

# Magnetic-ordering temperature in a Fermi liquid close to a phase transition

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(Submitted 4 January 1979)  
Zh. Eksp. Teor. Fiz. 76, 1834–1836 (May 1979)

Stability conditions are found for a Fermi liquid close to a phase transition at finite temperatures. The critical temperatures  $T_c^{(l)}$  of the magnetic phase transitions are calculated. Under certain conditions high temperatures  $T > T_c^{(l)}$  correspond to a magnetically ordered phase.

PACS numbers: 75.40.Bw

According to the Pomeranchuk conditions,<sup>1</sup> the ground state of a paramagnetic Fermi liquid at zero temperature ( $T=0$ ) is unstable against the appearance of magnetic order in the system if even just one of the inequalities  $Z_l + (2l+1) < 0$ ;  $l=0, 1, 2, \dots$ , is fulfilled, where  $Z_l$  are the coefficients, defined in the usual way, in the harmonics of the Fermi-liquid function. If the system is close to a magnetic phase transition, i.e.,  $|Z_l + (2l+1)| \ll 1$ , even extremely small thermal effects can lead to instability of the paramagnetic Fermi liquid and to the appearance or disappearance of magnetic order at a certain temperature  $T_c^{(l)}$ . The phase-transition temperature  $T_c^{(l)}$  turns out to be considerably lower than the Fermi degeneracy temperature  $T_F$ , and this ensures that it is possible to apply Fermi-liquid theory to calculate  $T_c^{(l)}$ . In practice the situation  $T_c^{(l)}/T_F$  is realized in, e.g., ferromagnets of the Fe group. We shall confine ourselves here to studying an isotropic Fermi liquid, in which the Fermi surface of the quasi-particles in the disordered phase is a sphere with weak thermal diffuseness:  $T/T_F \ll 1$ .

As in the BCS theory, the magnetic phase-transition temperature can be determined only by starting from the properties of the Fermi liquid in the disordered phase. At  $T=T_c^{(l)}$ , in the spin part of the vertex function of the disordered Fermi liquid an imaginary pole should appear, corresponding to the growth of an arbitrarily small magnetization fluctuation and to instability of the paramagnetic state of the system. In a paper by Éliashberg<sup>2</sup> it was shown that in order to determine the spectrum of the spin fluctuations in a paramagnetic Fermi liquid at finite temperatures  $T/T_F \ll 1$  we can use the Landau equation for the vertex function at  $T=0$  (Ref. 3). Here, of course, the relationship between the vertex function and the Fermi-liquid function is renormalized on account of the temperature terms, and a collision term due to the mutual scattering of the quasi-particles is added to the kinetic equation. The kinetic equation determining the poles of the vertex function in the disordered Fermi liquid has the form

$$(\omega - kv)\lambda(\mathbf{p}) + kv \frac{\partial n}{\partial \epsilon} \int \zeta(\mathbf{p}, \mathbf{p}') \lambda(\mathbf{p}') d\Gamma' = I(\lambda), \quad (1)$$

where  $\omega$  and  $\mathbf{k}$  are the frequency and wave vector of the spin fluctuation,  $\mathbf{v} = \partial \epsilon / \partial \mathbf{p}$  is the velocity of the Fermi liquid quasiparticle,  $\zeta(\mathbf{p}, \mathbf{p}')$  is the spin part of the Fermi liquid function,  $d\Gamma = 2d^3p / (2\pi\hbar)^3$ ,  $I(\lambda)$  is the collision integral,

$$\frac{\partial n}{\partial \epsilon} = -\delta(\epsilon - \mu) - \frac{\pi^2}{6} T^2 \frac{\partial^2}{\partial \epsilon^2} \delta(\epsilon - \mu), \quad (2)$$

and  $\mu$  is the chemical potential.

We shall seek the eigenvalue  $\omega = ikv\gamma$  of Eq. (1) that are pure-imaginary ( $\text{Im}\gamma = 0$ ) and small ( $|\gamma| \ll 1$ ). In this case, as can be seen from the results of Éliashberg's paper,<sup>2</sup> the collision integral can be neglected and, in the linear approximation in  $\gamma$ , Eq. (1) reduces to a system of independent equations for fluctuations with different azimuthal angular momenta  $l$ :

$$1 + \frac{(2l-1)!!}{(2l)!!} \int Z_l \frac{\partial n}{\partial \epsilon'} \frac{\cos \theta'}{i\gamma - \cos \theta'} \sin^{2l} \theta' d\epsilon' \frac{d\theta'}{4\pi} = 0. \quad (3)$$

Neglecting terms of order  $(T/T_F)^2 \gamma$ , which is justified by the result obtained, from (3) we obtain

$$(2l+1) + Z_l + \frac{\pi^2}{6} \left(\frac{T}{T_F}\right)^2 \mu^2 \frac{\partial^2 Z_l}{\partial \mu^2} = \frac{\pi}{2} \frac{(2l+1)!!}{(2l)!!} Z_l |\gamma|. \quad (4)$$

This equality determines the critical temperature  $T_c^{(l)}$  at which an imaginary pole appears in the vertex part of the disordered Fermi liquid and a magnetic phase transition occurs:

$$T_c^{(l)} = \frac{T_F}{\pi} (6\alpha_l)^{1/2}, \quad \alpha_l = -\frac{Z_l + (2l+1)}{\mu^2 \partial^2 Z_l / \partial \mu^2}, \quad |\alpha_l| \ll 1. \quad (5)$$

Closeness of the system to the phase transition gives rise to a small value of the critical temperature  $T_c^{(l)}$ . At  $T=0$ , Eq. (4) leads to the Pomeranchuk conditions.<sup>1</sup>

It can be seen from the expression (5) that in the case  $Z_l + (2l+1) > 0$ ,  $\mu^2 \partial^2 Z_l / \partial \mu^2 < 0$  we have the unusual situation in which the higher temperatures  $T_F \gg T > T_c^{(l)}$  correspond to a magnetically ordered phase while for  $T < T_c^{(l)}$  the Fermi liquid is paramagnetic. The Fermi-liquid stability conditions for the spin-averaged interaction of the quasi-particles are changed entirely analogously:

$$\frac{F_l + (2l+1)}{\mu^2 \partial^2 F_l / \partial \mu^2} = \beta_l < 0, \quad |\beta_l| \ll 1. \quad (6)$$

The ferromagnetic-transition temperature  $T_c^{(0)}$  can also be obtained easily when the magnetic susceptibility of the Fermi liquid is calculated in the usual way<sup>3</sup> with allowance for the temperature corrections (2). In the paramagnetic phase near the Curie temperature  $T_c^{(0)}$  we have for the susceptibility  $\chi$ :

$$\chi = \frac{9}{2\pi^2} \frac{T_F}{T_c^{(0)}} \frac{\beta^2 N}{\mu^2 \partial^2 Z_0 / \partial \mu^2} \frac{1}{T - T_c^{(0)}}, \quad (7)$$

where  $\beta$  is the fermion magnetic moment and  $N$  is the atomic density. The critical temperatures  $T_c^{(1)}$  for magnetic transitions from the disordered phase can also be found thermodynamically from the conditions for the stability of a Fermi sphere with thermal diffuseness against small deformations.

Model calculations of the temperature dependence of the magnetization in the ordered phase and  $T_c^{(1)}$  were performed earlier by Akhiezer *et al.*<sup>4</sup> These calculations did not take into account, e.g., the dependence of the Fermi-liquid function on the magnetic moment, of the form  $\varphi(\sigma + \sigma')M$ , which gives a contribution to the free energy of the same order as that in Ref. 4.

In liquid  $^3\text{He}$  the quantity  $1 + Z_0 \approx 0.3$ . Crude estimates from the available experimental data show that  $\partial^2 Z_0 / \partial \mu^2 > 0$ , i.e.,  $\alpha_0 < 0$ , and the paramagnetic state of  $^3\text{He}$

is evidently stable in the entire region of existence of the liquid phase.

The author is grateful to A. F. Andreev, I. E. Dzyaloshinskiĭ, M. I. Kaganov, and L. P. Pitaevskii for very useful discussions and consultations.

<sup>1</sup>I. Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. 35, 524 (1958) [Sov. Phys. JETP 8, 361 (1959)].

<sup>2</sup>G. M. Éliashberg, Zh. Eksp. Teor. Fiz. 41, 1241 (1961); 42, 1658 (1962) [Sov. Phys. JETP 14, 886 (1962); 15, 1151 (1962)].

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<sup>4</sup>I. A. Akhiezer, I. T. Akhiezer, and E. M. Chudnovskii, Fiz. Nizk. Temp. 2, 1321 (1976) [Sov. J. Low Temp. Phys. 2, 645 (1976)].

Translated by P. J. Shepherd

## Natural optical activity in semiconductors with wurtzite structure

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(Submitted 12 January 1979)

Zh. Eksp. Teor. Fiz. 76, 1836–1855 (May 1979)

A theory is constructed of natural optical activity of crystals with wurtzite structure in the exciton-resonance frequency region. The reflection spectra are calculated for parallel and crossed polarizations of the incident and reflected light. The method of calculating the reflection coefficient is generalized to include the case of oblique incidence of the light for a generate exciton level, with allowance for dipole-forbidden states. Analysis of the limiting transition to the nonresonant region has made it possible to compare the conclusions of the developed microscopic theory with the results of the phenomenological approach. The reflection spectra of CdS at oblique incidence of light on the crystal boundary, in the region of the exciton resonance  $B_{n=1}$  are experimentally investigated for the first time ever. The theoretical and experimental results are compared.

PACS numbers: 78.20.Ek, 78.20.Dj

### INTRODUCTION

Natural optical activity (NOA) can be possessed by crystals whose symmetry admits of linear terms in the expansion of the dielectric tensor  $\epsilon_{ij}(\omega, \mathbf{k})$  in powers of the wave vector  $\mathbf{k}$ .<sup>1,2</sup> The best known consequence of NOA is the rotation of the plane of polarization of a linearly polarized light wave when it propagates in an optically active medium. At the same time, there exist crystal classes  $C_{3v}$ ,  $C_{4v}$  and  $C_{6v}$ , which admit of NOA but do not have a rotating ability for any of the light-propagation directions.<sup>3</sup> In such crystals, just as in uniaxial inactive crystals, ordinary (transverse) and extraordinary (mixed) waves can be excited. However, as a result of the NOA the extraordinary waves in these crystals are elliptically polarized. The polarization ellipse lies in this case in the plane containing the hexagonal axis  $C_6$  and the wave vector  $\mathbf{k}$ . The feasibility in principle of the existence of such waves and their experimental manifestations was apparently first pointed out in Refs. 3 and 4. Recently experimental observation of NOA was reported in the crystals CdS (Ref. 5) and AgI (Ref. 6) (crystal

class  $C_{6v}$ ).

The present paper is devoted to a theoretical and experimental investigation of NOA in the region of exciton resonance of crystals of the wurtzite type (symmetry  $C_{6v}$ ). In the first part of the article we derive a system of material equations and obtain the energy spectrum of optical excitons in the region of exciton states  $B_{n=1}$  with account taken of the terms linear in  $\mathbf{k}$  in the exciton Hamiltonian (Secs. 1 and 2), we analyze the supplementary boundary conditions for a degenerate excitonic level, which are applicable for oblique incidence of light (Sec. 3) establish, by a limiting transition to the nonresonant region, the connection between the microscopic and phenomenological theories of NOA (Sec. 4), and present the results of a theoretical calculations of the reflection spectra with account taken of the NOA of crystals with wurtzite structure (Sec. 5). In the second part we describe the experimental procedure (Sec. 1), the results of the experimental investigation of the NOA in CdS crystals (Sec. 2), discuss the role of each of the parameters of the theory in the formation of the reflection