Propagation of transverse zero sound in He^3 and of spin waves in He^3–He II solutions

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(Submitted 6 March 1979)
Zh. Eksp. Teor. Fiz. 77, 383–395 (July 1979)

The conditions of propagation of high-frequency Fermi-liquid modes are determined by taking into account both relaxation and collisionless damping. Cherenkov absorption may dominate if the wave propagation velocity is close to the Fermi value. It is shown that the propagation of transverse zero sound in He^3 is possible even if the wave velocity is somewhat smaller than the Fermi velocity. The dispersion law for transverse sound is found without resorting to model concepts regarding the type of the f function, and all dispersion characteristics are expressed in terms of the propagation velocity of the oscillations. Due to collisionless absorption, the observation of spin waves in an He^3–He II solution is possible only at sufficiently low temperatures. In a magnetic field, the transverse spin waves with small wave vectors become weakly damped even in the Boltzmann temperature range.

PACS numbers: 67.50.Fi, 67.60.Fp

According to the Landau theory of a Fermi liquid,[1] weakly damped high-frequency $\omega\tau \gg 1$ (ω is the frequency of oscillation, τ is the characteristic relaxation time) zero-sound and spin waves can propagate in normal He^3 and in degenerate solutions of He^3 in superconducting He^4. The absorption of these modes is usually determined by collisions of the quasiparticles of the Fermi liquid with one another and the absorption falls off in proportion to T^2 upon decrease in the temperature T (see, for example, Ref. 2).

There is also a collisionless damping mechanism for the high-frequency oscillations, connected with the Cherenkov absorption of the wave by the quasiparticles. The probability of such processes depends on the number of quasiparticles moving in phase with the wave. If the velocity of propagation of the high-frequency modes significantly exceeds the Fermi velocity v_0, then at $T \ll T_0$ (T_0 is the degeneracy temperature) the collisionless damping is exponentially small. The described Landau damping mechanism turns out to be dominant in those causes in which the wave velocity is so close to the Fermi that the energy of the quasiparticles moving with such velocities lies in the region of thermal smearing out of the Fermi step.

Just such a situation exists for transverse zero sound in normal He^3 and spin waves in weak He^3–He^4 solutions. The possibility of propagation of zero sound in He^3 (high-frequency, nonsymmetric oscillations with azimuthal number $m=1$), observed experimentally by Roach and Ketterson,[3] Lea, Butcher and Dobbs,[4] is roughly determined by the inequality at the first harmonic of the Fermi-liquid function $F_1 > 6$. It is known from thermodynamic measurements (see, for example, Ref. 5) that the quantity $F_1 - 6$ is very small at low pressures. The small difference $F_1 - 6$ determines the closeness of the propagation velocity of the transverse sound to the Fermi velocity. The experimental difficulties in the study of transverse sound are to a significant degree associated with this circumstance. There are also some grounds[4] for assuming that at very low pressures the account of the principal harmonics of the f function leads to the result that the velocity of the wave would turn out to be smaller than the Fermi, which would generally make the propagation of undamped oscillations impossible. This also confirms the fact that there exists a region of pressures at which the velocity of transverse sound is close to the Fermi velocity. It is in just this region that the role of the collisionless damping studied in this paper is important.

The Cherenkov absorption is the basic mechanism of dissipation also in the case of spin waves in a weak He^3–He II solution, the velocity of propagation in which is exponentially close to v_0 in the expansion in terms of the He^3 concentration raised to the $\frac{1}{3}$ power.[6] There is great interest in the study of collisionless damping of the high-frequency modes in an He^3–He^4 solution in a constant magnetic field: the onset of this damping changes greatly the dispersion laws and the types of possible oscillations.[7,8] In a weak He^3–He^4 solution, it turns out to be possible to determine the velocity of propagation and the absorption coefficients of the spin waves at arbitrary temperatures, including the Boltzmann region $T \gg T_0$.

The basic experimental difficulty in the study of high-frequency oscillations that propagate with a velocity u close to the Fermi velocity lies in the identification of the collective mode against the background of the signal of free quasiparticles of the Fermi liquid. Time-of-flight measurements are the most accurate method for the determination of the wave velocity. To resolve the signals of the wave and of the free fermions it is necessary that the time difference between the signals $\delta t \sim L(u-v_0)/v_0^2$ (L is the distance between the receiver and the radiator) be large in comparison with the width of the pulse of high-frequency oscillations $\delta t \sim 1/\omega$, i.e., $L > v_0^2/\omega(u-v_0)$. On the other hand, for the receiver to record a pulse it is necessary that the signal not be damped over a distance $L < v_0/\omega$, where $\omega'' = \mathrm{Im}\,\omega$ is the absorption coefficient of the high-frequency mode. It is then seen that to separate experimentally at the wave contribution it is necessary to satisfy the condition

$$\omega''/\omega < (u-v_0)/v_0 \ll 1, \tag{1}$$

which is stronger than the simple condition of weak damping $\omega'' < \omega$.

Another method of determination of the signal of the

 0038-5646/79/070196-07$02.40

collective mode is possible in principle. It is connected with the fact that incoherence in the radiation of free quasiparticles leads to a decrease in the amplitude of their signal in proportion to $1/L^2$ even upon neglect of the relaxations.[9] This circumstance can turn out to be important in the case in which the velocity of transverse zero sound in He^3 becomes somewhat smaller than the Fermi velocity (see below).

1. TRANSVERSE ZERO SOUND

In the study of transverse sound in liquid He^3 we are usually limited to the first two harmonics of the Fermi-liquid function, while to take account of the damping we use the collision integral in the τ approximation.[2,9-14] This limitation is connected with the fact that for transverse oscillations, in contract to the known case of a symmetrical ($m=0$) mode[1] it has not been possible to express the propagation velocities of the wave in terms of the f function in explicit form, even with logarithmic accuracy under the condition of closeness of the velocity of zero sound to the Fermi velocity. This is due to the same causes as those for which, in an almost ideal Fermi gas, allowance for the factor of the exponential in the expression for the velocity of the symmetric wave is an exaggeration of accuracy. Nevertheless, there exists a general method of determining the dispersion of a transverse wave, similar to that proposed by the authors earlier,[8] in which all the characteristic quantities are expressed in terms of the value of the propagation velocity of the oscillations in the absence of damping $T=0$ and of magnetic field $\mathbf{H}=0$.

The collisionless kinetic equation, which describes the propagation of zero sound in the absence of a magnetic field, reduces to the form

$$(\mathbf{kv}-\omega)\delta n(\mathbf{p})-\mathbf{kv}\frac{\partial n_0}{\partial\varepsilon}\int 2\psi(\mathbf{p},\mathbf{p}')\delta n(\mathbf{p}')d\Gamma'=0, \qquad (1.1)$$

where $\mathbf{k}$ and ω are the wave vector and the frequency of the oscillations, $\mathbf{v}$, $\mathbf{p}$ and ε are the velocity, momentum and energy of the quasiparticles of the Fermi liquid, $\delta n(\mathbf{p})$ is the deviation of the quasiparticle distribution function from the equilibrium Fermi function $n_0(\varepsilon)$, $\psi(\mathbf{p},\mathbf{p}')$ is the Fermi-liquid function averaged over the spins, and $d\Gamma = d\mathbf{p}/(2\pi\hbar)$.[3] For an isotropic liquid, the function ψ depends only on the value of the momenta $|\mathbf{p}|$ and $|\mathbf{p}'|$ and on the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$. Since the quantity

$$\cos\chi=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\varphi-\varphi')$$

(θ, φ and θ', φ' are the polar angles of the vectors $\mathbf{p}$ and $\mathbf{p}'$) depends only on the difference $\varphi-\varphi'$, the equations which determine the laws of propagation of oscillations with different azimuthal numbers turn out to be independent. For transverse zero sound $m=1$ we have

$$n_1(p,x)-\frac{x}{x-s}\frac{\partial n_0}{\partial\varepsilon}\int\psi_1(p,p';x,x')n_1(p',x')d\varepsilon'\frac{dx'}{2}=0 \qquad (1.2)$$

Here $s=\omega/kv$, $x=\cos\theta$, $x'=\cos\theta'$, and the functions n_1 and ψ_1 are the first harmonics of δn and ψ:

$$\psi_1(p,p';\theta,\theta')=\int_0^{2\pi}2\psi(p,p';\cos\chi)\frac{d\Gamma}{d\varepsilon}e^{i\varphi}\frac{d\varphi}{2\pi},$$
$$n_1(p,\theta)=\int_0^{2\pi}\delta n(p,\theta,\varphi)e^{i\varphi}\frac{d\varphi}{2\pi}. \qquad (1.3)$$

Since the propagation velocity of transverse zero sound $u=\omega/k$ in degenerate He^3 at $T/T_0\ll 1$ is close to the Fermi velocity $0<(u-v_0)/v_0\equiv\alpha\ll 1$, the functions $n_1(p,\theta)$ and $\partial n_0/\partial\varepsilon$ differ from zero only in a narrow range of momenta near the Fermi momentum p_0 and of small angles θ. The accuracy of the further calculations corresponds to the neglect of terms of the order of α in comparison with the terms $\alpha\ln\alpha$. This allows us to set $p=p'=p_0$ in the argument of the function ψ_1, which is a smooth function of the momenta p and p'. Equation (1.2) is then considerably simplified:

$$\nu(x)-x\int\psi_1(x,x')\nu(x')\frac{dx'}{2}\int\frac{\partial\varepsilon}{x-s}\frac{\partial n_0}{\partial\varepsilon}=0,\quad \nu=\int n_1 d\varepsilon. \qquad (1.4)$$

We are interested in the case of small damping (1): $|s''|\ll 1-s'$ ($s''=\mathrm{Im}s<0$, $s'=\mathrm{Re}s$). Only under such conditions can the zero sound signal be distinguished from the signal of the free particles. Here the propagation velocity of the oscillations u changes only insignificantly as a function of the temperature within the limits of accuracy, $u(T)\approx u(0)\equiv u_0$, i.e., $s'\approx u_0/v_0$. Here $\alpha_0\equiv u_0/v_0-1\ll 1$, $|s''|\ll\alpha_0$, and the main contribution is made by the region of small angles θ and θ', while Eq. (1.4) can be solved by expanding the function ψ_1 in powers of $(1-x)^{1/2}$ and $(1-x')^{1/2}$. The integral equation (1.4) reduces to a system of linear algebraic equations for the quantities

$$\nu_n=\int\nu(x)(1-x)^{n/2}dx$$

with coefficients containing integrals of the form

$$I_n=\int\frac{(x-1)^n}{x-s}\frac{\partial n_0}{\partial\varepsilon}d\varepsilon\,dx,\quad n>0. \qquad (1.5)$$

The integrals I_0 do not enter into the equation since, according to (1.3), $\psi_1(1.1)=0$. Since we are interested in terms of the order $\alpha\ln\alpha$ in all the equations, we have this accuracy,

$$I_n(n>1)\approx-(-2)^n/n, \qquad (1.6)$$

while the value of I_1 is easily calculated with the help of the usual rule of passing around the poles:

$$\int\frac{d\varepsilon}{x-s}\frac{\partial n_0}{\partial\varepsilon}=\int\frac{d\varepsilon}{x-s}\frac{\partial n_0}{\partial\varepsilon}-i\pi\frac{\partial n_0}{\partial x},$$
$$\frac{\partial n_0}{\partial x}=\left(\frac{\partial n_0}{\partial\varepsilon}\frac{d\varepsilon}{ds'}\right)_{s'=x}.$$

In the principal value of the integral, it suffices to set $\partial n_0/\partial\varepsilon=-\delta(\varepsilon-\varepsilon_0)$. Correspondingly,

$$I_1\approx-2-(s'-1)\ln|s'-1|-is''\ln|s'-1|-i\pi\int_0^1(x-1)\frac{\partial n_0}{\partial x}dx. \qquad (1.7)$$

After integration by parts and the substitution $z=(2T_0/T)[(\omega/kvx)-1]$ the integral in (1.7) reduces to the form

$$I_1\approx-2-\alpha_0\ln|\alpha_0|-is''\ln|\alpha_0|+\frac{i\pi T}{2T_0}\ln\left[1+\exp\left(-2\alpha_0\frac{T_0}{T}\right)\right], \qquad (1.8)$$

where $\alpha_0=s'-1$.

Since the imaginary parts of both integrals $I_n(n>1)$ are small in comparison with the imaginary part of I_1, the collisionless damping of the oscillations is determined by the equation $\mathrm{Im}(I_1)=0$:

$$\omega''=\frac{\pi T}{2T_0\ln|\alpha_0|}kv_0\ln\left[1+\exp\left(-\frac{2\alpha_0T_0}{T}\right)\right]. \qquad (1.9)$$

For the determination of the propagation velocity of the oscillations, all the coefficients of (1.6) and (1.8) are important, and the quantity u_0 is given with the specified accuracy by an equation of the form

$$A+\alpha_0 \ln|\alpha_0|=0, \tag{1.10}$$

where the constant A is a combination of derivative of all orders of the Fermi-liquid function at the point $\chi=0$ and the constants I_n [(1.6), (1.8)] at $s=1$. The calculation of the combination in general form is impossible.

The proposed method of determination of the dispersion of transverse sound is He^3 for a wave propagation velocity close to the Fermi velocity is formally equivalent to the solution of the kinetic equation in the single harmonic approximation, in which the role of the coefficient F_1 of the first harmonic of the f function is played by the quantity $6(1+3A)$. The value of A can be found with the help of (1.10) from experimental data on the velocity of transverse oscillations in He^3.

We note that the collisionless damping (1.9) can turn out to be small even in the case in which the velocity of wave propagation u_0 is a little less than the Fermi velocity v_0, i.e., at $0<-\alpha_0\ll 1$. This fact, which is connected with the necessity of satisfaction of the law of conservation of the angular momentum in the case of absorption of quanta of transverse zero sound by the quasiparticles of the Fermi liquid, does not occur in the propagation of the symmetrical modes with $m=0$, when at $u_0<v_0$ the damping is always large.

In the case when the velocity of transverse sound is greater than the Fermi velocity, $\alpha_0>0$, the condition of smallness of the collisionless damping (1) $|s''|\ll\alpha_0$ has the form

$$T/T_0\ll\alpha_0|\ln\alpha_0|$$

and the restriction it imposes on the temperature at which observation of the signal of high-frequency oscillations is possible is weaker than the condition $T/T_0\ll\alpha_0$, which is necessary for isolation of the symmetric waves propagating with a velocity close to the Fermi velocity. At low temperatures $T/T_0\ll\alpha_0$ the collisionless damping (1.9) is exponentially small, as noted in the Introduction:

$$\omega''=\frac{kv_0}{\ln\alpha_0}\frac{\pi T}{2T_0}\exp\left(-\frac{2\alpha_0 T_0}{T}\right).$$

At higher temperatures, $\alpha_0|\ln\alpha_0|\gg T/T_0\gg\alpha_0$, the damping is equal to

$$\omega''=\frac{kv_0}{\ln\alpha_0}\frac{\pi\ln 2}{2}\frac{T}{T_0}. \tag{1.11}$$

If the velocity of transverse sound turns out to be less than the Fermi velocity, $\alpha_0<0$, then, at low temperatures, $T/T_0\ll|\alpha_0|$, the absorption coefficient

$$\omega''=\pi kv_0\frac{|\alpha_0|}{\ln|\alpha_0|}$$

does not depend on the temperature and is small upon approach of the wave velocity to the Fermi velocity $|\ln(-\alpha_0)|\gg 1$. At higher temperatures, $\alpha_0\ln|\alpha_0|\gg T/T_0\gg|\alpha_0|$, the absorption is still small and the expression for ω'' has a form similar to (1.11).

The expressions obtained take into consideration in implicit form all the harmonics of the Fermi-liquid function, and the measurement of u_0 can give information on the principal harmonics of the f function. If we limit ourselves to the two-harmonic approximation, then, with our accuracy, at $|\ln|\alpha_0||\gg 1$[cf. (1.10)][8,9]

$$-A=\alpha_0\ln|\alpha_0|=-\frac{F_1-6+3F_2/(1+F_2/5)}{3F_1+9F_2/(1+F_2/5)}. \tag{1.12}$$

We emphasize that the propagation of transverse zero sound turns out to be possible if the quantity on the right side of expression (1.12) is positive, i.e., in fact if $F_1<6$. In this case the oscillations are weakly damped at low temperature, to the extent of the smallness of the quantity $|\ln(1-F_1/6)|^{-1}$.

We now consider the role of collisional damping in the case of closeness of the wave velocity to the Fermi velocity. The coefficient of collisional damping of the high-frequency mode, with account of the quantum properties of the zero-sound excitation, is expressed in terms of the relaxation time τ in the following manner[1]:

$$\omega''_{coll}=-\frac{1}{\tau}\left[1+\left(\frac{\hbar\omega'}{2\pi T}\right)^2\right]. \tag{1.13}$$

For observation of the signal of high-frequency sound, satisfaction of the condition (1) is necessary. In the case of collisional absorption (1.13), this condition is satisfied at frequencies

$$\omega_{1,2}=2|\alpha_0|\frac{\pi^2T^2\tau}{\hbar^2}\left\{1\pm\left[1-\left(\frac{\hbar}{\pi T\tau|\alpha_0|}\right)^2\right]^{1/2}\right\}. \tag{1.13'}$$

For the existence of such a range of frequencies, it is necessary that the temperature be sufficiently small:

$$T<|\alpha_0|\pi b/\hbar,\quad b=\tau T^2=\text{const},$$

where it is taken into account that $\tau\propto T^{-2}$ in the normal Fermi liquid. By use of experimental data,[5] we can obtain the numerical estimate

$$T[\mathrm{K}]<|\alpha_0|,\quad \omega_{1,2}[\sec^{-1}]\approx 10^{11}|\alpha_0|\left\{1\pm\left(1-\frac{T^2[\mathrm{K}]}{\alpha_0^2}\right)^{1/2}\right\}.$$

It is easy to show that in this range of frequencies and temperatures, in which the oscillations are weakly damped and the condition of the expansion (1) is satisfied, the collisional absorption of zero sound is more significant than the Landau damping in the case of a wave velocity much greater than the Fermi velocity. Collisionless damping can turn out to be important dissipative mechanism in the case $u_0<v_0$, $T[\mathrm{K}]\lesssim\alpha_0/\ln|\alpha_0|$.

Transverse zero sound in a magnetic field

As a consequence of the closeness of the oscillation velocity to the Fermi velocity, the conditions of propagation of transverse zero sound turn out to be sensitive to the application of even a weak magnetic field **H**.[8] In He^3 in a magnetic field, the Fermi surfaces of the quasiparticles with different spin orientations are two Fermi spheres, the radii of which $v_\pm$ in a weak magnetic field

$$h=\beta H/T_0(1+Z_0)\ll 1$$

(β is the magnetic moment of the He^3 nucleus, Z_0 is the zeroth harmonic of the exchange part of the f function and determines the static susceptibility of He^3 in weak fields) are equal to

$$(v_\pm - v_0)/v_0 = \pm h/2.$$

In an isotropic Fermi liquid in the exchange approximation, the f function in a weak magnetic field can always be represented in the form of the following linear combination of spin operators σ (see, for example, Ref. 7):

$$f(\mathbf{p}, \mathbf{p}') = \psi(\mathbf{p}, \mathbf{p}') + \zeta(\mathbf{p}, \mathbf{p}')\sigma\sigma' + \varphi(\mathbf{p}, \mathbf{p}')\mathbf{H}(\sigma + \sigma').$$

The propagation of high-frequency oscillations is determined by the collisionless kinetic equation for the single particle density matrix $\hat{n}$. In the given case, we are interested in two coupled equations, for the scaler distribution function $n = \mathrm{Sp}_\sigma\, \hat{n}$ and for the longitudinal magnetization $n^z = \mathrm{Sp}_\alpha\, \hat{n}\sigma^z$ (cf. Refs. 8 and 9):

$$\begin{aligned}
&(\omega - \mathbf{kv})\delta n + \mathbf{kv}\left(\frac{\partial n_+}{\partial \varepsilon_+} + \frac{\partial n_-}{\partial \varepsilon_-}\right)\left[\int \psi \delta n'\, d\Gamma' + H\int \varphi \delta n^{z\prime}\, d\Gamma'\right] + \mathbf{kv}\left(\frac{\partial n_+}{\partial \varepsilon_+} - \frac{\partial n_-}{\partial \varepsilon_-}\right)\int \zeta \delta n^{z\prime}\, d\Gamma' = 0,\\
&(\omega - \mathbf{kv})\delta n^z + \mathbf{kv}\left(\frac{\partial n_+}{\partial \varepsilon_+} + \frac{\partial n_-}{\partial \varepsilon_-}\right)\left[\int \zeta \delta n^{z\prime}\, d\Gamma' + H\int \varphi \delta n'\, d\Gamma'\right] + \mathbf{kv}\left(\frac{\partial n_+}{\partial \varepsilon_+} - \frac{\partial n_-}{\partial \varepsilon_-}\right)\int \psi \delta n'\, d\Gamma' = 0,
\end{aligned} \tag{1.14}$$

where $n_\pm$ $(\varepsilon_\pm)$ are the equilibrium Fermi distribution functions for particles polarized along and against the direction of the field, $\delta\hat{n}$ is the deviation of the single-particle density matrix from equilibrium.

Since spin waves cannot propagate in normal He^3 in the absence of a magnetic field, the effect of the oscillations of the longitudinal magnetization δn^z on the solution of zero sound type in weak fields $h \ll 1$ is proportional to h^2, in accord with (1.14). This statement would remain valid even if undamped spin modes could be propagated in the liquid, under the condition that its velocities differ greatly from the velocity of zero sound. In what follows, we shall limit ourselves in the dispersion equation only to terms of the order $h \ln h$. Therefore, the oscillations of the longitudinal magnetization can be disregarded:

$$(\omega - \mathbf{kv})\delta n + \mathbf{kv}\left(\frac{\partial n_+}{\partial \varepsilon_+} + \frac{\partial n_-}{\partial \varepsilon_-}\right)\int \psi(\mathbf{p}, \mathbf{p}')\delta n'\, d\Gamma' = 0. \tag{1.15}$$

In the limits of accuracy, the differences of the momenta p and p' from p_0 in the argument of the f function can be neglected (cf. Ref. 8). The method of solution of Eq. (1.15) is virtually unchanged from that already used for the solution of Eq. (1.1), and the dispersion relations in the case of small damping (1) take the form

$$\begin{aligned}
&2A + \alpha_+ \ln|\alpha_+| + \alpha_- \ln|\alpha_-| = 0,\\
&\omega'' = \frac{\pi T}{2T_0} kv_0 \frac{\ln[1 + \exp(-2\alpha_+ T_0/T)] + \ln[1 + \exp(-2\alpha_- T_0/T)]}{\ln|\alpha_+| + \ln|\alpha_-|}
\end{aligned} \tag{1.16}$$

Here $\alpha_\pm = \omega/kv_\pm - 1 = \omega/kv_0 - 1 \mp h/2$, and A is the same constant as in the expression (1.10).

The velocity of propagation of transverse zero sound, given by the first of the relations (1.16), does not depend on the magnetic field in weak fields at the logarithmic accuracy with which the expressions (1.16) themselves are valid:

$$\omega/k = u_0 = v_0(1 + \alpha_0), \quad \alpha_\pm = \alpha_0 \mp h/2. \tag{1.17}$$

The condition of collisionless propagation of zero sound (at $T = 0$) $v_+ < u$ ceases to be satisfied[8] in a critical field h_c such that $\alpha_+(h_c) = 0$ and $\alpha_-(h_c) = h_c$. Correspondingly, the critical field (at $u_0 > v_0$)

$$h_c = 2\alpha_0 \ll 1 \tag{1.18}$$

turns out to be very small, which allows us to assume a weak magnetic field in all the calculations. In fields greater than h_c, damping arises even at $T = 0$. The wave is ever more weakly attenuated as $h - h_c$ becomes smaller.

The determination of the magnitude and temperature dependence of the sound absorption coefficient in the magnetic field (1.16) with account of (1.17) is carried out in similar fashion and leads to similar results as for the absorption coefficient in the absence of a field (1.9).

Thus, for example, at $T = 0$ and $1 \gg (h - h_c)/h_c \geqslant 0$, the absorption coefficient of transverse sound

$$\omega'' = \pi k v_0 \frac{(h - h_c)/2}{\ln[(h - h_c)/2]}$$

vanishes if $h = h_c$ and increases with increase in the magnetic field.

All the dispersion characteristics of the transverse zero sound in He^3 given above were expressed in terms of an observed quantity—the velocity of propagation of the oscillations u_0. In a real experiment, it may be simpler to determine not u_0 but the critical magnetic field h_c. The relation (1.18) makes it possible to express the dispersion of the oscillations in terms of h_c also. With the help of experimental data on He^3 and the relations (1.12) and (1.18) for the value of the critical field at zero pressure we get a value of the order of 4 kOe.[8]

2. SPIN WAVES

The collisionless kinetic equation which describes the propagation of spin waves in a paramagnetic Fermi liquid in the absence of an external field $\mathbf{H} = 0$ can be represented in form[1]

$$(\omega - \mathbf{kv})\nu(\mathbf{p}) + \mathbf{kv}\frac{\partial n_0}{\partial \varepsilon}\int 2\zeta(\mathbf{p}, \mathbf{p}')\nu(\mathbf{p}')\, d\Gamma' = 0, \tag{2.1}$$

where $\zeta(\mathbf{p}, \mathbf{p}')$ is the exchange part of the Fermi-liquid function. It follows from experimental data on the magnetic susceptibility of normal He^3 that the propagation in it of spin oscillations is most probably impossible. In the case of degenerate He^3–He II solutions, there exists only the single undamped spin mode—a symmetric wave whose propagation velocity u_0 at $T = 0$ proves to be exponentially close to the Fermi velocity[6]:

$$\frac{u_0}{v_0} = \frac{\omega}{kv_0} = 1 + \exp\left(-\frac{1}{|\lambda|}\right). \tag{2.2}$$

For weak He^3–He^4 solutions, all the calculations can be carried out in explicit fashion, since an exact expression is known for the Fermi-liquid function in the form of an expansion in a power series in the small parameter $\lambda = p_0 a/\pi\hbar \ll 1$ ($a = -1.5$ Å is the-scattering length of the He^3 quasiparticles by one another), which is proportional to the concentration raised to the $\frac{1}{3}$ power. The expression for the velocity of spin waves (2.2) was written with logarithmic accuracy. In the calculation of the pre-exponential factor, it is necessary to take into account the values of the derivatives of the f functions at $\chi=0$. The exponential closeness of the propagation velocity of the oscillations to v_0 is the cause of the importance of the collisionless Landau damping even at very low temperatures.

In the Born approximation, Eq. (2.1) is equivalent to the following dispersion equation:

$$1+\zeta_0\int\frac{2(\partial n_0/\partial\varepsilon)\mathbf{kv}}{\omega-\mathbf{kv}}d\Gamma\equiv\chi(\omega,\mathbf{k})=0, \tag{2.3}$$

where $\zeta_0=\zeta(\mathbf{p}_0,\mathbf{p}_0')=-2\pi a\hbar^2/M$, ${}^6M=2.33m_s$ is the effective mass of the bare He^3 excitation, m_3 is the mass of the He^3 atom. We bypass the pole of the integrand in (2.3) in the usual fashion, just as in the previous part of the work:

$$\int\frac{(\partial n_0/\partial\varepsilon)\mathbf{kv}\,d\Gamma}{\omega-\mathbf{kv}+i0}=\int\frac{(\partial n_0/\partial\varepsilon)\mathbf{kv}\,d\Gamma}{\omega-\mathbf{kv}}-i\pi\int\frac{\partial n_0}{\partial\varepsilon}\delta(\omega-\mathbf{kv})\mathbf{kv}\,d\Gamma. \tag{2.4}$$

For the determination of the spectrum of weakly damped oscillations, we seek a solution of Eq. (2.3) in the form $\omega\,\omega'-i\gamma$, $0<\gamma\ll\omega'$. In the case of small damping, it follows from the dispersion relation (2.3) that the real part of the propagation velocity ω'/k and the absorption coefficient γ of the spin wave are determined by the relations

$$\begin{gathered}\operatorname{Re}\chi(\omega',\mathbf{k})=0,\\ \gamma=\operatorname{Im}\chi(\omega',\mathbf{k})\Big/\frac{\partial\operatorname{Re}\chi(\omega',\mathbf{k})}{\partial\omega'}.\end{gathered} \tag{2.5}$$

At low temperatures $T/T_0\ll1$, the derivative $\partial n_0/\partial\varepsilon$ can be represented in the form

$$\frac{\partial n_0}{\partial\varepsilon}=-\delta(\varepsilon-\mu)-\frac{\pi^2}{6}T^2\frac{\partial^2}{\partial\varepsilon^2}\delta(\varepsilon-\mu), \tag{2.6}$$

where μ is the chemical potential of He^3 in the solution. The dispersion equation is given here in the following form:

$$\begin{gathered}\operatorname{Re}\chi(\omega',\mathbf{k})=1-Z_0w(s_0)-\frac{\pi^2}{3}\zeta_0T^2\frac{\partial^2}{\partial\varepsilon^2}\left[w(s)\frac{d\Gamma}{d\varepsilon}\right]_{\varepsilon=\mu},\\ \operatorname{Im}\chi(\omega',\mathbf{k})={}^1/{}_2\pi Z_0s_0n_0(s_0).\end{gathered} \tag{2.7}$$

In the expressions (2.7), $s_0=\omega'/kv_0=u'/v_0$, $n_0(s_0)$ is the value of the distribution function for the He^3 quasiparticles moving with a velocity equal to the velocity of the spin wave, i.e., at $v=\omega'/k$ we have

$$\begin{gathered}Z_0=2\left(\frac{d\Gamma}{d\varepsilon}\right)_{\varepsilon=\mu}\quad\zeta_0=-2\lambda,\\ w(s)=\frac{s}{2}\ln\frac{s+1}{s-1}-1,\qquad s=\frac{u'}{v}.\end{gathered} \tag{2.8}$$

Since $|\lambda|\ll1$ in the degenerate He^3–He II solutions at sufficiently low concentrations of impurity atoms (in practice, below 3–4%), then the basic contribution to the function $w(s)$ is made by the large logarithm $\ln(s-1)$. With account of this fact, we determine the collisionless absorption of the symmetric spin mode with logarithmic accuracy from the formulas (2.5) and (2.7):

$$\gamma=\pi n_0(s_0)kv_0\exp\left(-\frac{1}{|\lambda|}\right) \tag{2.9}$$

and the temperature corrections to the wave propagation velocity at $T=0$:

$$\begin{gathered}u'=\frac{\omega'}{k}=u_0-\frac{\pi^2}{24}\left(\frac{T}{T_0}\right)^2v_0\exp\left(-\frac{1}{|\lambda|}\right),\\ \frac{T}{T_0}\ll\exp\left(-\frac{1}{|\lambda|}\right)\ll1.\end{gathered} \tag{2.10}$$

We note that the second term in (2.10) is small in comparison with the difference between the real value of u_0 and that given by formula (2.2) with logarithmic accuracy. However, this term at $T/T_0\ll\exp(-1/|\lambda|)$ completely determines the temperature dependence of the spin wave velocity.

The condition of the smallness of the damping, which was used in the derivation of the equalities (2.5) in the given case, implies, as a consequence of the effect of the closeness of the propagation velocity of the spin wave to the Fermi velocity, that $\gamma\ll\omega'-kv_0\sim kv_0\times\exp(-1/|\lambda|)$. This means that the region of applicability of the expression for γ (2.5) and (2.9) is restricted by the condition

$$n_0(s_0)=\exp\left(-2e^{-1/|\lambda|}\frac{T_0}{T}\right)\ll1. \tag{2.11}$$

The dispersion characteristics of the symmetric spin mode can be expressed in terms of u_0 with the help of a method similar to that used in the previous section of the paper. In the corresponding notation, the dispersion equation has the form

$$\ln\alpha-i\pi n(\alpha)=\ln\alpha_0, \tag{2.12}$$

where $\alpha=\omega/kv_0-1$, $\alpha_0=\exp(-1/|\lambda|)$. In the case of weak damping $n(\alpha)\ll1$, the absorption coefficient is determined by Eq. (2.9). At higher temperatures $\alpha_0T_0/T\ll1$, the quantity $n(\alpha)\sim\frac{1}{2}$ and the condition (1) is violated. Thus, the spin waves in the He^3–He^4 solution can be recorded only at sufficiently low temperatures $T\ll T_0\exp(-1/|\lambda|)$. This is equivalent to the numerical inequality

$$T[\mathrm{mK}]\ll4.2\cdot10^3x^{2/3}\exp(-2.4/x^{1/3}),$$

where x is the molar concentration of the He^3 in the solution.

For collisional absorption (1.13), the resolution condition (1) is satisfied in the given case in the range of frequencies (1.13′)

$$\omega_1<\omega<\omega_2\ll\frac{T_0}{\hbar},\qquad\omega_{1,2}=10^{12}\alpha_0\left\{1\pm\left(1-\frac{T^2[\mathrm{K}]}{10\alpha_0^2}\right)^{1/2}\right\}$$

at temperatures $T[\mathrm{K}]\lesssim\alpha_0$. This restriction on the temperature is weaker than that obtained above for the Landau damping. From a comparison of the coefficient of Cherenkov absorption (2.9) with ω_{coll} (1.13), with account of (1), it is easy to determine the range of frequencies included in the interval between ω_1 and ω_2, and the condition on the temperature, $T[K]<10\alpha\times\exp(-2\alpha_0T_0/T)$, at which the collisionless damping dominates.

Spin waves in a magnetic field

The action of an external magnetic field H leads to a change in the spectrum of the spin waves in the degenerate He^3–He II solution. Two transverse slit modes appear here for the oscillations of the magnetic-moment components perpendicular to the direction of the field H, and a longitudinal mode of coupled high-frequency spin-sound oscillations.[7] Since the longitudinal spin mode in the solution appears even in an exponentially weak magnetic field[7] and at very low temperatures, we shall in what follows be interested in the conditions of propagation only of transverse oscillations of the magnetization.

The equation for the circular component of the magnetic moment $M_x + iM_y$ can be represented in the following form[7]:

$$(\omega-\mathbf{kv})\nu+\zeta_0\left[\mathbf{kv}\left(\frac{\partial n_+}{\partial\varepsilon_+}+\frac{\partial n_-}{\partial\varepsilon_-}\right)-\frac{2}{\hbar}(n_+-n_-)\right]\int\nu\,d\Gamma-\frac{2BH}{\hbar}\nu=0, \quad (2.13)$$

where $n_\pm$ are the equilibrium distribution functions of the quasiparticles with spin orientations along and counter to the direction of the magnetic field H, and energies $\varepsilon_\pm$, respectively. In Eqs. (2.13) we have used the value of the exchange part of the Fermi-liquid function $\zeta(\mathbf{p}, \mathbf{p}')$ in the Born approximation $\zeta(\mathbf{p}, \mathbf{p}') = \zeta_0 = -2\pi a\hbar^2/M$. In this case, $BH = \beta H - \zeta_0(N_+ - N_-)$, $N_\pm$ is the number of impurity atoms of He^3 with different spin orientations per unit volume of solution, and $N = N_+ + N_-$ is the total number of He^3 atoms per unit volume.

After simple transformations of (2.13) we find the dispersion equation for the spectrum of a transverse spin wave:

$$1+\zeta_0\int\frac{\mathbf{kv}(\partial n_+/\partial\varepsilon_+)d\Gamma}{\Omega-\mathbf{kv}}+\zeta_0\int\frac{\mathbf{kv}(\partial n_-/\partial\varepsilon_-)d\Gamma}{\Omega-\mathbf{kv}}$$
$$-\frac{2\zeta_0}{\hbar}\int\frac{n_+-n_-}{\Omega-\mathbf{kv}}d\Gamma\equiv\chi(\omega,\mathbf{k})=0. \quad (2.14)$$

Here $\Omega = \omega - 2BH/\hbar$. The equation for the conjugate circular component can be obtained from (2.14) by the formal substitution $\omega \to -\omega$, $\mathbf{k} \to -\mathbf{k}$.

We seek the weakly damped solution of Eq. (2.14) in the form $\omega = \omega' - i\gamma$, $0 < \gamma \ll \omega'$ according to the formulas (2.5). After integration in (2.14) we get with the aid of (2.6)

$$\operatorname{Re}\chi(\omega',\mathbf{k})=1+\lambda\left\{\frac{v_+}{v_0}w(s_+)+\frac{v_-}{v_0}w(s_-)\right.$$
$$+\frac{4T_0}{\hbar\Omega}\left[-\frac{N_+}{N}(s_+^2-1)w(s_+)+\frac{N_-}{N}(s_-^2-1)w(s_-)+\frac{N_+-N_-}{N}\right]$$
$$\left.-\zeta_0\frac{\pi^2}{6}T^2\left\{\left[\frac{\partial^2}{\partial\varepsilon^2}w(s)\frac{d\Gamma}{d\varepsilon}\right]_{\varepsilon_+=\mu}+\left[\frac{\partial^2}{\partial\varepsilon^2}w(s)\frac{d\Gamma}{d\varepsilon}\right]_{\varepsilon_-=\mu}\right\}\right\}, \quad (2.15)$$

$$\operatorname{Im}\chi(\omega',\mathbf{k})=-\frac{\pi}{2}Z_0s_0[n_+(s_0)+n_-(s_0)]$$
$$+\frac{\pi}{2}\frac{Z_0}{\hbar kv_0}\int_{T_0s_0^2}^{\infty}[n_+(\varepsilon)-n_-(\varepsilon)]d\varepsilon. \quad (2.16)$$

In Eqs. (2.15) and (2.16), the function $w(s)$ is determined by the formula (2.8), $s_\pm = \Omega/kv_\pm$, $n_\pm(s_0)$ is the value of the distribution function $n_\pm(\varepsilon_\pm)$ at $\varepsilon_\pm = T_0 s_0^2$, $s_0 = \Omega/kv_0$.

In the region of small wave vectors, the magnon spectrum is quadratic in $\mathbf{k}$[7]:

$$\omega'=\frac{2\beta H}{\hbar}+\frac{\hbar k^2}{2M}\frac{(36\pi)^{1/3}}{10|a|}\frac{N_+^{5/3}-N_-^{5/3}}{(N_+-N_-)^2}, \qquad kv_+\ll\Omega. \quad (2.17)$$

Here the Cherenkov damping of the spin wave is exponentially small:

$$\gamma\propto\left(\frac{\omega_{int}}{kv_0}\right)\exp\left[-\frac{T_0}{T}\left(\frac{\omega_{int}}{kv_0}\right)^2\right], \quad \omega_{int}=\frac{2\zeta_0}{\hbar}(N_+-N_-) \quad (2.18)$$

and the fundamental dissipation mechanism is the collisional relaxation absorption.

In the shortwave region $kv_- \gg \omega_{int}$, the dispersion equation (2.13) takes a form similar to (2.3). Therefore, all the results of the previous section of the paper can be extended directly to this case. It should be noted that the region of existence of the shortwave solution of Eq. (2.13) is limited by the condition of applicability of the quasi-classical kinetic equation used in the derivation of (2.13).

At higher temperatures of the solution, $T \gtrsim T_0$, the damping connected with the finite lifetime of the quasiparticles of the Fermi liquid is important. However, thanks to the low concentration of the solution, the damping of the excitation of He^3 is small to the extent of the smallness of the gas parameter λ.[15] In the Born approximation, the damping of the quasiparticles is generally absent. This allows us to solve the problem in first order perturbation theory of the propagation of waves of magnetization in the weak He^3–He^4 solution at arbitrary temperatures.

At $T \gg T_0$, the impurity atoms of He^3 obey Boltzmann statistics. In the Boltzmann region $T \gg T_0$, in the absence of a magnetic field $H = 0$, the dispersion equation (2.3), which describes the symmetric spin mode is easily transformed to the form (compare, for example, Ref. 16)

$$1+\frac{\mu_{int}}{T}\left[1-J\left(\frac{\omega}{kv_T}\right)\right]=0, \quad (2.19)$$

where $\mu_{int} = \zeta_0 N$ is the contribution to the chemical potential of He^3 in the solution due to interaction of the impurity excitations with one another, $v_T = (T/M)^{1/2}$ is the thermal velocity of the quasiparticles of He^3, and the quantity $J(x)$ is determined by the expression

$$J(x)=xe^{-x^2/2}\int_{i\infty}^{x}dz\,e^{z^2/2}=i\left(\frac{\pi}{2}\right)^{1/2}x\operatorname{erf}\left(\frac{x}{2^{1/2}}\right). \quad (2.20)$$

It is not difficult to convince oneself that, because of $\mu_{int}/T \ll 1$, the dispersion equation (2.19) has only a highly damped solution with $|\operatorname{Im}\omega| \gg |\operatorname{Re}\omega|$:

$$\frac{\omega}{kv_T}\approx\frac{\pi}{[2\ln(T/\mu_{int})]^{1/2}}-i\left[2\ln\frac{T}{\mu_{int}}\right]^{1/2}.$$

This agrees with the result obtained above, that the symmetric spin oscillations are no longer observed in practice at very low temperatures $T \sim T_0\exp(-1/|\lambda|) \ll T_0$.

An entirely different situation arises when an external magnetic field H is turned on. After calculation of the integrals with the Boltzmann equilibrium distribution functions n_+ and n_-, the dispersion equation (2.14) for the transverse oscillations of the magnetization takes the form

$$\Omega+\left[1-J\left(\frac{\Omega}{kv_T}\right)\right]\left[\frac{\mu_{int}}{2T}\Omega+\omega_{int}\right]-\omega_{int}=0, \qquad (2.21)$$

where $\Omega=\omega-2BH/\hbar+\omega_{int}$ and the functions ω_{int} and $J(x)$ are determined by the expressions (2.18) and (2.20), respectively.

We seek a long-wave weakly damped solution of Eq. (2.21) in the form of an expansion in powers of kv_T/ω_{int}. In the zeroth approximation $N_+-N_-=N\tanh(BH/T)$ corresponds to a uniform free precession of the nuclear spins of He^3. With the help of the asympototic representation (2.20) of the function $J(x)$, for homogeneous solution $\omega'=\omega_0$ corrections that are quadratic in kv_T/ω_{int}:

$$\omega'=\omega_0+\frac{\omega_{int}}{\omega_0^2}k^2v_T^2=\omega_0+\frac{2\mu_{int}}{\hbar}\frac{k^2}{M\omega_0^2}T\,\mathrm{th}\left(\frac{\beta H}{T}\right), \qquad (2.22)$$
$$\mu_{int}=-2\pi a\hbar^2N/M.$$

It was taken into account in (2.22) the fact that in the Born approximation, it is necessary to use for the difference in populations N_+-N_-, which enters into ω_{int}, its value in the ideal Boltzmann gas with neglect of the Fermi-liquid interaction: $N_+-N_-=N\tanh(BH/T)$. The absorption coefficient γ of the wave is equal to

$$\gamma=\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_{int}}{kv_T}\exp\left[-\frac{1}{2}\left(\frac{\omega_{int}}{kv_T}\right)^2-\frac{\omega_{int}}{\omega_0}\right]. \qquad (2.23)$$

At not too small vectors k the Cherenkov damping (2.23) can turn out to be greater than the collisional damping $\gamma_{coll}\sim Na^2v_T$.

In the short-wave region, when the presence of a gap in the dispersion law becomes unimportant and the oscillations of the magnetization are described by an equation of the form (2.19), strong collisionless damping makes propagation of the spin wave impossible.

Thus, the observation of spin waves in the He^3–He II solution in the absence of a magnetic field required the obtaining of very low temperatures. The turning on of the magnetic field makes possible the propagation of weakly damped spin modes over a wide range of temperatures. The increase in the temperature decreases the width of the region of wave vectors in which undamped magnons exist.

We express our gratitude to A. F. Andreev, M. I. Kaganov and I. A. Fomin for useful discussions.

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Translated by R. T. Beyer