. Hence to evaluate Γ_{np} we may use the asymptotic expression for the radial wave function of the quasistationary level (in atomic units of length):

$$\Phi_{np}^* \approx 2^{n+1/2} \pi^{-1} e^{-r/a_B} / (2n!)^{1/2} n^{n+1/2}. \quad (8)$$

As a result we obtain the following estimate for Γ_{np} (for $n \gg 1$ and $kn a_B \ll 1$):

$$\hbar \Gamma_{np} \approx 2 / \pi^{1/2} n^{1/2} (e E_0 a_B)^2 a_B^3 |p_0| m / \hbar^2, \quad (9)$$

where a_B is the Bohr radius. In deriving (9) it was assumed that the transition to the bound state takes place from an S state of the continuum.

For mesic atoms, binding energies in the optical range correspond to $n \geq 30$. In this case we obtain

$$\hbar \Gamma \sim 10^{-13} (e E_0)^2 (kT)^{1/2} \text{ eV}. \quad (10)$$

for thermalized muons. The reaction rate λ , averaged over the Maxwell distribution of the colliding particles, is given by

$$\hbar \lambda = \hbar n_0 \int v d\sigma = \left(\frac{2}{\pi}\right)^{1/2} n_0 \frac{\gamma}{kT} \frac{|V_{np}|^2}{kT} \int_0^\infty x^{1/2} e^{-x} [(x-\Delta)^2 + b^2]^{-1} dx, \quad (11)$$

where n_0 is the particle density, $x = \varepsilon_{p_0} / kT$, $\Delta = (\omega - |I_0|) / kT$, and $b = \Gamma / kT$. For $\Delta \gg 1$ and $b \ll \Delta$, we obtain

$$\hbar \lambda \approx 2(2\pi)^{1/2} n_0 \frac{|V_{np}|^2}{kT} \Delta^{1/2} e^{-\Delta} \frac{\gamma}{\Gamma} \text{ eV}, \quad (12)$$

from Eq. (11), while for $b \gg 1$ and $\Delta \leq 1$, we obtain

$$\hbar \lambda \approx n_0 \frac{|V_{np}|^2}{\Gamma} \frac{\gamma}{\Gamma} \text{ eV}. \quad (13)$$

Thus, the reaction rate for the production of bound systems with Coulomb interaction in the field of an intense electromagnetic wave ceases to be temperature dependent when the field strength is high enough.

Let us obtain numerical estimates of the reaction rate for the production of mesic hydrogen atoms. Using (12), we find

$$\hbar \lambda \approx 10^{-20} n^{1/2} \frac{e^2 E_0^2}{kT} \frac{\gamma}{\Gamma} n_0 \text{ eV}. \quad (14)$$

At the temperature and density of liquid hydrogen we have $kT \sim 10^{-3}$ eV and $n_0 = 4.25 \times 10^{22}$ cm⁻³; then assuming an electromagnetic field strength of $E_0 \sim 3 \times 10^4$ V/cm and using the values $n \sim 30$ and $\gamma \sim 10^{-4}$ eV, we obtain $\hbar \lambda \sim 10^{-5}$ eV. This estimate shows that the time for spontaneous production at very moderate strengths of the electromagnetic field.

4. Now let us consider the production of mesic deuterium molecules in the field of an intense electromagnetic wave. In this case we must find the matrix element V_{mp} for the transition from the continuous spectrum of the $a + d\mu$ system to a weakly bound state of the $dd\mu$ molecule.⁷ The cross section for this process is given by Eq. (7), and the reaction rate, averaged over the Maxwell distribution, is given by Eq. (11), in which the matrix element is taken from Ref. 5. According to Ref. 5, the level of the mesic molecule $dd\mu$ with quantum numbers $J=1$ and $v=1$ has the binding energy $|I_0| = 2.196$ eV and the corresponding matrix element is $V_{mp} = 400$ mesic atomic units. We accordingly obtain the following estimate for the stimulated photoassociation width Γ_{np} (at liquid-hydrogen density):

$$\hbar \Gamma_{np} \approx 10^{-20} (e E_0)^2 (kT)^{1/2} \text{ eV} \quad (15)$$

If $\Gamma_{np} \leq \gamma$, then $\gamma / \Gamma \sim 1$, and Eq. (12) therefore gives the following estimate of λ (using $\Delta \sim 1$ and $n_0 = 4.25 \times 10^{22}$ cm⁻³):

$$\hbar \lambda \approx 10^{-25} \frac{e^2 E_0^2}{kT} \text{ eV}. \quad (16)$$

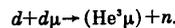
At liquid hydrogen temperature with an electric field of strength $E \sim 10^6$ V/cm, (16) yields $\lambda \sim 10^{-9}$ eV or $\lambda \sim 10^6$ sec⁻¹. This is an order of magnitude larger than the measured probability for spontaneous production of mesic molecules in liquid hydrogen.⁵

5. The production in the field of an intense electromagnetic wave of bound states in positronium and muonium atoms, mesic atoms, and mesic molecules at normal temperatures takes place from an S state of the continuum. Hence the formulas derived above are not directly applicable to these cases. Several partial waves of the continuum wave function may take part in the production of molecules in collisions of ordinary atoms. This case therefore requires some refinement.

We also note that the case of elastic scattering in the field of an electromagnetic wave requires special treatment. Thus, the negative level leads to the appearance in the elastic scattering amplitude of a pole term that may interfere with the potential-scattering amplitude. Interference in the inelastic channel may be neglected if the condition $e E_0 v / \hbar \omega^2 \ll 1$ is satisfied.

The above estimates of the probabilities for the production of mesic atoms in the field of an intense electromagnetic wave are valid provided the muons are thermalized. If the capture of a muon to form a mesic atom as a result of the Auger effect on bound electrons takes place from continuum states with energies $E_k \gg 1$ eV and the muon has not been thermalized, then one will not be able to significantly alter the probability for the production of mesic atoms by the use of optical lasers. But if the muons are thermalized, one can considerably affect the production of mesic atoms (and of other exotic atoms) by the use of lasers, and in particular, it becomes possible to control the cascade of mesic x rays and the populations of the states of the mesic atoms.

The stimulated enhancement of the probability for the production of the mesic molecule $dd\mu$ can be observed via the neutron yield from the nuclear reaction



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APPENDIX

Let us find the explicit form of $C_\lambda(t)$. Using Eqs. (3) and (5), we obtain

$$C_\lambda(t) = H_\lambda V_{np} \left[\frac{1}{(E_\lambda - E_{pnn} + i\Gamma/2)(E_\lambda + \omega - I_0 + i\Gamma_0/2)} + \frac{\exp[-i(E_{pnn} - E_\lambda)t - \Gamma t/2]}{[E_{pnn} - E_\lambda - i\Gamma/2][E_{pnn} + \omega - I_0 + i\Gamma/2 - i\Gamma_0/2]} + \frac{\exp[-i(I_0 - E_\lambda - \omega)t - \Gamma_0 t/2]}{[E_0 - E_\lambda - \omega + i\Gamma_0/2][E_{pnn} + \omega - I_0 + i\Gamma/2 - i\Gamma_0/2]} \right]. \quad (A.1)$$

The probability that the system will be found in the state Φ_λ is $|C_\lambda(t)|^2$. If we are interested in the probability without regard for the quantum numbers of the emitted photon, we must sum over the wave vector \mathbf{k} and polarization μ of the photon. There results

$$W(t) = \sum_{k,\mu} |C_{k,\mu}(t)|^2 = \gamma |V_{np}|^2 \frac{1}{(\epsilon_p + |I_0| - \omega)^2 + (\Gamma - \Gamma_0)^2/4} \times \left\{ \frac{1}{\Gamma} (1 - e^{-\Gamma t}) + \frac{1}{\Gamma_0} (1 - e^{-\Gamma_0 t}) - \frac{(\Gamma + \Gamma_0)K}{(\epsilon_p + |I_0| - \omega)^2 + (\Gamma + \Gamma_0)^2/4} \right\}, \quad (\text{A.2})$$

where

$$K = 1 - \exp(-1/2(\Gamma + \Gamma_0)t) [\cos(\epsilon_p + |I_0| - \omega)t - 2(\epsilon_p + |I_0| - \omega) \sin(\epsilon_p + |I_0| - \omega)t].$$

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Quasiclassical theory of direct chemical reactions in rapid collisions

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The amplitude of the rearrangement reactions that accompany fast molecule collisions is analyzed within the framework of the eikonal approximation. Calculation of the eikonal integral by the stationary-phase method establishes the presence of different process mechanisms, each connected with a region of scattering angles and energies in which the mechanism makes the decisive contribution to the cross section. For the differential cross section in the vicinity of the principal directions connected with these mechanisms, simple analytic formulas are obtained and connect the angular distribution with the parameters of the binary potentials between the atoms. The rate constants of the chemical reactions are estimated on the basis of the obtained approximate expressions.

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The development of effective analytic methods for the estimate of the cross sections, and also of the constants of chemical reactions in the energy interval from several electron volts and upwards is of great interest for a large number of gas dynamic problems at high temperatures.

It is known that at such energies the dominant mechanism is in many cases the mechanism of direct reactions,¹ which means both the absence of an intermediate complex and a decrease of the role of the nonadiabatic transitions between the potential surfaces. These reactions were investigated most consistently in the impulse approximation² under the conditions.

$$\tau\omega \ll 1, \quad \mathcal{F} \ll \rho_0, \quad (1)$$

where ω is the characteristic frequency of the vibrations of the nuclei, τ is the collision time, \mathcal{F} is the amplitude of the pair scattering, and ρ_0 is the average distance between atoms. In addition, an additive model of interaction potentials was used. It must be stated that the second condition of (1) is too stringent, as is incidental-

ly also the first.

Much more accurate is the eikonal approximation developed for the rearrangement reactions in our preceding papers.³⁻⁵ It is not connected with the second condition of (1) and makes it possible to relax the first somewhat. The eikonal formulas take fuller account of the interaction between the particles and make it possible, as we shall show below, to describe correctly within the framework of a single expression the "stripping," "pickup," and "knock-out" mechanisms. In the present paper we obtain from the eikonal expression a general formula of quasiclassical character for the amplitude of the stripping and pickup reactions, which yields the contribution to the small-angle scattering. This formula at lower energies than in the impulse approximation. Next, for all three mechanisms at higher energies, we obtain simple analytic formulas for the scattering amplitudes, applicable for mass calculations of the cross sections and of the rate constants of reactions at high temperatures. We obtain for the rate constants of the reactions analytic expressions that depend on the principal parameters of the colliding partners.