Excitation of Langmuir solitons by monoenergetic electron beams

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We describe experiments in which we applied a beam of monoenergetic electrons to produce Langmuir solitons in a magnetized collisionless plasma, in contrast to a previous work [Zh. Eksp. Teor. Fiz. **74**, 965 (1978); Sov. Phys. JETP **47**, 506 (1978)] where we used an electron beam with a spread of velocities. This change in the experimental method turned out to be of principal importance. As a result for the first time in experiments producing solitons satisfactorily reducible results were obtained. The picture obtained of the self-compression of large-amplitude Langmuir waves into solitons corresponds on the whole to the theoretical ideas about the aperiodic modulational or oscillating two-stream instability (OTSI). One of the manifestations of this instability is the experimentally observed periodic pattern of the solitons (or of their "bunches") with a distance λ_M between them equal to the wavelength λ of the initial Langmuir wave which is excited by the beam in the plasma: $\lambda_M \approx \lambda = u/f_p$ (u is the velocity of the beam electrons and f_p the electron Langmuir frequency of the plasma) or equal to one half of λ. In both cases the initial wave turns out to be the *soliton envelope* rather than the soliton of the envelope corresponding to a low oscillation energy. The most intense solitons are characterized by a very steep depression of the plasma density: the relative depth of the density wells $\delta n/n$ reaches 0.3 while the longitudinal size of the solitons does not exceed 5 to 6 electron Debye radii. In such a state (which may be called a one-dimensional induced quasicollapse of Langmuir waves) the soliton amplitude may be limited by dissipative processes [V. I. Petviashvili, Kurchatov Institute Thesis, 1978; Yu. S. Sigov and Yu. V. Khodyrov, Dokl. Akad. Nauk SSSR **229**, 833 (1976); Sov. Phys. Doklady **21**, 444 (1976); T. A. Gorbushina, L. M. Degtyarev, R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Preprint No. 17, Inst. Appl. Math., Acad. Sc. USSR (1976); A. Y. Wong and B. H. Quon, Phys. Rev. Lett. **34**, 1499 (1975)]. The observed instability threshold corresponds to the theoretical Eq. (3) for the OTSI: $W/nT \approx (Kr_D)^2$.

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INTRODUCTION

In contrast to an earlier work by us[1] we apply in the present work for the production of Langmuir solitons in a magnetized collisionless plasma a monoenergetic electron beam and this enabled us to be the first to obtain satisfactorily reproducible results. In the experiments of Ref. 1 the solitons—plasma density "wells" with Langmuir oscillations localized in them—were observed very clearly, but rarely. It was therefore necessary to answer the question: why is it only relatively rarely possible to observe Langmuir solitons—is it due to their physical features or due to the imperfection of the experimental conditions?

The present work was undertaken with the aim of obtaining first and foremost an answer to that problem. The change in the experimental conditions made in this work revealed the possibility to produce, reliably and reproducibly, Langmuir solitons by an electron beam, and led to the observation of a picture of forming solitons which are typical for the aperiodic modulational instability of large-amplitude Langmuir waves.

§1. STATEMENT OF THE PROBLEM AND EXPERIMENTAL SETUP

The analysis of the conditions of the experiments carried out in Ref. 1 on the beam excitation of Langmuir solitons led us to the following working hypothesis. In the conditions of the above-mentioned paper the electron beam and the plasma are produced at the same end of the experimental setup and they interact for rather a long time along the path to the working volume (where the solitons are produced)—see Fig. 1a. The beam therefore arrives at the beginning of the working volume

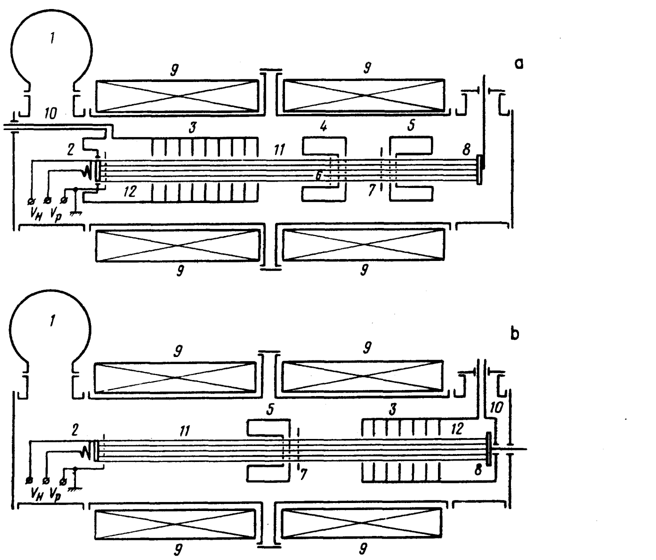

FIG. 1. Experimental setup: a) earlier variant,[1] b) new variant; 1—vacuum pump, 2—electron gun, 3—gas trapping line, 4—pumping oscillator, 5—diagnostic oscillator, 6—"modulating" grid, 7—rf probe, 8—beam collector, 9—magnetic field coils, 10—gas supply for the discharge chamber 12, 11—plasma filament.

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with a relatively large spread in velocities and excites Langmuir solitons appreciably more weakly than if it were mono-energetic. Indeed, the relaxation length l of a mono-energetic electron beam of density n_1 and velocity u in a plasma of density n can be estimated from the expression

$$l \approx 10u/\gamma, \quad \gamma \approx \omega_p (n_1/n)^{1/3}.$$

If $n_1/n \approx 10^{-2}$ (which is typical for our experiments), $u \approx 10^9$ cm/s, and $\omega_p = 2\pi \times 5 \times 10^8$ s^{-1} the relaxation length $l \approx 15$ cm. Over such a distance the beam electrons acquire an appreciable spread in velocities: $\Delta u/u \approx (n_1/n)^{1/3} \approx 0.2$. At the same time the distance from the electron gun to the working volume was in the experiments of Ref. 1 ~ 100 cm $\gg l$. It is thus necessary for the buildup of Langmuir waves of a sufficiently large amplitude (larger than the threshold for the formation of solitons) to increase the beam current so much that in the plasma (as the result of electron-ion beam instabilities) strong "inherent" density fluctuations occur and it turned out to be extremely difficult to distinguish the very thin soliton wells against their background.

Starting from these considerations, we changed the experimental conditions as follows. The discharge chamber in which the plasma was produced (by a beam, as before) was shifted to the opposite end of the apparatus with respect to the gun: Fig. 1b. For such a geometry the plasma spreads towards the beam and the electrons in the beam entering the plasma, when it reaches the working volume, can be considered to be monoenergetic. The second change in the experimental conditions was the increase of the beam diameter and of the plasma to 6 cm—through lowering the magnetic field in the working volume by a factor 3 and keeping the field in the region of the electron gun and the cathode diameter the same. The increase in the diameter of the plasma column was made in order to make the soliton which is produced (with longitudinal size Δ) to a large degree one-dimensional. In the geometry of the experiment of our earlier work[1] $\Delta \lesssim 1$ cm, the plasma radius $a = 1.5$ to 2 cm, and the ratio $\Delta/a \approx 0.5$ and the soliton is possibly "insufficiently one-dimensional." We note that in itself this second change without the shift of the discharge chamber would not lead to anything important.

In the new experimental geometry, in connection with the increase in the cross-sectional areas of the beam, it is necessary for an effective pumping of Langmuir waves with a sufficiently large amplitude to increase considerably the beam current strength: $I = 60$ to 80 mA for an electron energy $W_1 = 300$ to 1000 eV. The magnetic field strength usually was 500 to 1000 Oe. The electron oscillations in such a field are magnetized (as in Ref. 1) at a plasma density $n = 3 \times 10^9$ cm^{-3}: the electron Larmor frequency f_H is appreciably larger than their Langmuir frequency f_p.

The indicated changes led to a radical result: under the new conditions Langmuir solitons are very reliably and reproducibly produced. Thanks to this a few details of the apparatus were no longer necessary—under the new conditions the pumping oscillator and the modulating grids are not present. The diagnostics of the solitons remained as before: the plasma density wells and the density n itself are measured by a diagnostic oscillator and the Langmuir oscillations by an rf probe of the same construction as before (Fig. 1b, see also Ref. 1). In the experiments described we applied a single beam with a pulse length of ~500 μs. The beam produces a hydrogen plasma and after it has passed from the discharge chamber to the working volume (after 100 to 150 μs) it builds up Langmuir waves in it. The electron temperature T_e, measured by a probe, is 10 eV.

The new experimental conditions enabled us to observe new effects, not found in Ref. 1. Firstly, a rather distinct (energy) threshold appeared for the observation of Langmuir solitons and clearly decreased when the energy of the beam electrons increased. Secondly, we observed that the spatial structure of the observed modulational instability is characterized by a well-defined period (length) λ_M of the modulation of the non-linear Langmuir waves and this period is more or less uniquely connected with the wavelength λ of the initial (linear) Langmuir wave which is excited in the plasma by the beam. Indeed: either $\lambda_M = \lambda$ or $\lambda_M \approx \frac{1}{2}\lambda$. Such a picture agrees with the theory typical for the aperiodic variant of the modulational instability[3-9] also called the oscillating two stream instability (OTSI).[7,1] Thirdly, we were able to observe very strong solitons for which the plasma density depression $\delta n/n$ reaches 0.3 while the longitudinal size Δ is not more than five to six electron Debye radii, i.e., very close to the smallest possible value. The relation between the size Δ and the magnitude $\delta n/n$ of the density depression was traced in much wider limits than in Ref. 1.

§2. EXPERIMENTAL DATA

In experiments with a plasma moving towards a beam the propagation velocity of the plasma was measured first of all. To do this the oscillograms of the signal from the loop of the diagnostic oscillator adjustable to the controlled plasma density were plotted for various displacements Δz of the oscillator with respect to the discharge chamber in the direction of the electron gun (we take $\Delta z = 0$ to be the point at a distance of 25 cm from the discharge chamber in the direction of the electron gun). Using these oscillograms we constructed a family of curves $n(t)$ for different Δz (Fig. 2). From these curves we determined the velocity of motion of the plasma: for the set a of curves of Fig. 2 this velocity was $v = 0.35 \times 10^6$ cm/s and for the set b of curves it equals $v = 0.45 \times 10^6$ cm/s. In accordance with these data we take for further estimates $v = 0.4 \times 10^6$ cm/s, although it is clear from Fig. 2 that the propagation speed of the plasma depends on the experimental conditions and, in particular, on the time.[1)] One should note that the measured magnitude of v is appreciably less than the flow velocity of the plasma obtained in Ref. 1 where it corresponded to the ion-sound speed. This is apparently caused by the smaller plasma density gradients under the new experimental conditions: the discharge chamber has a smaller internal diameter D and a larger aperture diameter d—in the case of Fig. 1a $D = 14$ cm, $d = 4.4$ cm and in the case of Fig. 1b $D = 10$ cm, $d = 7$ cm.

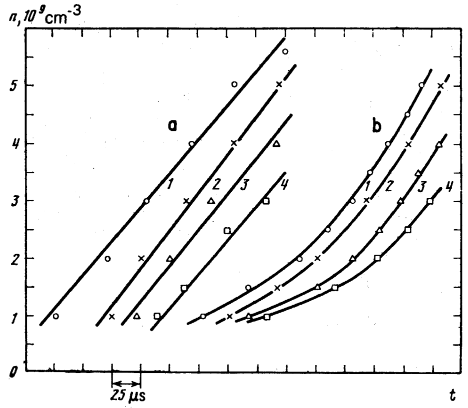

FIG. 2. Time-dependence of the plasma density in the diagnostic oscillator space in two different regimes for obtaining plasma, differing in the electron energy, current, and length of the beam pulse. The curves 2 to 4 differ from the curve 1 by the value of the shift Δz of the oscillator in the direction of the plasma propagation reckoned from some initial position corresponding to the curve 1: curves 1—$\Delta z = 0$, curves 2—$\Delta z = 10$ cm, curves 3—$\Delta z = 20$ cm, curves 4—$\Delta z = 30$ cm. From the shifts of the curves we determine the average velocity of the motion of the plasma: a) $v = 0.35$ cm/μs, b) $v = 0.45$ cm/μs.

Knowing the velocity v we can easily transform the time-scale of the change in the plasma density into a spatial scale. The signal from the loop of the diagnostic oscillator can thus be called either the time or the space profile of the plasma density. For a comparison of this profile with the oscillogram of the signal from the rf probe—the indicator of the Langmuir wave electric field bunches—it is necessary to take the following fact into account. The diagnostic oscillator is displaced relative to the rf probe in the direction of propagation of the plasma by 2.5 cm (Fig. 1b). If a soliton therefore is, for instance, stationary in the plasma it will be registered by the oscillator (as a density well) later than by the rf probe (as a field bunch). For this reason and also due to a certain inertia of the applied receivers of rf oscillations P5-19 and P5-20 (~1 μs) the observed delay of the well relative to the field bunch is (on average) 5 tp 6 μs (this delay will be shown in detail in what follows).

We recall that as in Ref. 1 the plasma in our experiments was collisionless: the (hydrogen) gas pressure was 4 to 6×10^{-6} mm Hg and the plasma density $n = 1$ to 4×10^9 cm^{-3}.

Before describing the results obtained we dwell on a few diagnostic problems. We note first of all that in the series of experiments considered the rectified signal from the rf probe which serves as an indicator for the field strength of the Langmuir waves was measured in two independent ways: either, as before,[1] by selective P5-19 and P5-20 receivers selecting a fixed frequency in the 250–1000 MHz band (with a band width of 0.8 MHz), or by a "wide-range" amplifying device. In the latter case the measured signal was given to a detector working in the 50–10000 MHz frequency band and then to a wide-band UZ-7A amplifier. Such a signal, which includes in it rf oscillations at practically all frequencies (and not just at one chosen one), will be called an integral signal.

We recall further that we apply two regimes for registrating the plasma density by means of a diagnostic oscillator.[1] In one of them the frequency of the rf generator supplying the oscillator is chosen somewhat higher than the maximum value of the eigenfrequency of the oscillator (corresponding to the moment of maximum density in the oscillator). In that regime the rectified signal from the loop of the oscillator duplicates in shape the behavior of the plasma density: it is a maximum at the moment of maximum density. In the second regime the frequency of the generator which supplies the oscillator is less than the above-mentioned maximum value of the eigenfrequency of the oscillator but larger than the "vacuum" value of that frequency (when there is no plasma). The maximum of the plasma density then lies between the pulses of the leading and trailing edges of the plasma column, i.e., at the place of the minimum of the oscillator signal. It is therefore necessary in the analysis of the oscillograms given below to bear in mind that the plasma density well corresponds either to a decrease in the signal from the oscillator loop—if that frequency (averaged over the fast deviation) does not pass through the smooth maximum, or to an increase in the oscillator signal—if the well is measured during the indicated maximum (for details see Ref. 1).

We now elucidate the method for an absolute measurement of the depth of the plasma density well $\delta n/n$. The density well causes a change in the signal of the diagnostic oscillator. When this signal is changed we find by using the resonance curve of the oscillator measured earlier the change in its eigenfrequency which is connected with the density well. Moreover, from the calibrated graph giving the dependence of the eigenfrequency of the resonator on the plasma density in its gap we determine the absolute change δn in the density which is connected with the well. By using the same calibrated graph giving the value of the frequency of the generator which supplies the oscillator we determine the absolute value of the plasma density n. We give below the well depth $\delta n/n$ measured in this way.

Finally, we note that in all photographs given below the following quantities are depicted: in the lower oscillogram (trace deflection upwards) we have the signal of the plasma density indicator (rectified signal from the loop of the diagnostic oscillator) and in the upper oscillogram (trace deflection downward) we have the signal of the indicator of the Langmuir wave electric field (rectified signal from the rf probe).

After these remarks we turn to a description of the basic experimental data.

We give in Figs. 3 to 6 typical oscillograms of the indicator of the temporal behavior of the plasma density and of the envelope of the amplitude of the electric field of the Langmuir waves measured by both the selective and the integral methods. The following facts call for attention:

1) The registered oscillations are, indeed, Langmuir oscillations: their frequency f is determined by the relation $f \approx 10^4 n^{1/2}$ (see the lengths of the figures).

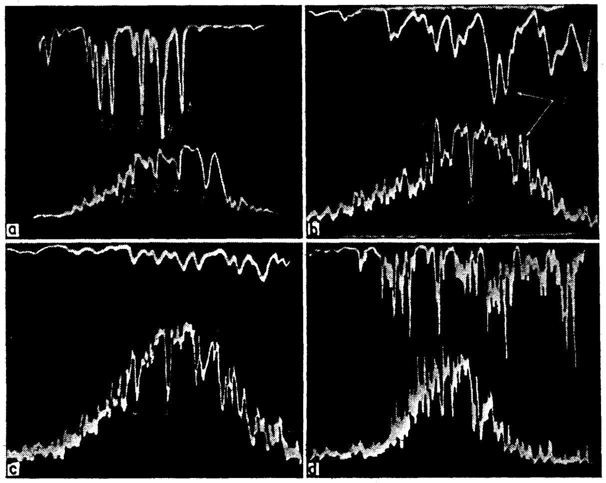

FIG. 3. Oscillograms of the signals from the rf probe (upper curve, trace deflection downwards) and the diagnostic oscillator (lower curve, trace deflection upwards). Time base (in μs/division): a—10, b—5, c—5, d—25 (here and everywhere we have in view the large division consisting of five small ones). The plasma density n (here and henceforth at the maximum of the oscillator signal) is equal to: a—1.8×10^9 cm^{-3}, b, c—3 to 3.5×10^9 cm^{-3}, d—2.2×10^9 cm^{-3}; the observed frequency f of the oscillations: a—350 MHz, b to d—500 MHz; the beam electron energy W_1: a, d—900 eV; b, c—300 eV. Sharply evident are in Figs. 3b, c very narrow density wells of a length $\tau < 1$ μs. The pulse[1] in Fig. 3c and the pulse 3 in Fig. 3b have a length of ~1 μs at the base; a relative depth of the density well of $\delta n/n = 0.1$ corresponds to these pulses. The pulses 2 and 3 in Fig. 3c and the pulses 1 in Fig. 3b are batches of very narrow wells, shifting with the plasma bunches.

2) The Langmuir waves are grouped in bunches.

3) The set of rf electric field bunches coincide approximately (Figs. 3; 5c; 6b) or exactly (Figs. 4, 5a, b) with the set of plasma density wells.

4) If we assume that the field bunches and wells move with the plasma and take into account the above mentioned delay in the registration of the wells relative to the registration of the bunches (~5 to 6 μs) we can see that in the conditions of the experiments described the

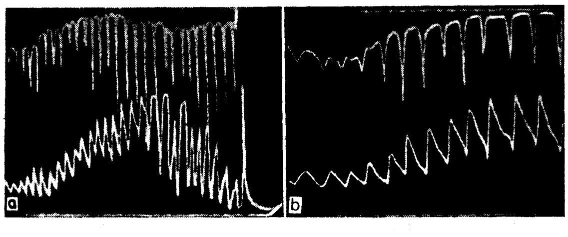

FIG. 4. Oscillograms of the integral signal of the rf probe (upper curve) and of the signal from the diagnostic oscillator. Time base (in μs/division): a—25, b—10. Plasma density $n = 1\times10^9$ cm^{-3}, W_1 = 900 eV. The maximum intensity in the spectrum of the oscillations occurs at a frequency of 300 to 350 MHz. In the conditions of Fig. 4a the relative density depression in the deepest wells is $\delta n/n = 0.3$. The cutoff of the signals at the end of the oscillograms is connected with the switching-off of the beam.

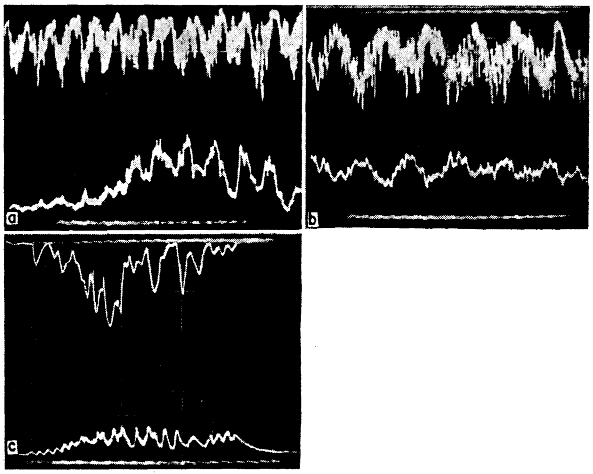

FIG. 5. Examples of the modulation of the integral signal of the rf probe (upper trace) and of the signal from the diagnostic oscillator (lower trace). Time base (in μs/division): a—10, b, c—5. Beam electron energy W_1 = 900 to 1000 eV; plasma density: a—4×10^9 cm^{-3}, b—4.5 $\times$ cm^{-3}, c—2.5×10^9 cm^{-3}.

field bunches are usually localized in the plasma density wells, i.e., they are Langmuir solitons.

To clarify this conclusion we consider in more detail some of the figures. If we move in Fig. 3a the lower ("delayed") oscillogram by approximately 6 μs to the left, the rf field pulses indicated by the numbers 1 to 5 practically coincide in time with the corresponding density wells (see footnote 1). In the case of Fig. 3b the coincidence of the prominent "two-hump" bunch 1 with a well of the same shape occurs for a shift to the left of the lower oscillogram by 4 to 5 μs. The coincidence of the field bunches with density wells for a shift of the lower oscillogram by 5 to 6 μs to the left is particularly clearly demonstrated by Fig. 4b. The same phenomenon is shown in Fig. 4a, registered under the same conditions as in Fig. 4b but for a slower time base (25 μs per division). Figure 4 shows the localization of all field bunches without exception in the corresponding density wells. Finally, the indicated coincidence of the wave bunches and the density wells for the same shift of 5 to 6 μs is also clearly evident from Figs. 5a, b.

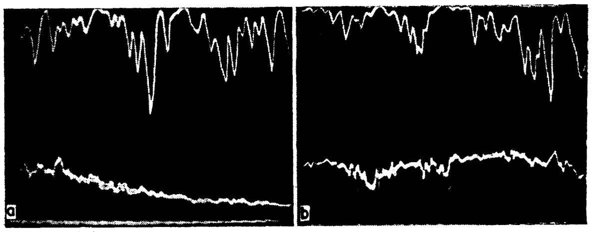

FIG. 6. Effect of the beam electron energy on the depth of the plasma density modulation: a) W_1 = 400 eV, beam current strength I = 60 mA; b) W_1 = 800 eV, I = 70 mA. The observed frequency of the oscillations is f = 500 MHz. Plasma density: a—2.3×10^9 cm^{-3}, b—2.5×10^9 cm^{-3}. Time base—5 μs/division. The depth of the wells 1 and 2 are respectively equal to $\delta n/n = 5\times10^{-2}$ and 4×10^{-2}.

The most reliable interpretation of the experimental data considered thus seems to us to be the following one: the wave formations studied are Langmuir solitons moving with the plasma (they are at rest in the frame of the plasma).

5) Under well-defined conditions (in particular, such as in Figs. 4 and 5) the integral signal of the indicator of the field of the Langmuir waves is modulated in time in the same way as the signal at a selected frequency (in the case of Fig. 4 at the frequency $f \approx f_p = 300$ MHz). This means that waves at all frequencies of the rf oscillation spectrum are localized in the density wells and are practically absent outside the wells. Under these conditions the modulational instability of the Langmuir waves manifests itself particularly clearly.

It is interesting to note that under different experimental conditions two different pictures of the wave modulation are observed: in the one case there occurs one (very narrow) soliton on each wavelength of the modulation—Fig. 4, in the other case a whole "stack" of such solitons is fitted into a wavelength of the modulation—Fig. 5 and the stacks are found not only for the wave bunches but also for the density wells. As we noted already earlier, the sharp modulation of the soliton amplitude observed in Figs. 4 and 5 which has a clearly expressed period is the distinguishing result of the present work.

6) The period τ_M of the modulation of the amplitude of the Langmuir wave bunches and the corresponding modulation wavelength $\lambda_M = v\tau_M$ (where $v = 0.4 \times 10^6$ cm/s is the velocity of the plasma) increases with increasing velocity of the electron beam.

This behavior is shown in Fig. 7; the data there are the result of an averaging over many beam pulses—for each value of the electron density W_1. We give in the same figure the W_1-dependence of the wavelength λ of the initial linear ("Cerenkov") Langmuir wave which is excited in the plasma by the beam:

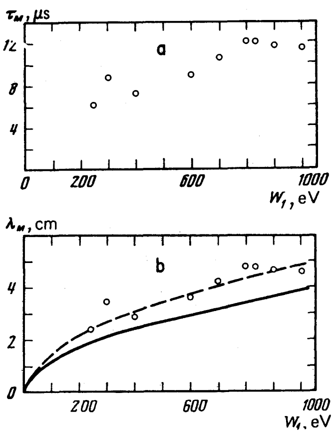

FIG. 7. Dependence on the energy of the electrons in the "pump beam" of: a—the period of the modulation of the electric field of the nonlinear Langmuir waves, b—the modulation length $\lambda_M = v\tau_M$, where $v = 0.4 \times 10^6$ cm/s dashed curve—experimental points; solid curve—wavelength of the linear Langmuir wave, $\lambda = u/f_p$, excited by the beam, as function of the beam electron energy $W_1 = \frac{1}{2}mu^2$.

$$\lambda = u/f_p, \qquad (1)$$

where u is the electron beam velocity (u [in cm/s] $= 6 \times 10^7$ (W_1 [in eV])$^{1/2}$). It is clear that $\lambda_M \approx \lambda$ under the conditions of Fig. 7, but not always. For instance, in the conditions of Figs. 4 and 5a, b we observe a different behavior: $\lambda_M \approx \frac{1}{2}\lambda$, but in the conditions of Figs. 5c and b we have again $\lambda_M \approx \lambda$. We note that a result close in sense to what we show in the present subsection was obtained also in numerical experiments[8,10] and in a theoretical paper.[11b]

7) In those cases when the field bunches can be compared with well-defined density wells it turns out that more intense bunches correspond to deeper and narrower wells—see, for instance, Figs. 3 and 4.

8) The plasma density depression (well depth) frequently has the magnitude $\delta n/n = 0.1$ to 0.3. Examples of deep wells are given in Fig. 4a (where $\delta n/n = 0.3$) and Figs. 3b, c (where $\delta n/n = 0.1$). We must note that if $\delta n/n \lesssim 1$ to 2×10^{-2} the well in our experiments is not registered above the background of the inherent fluctuations of the plasma density.

The longitudinal size (width) of the density wells, i.e., the length of the well pulse multiplied by the velocity of motion of the plasma ($v = 0.4 \times 10^6$ cm/s) is usually of the order of tenths of a centimeter. Examples of narrow wells are given in Fig. 3b (pulses 3, 1) and Fig. 3c (pulses 1 to 4). Figure 4 demonstrates one of the examples of more intensive (and narrower) bunches of the rf electric field. In the conditions of the indicated figures the width of the density wells (or the narrower field bunches) does not exceed 0.3 to 0.4 cm, i.e., 7 to 8 electron Debye radii r_D in the case of Figs. 3 and 5 to 6 radii r_D in the case of Fig. 4. It is then important to bear in mind that the resolving power of the applied apparatus for the registration of density wells is limited by the width of the gap in the diagnostic oscillator (2.5 mm for a mesh width of the grid of 2 mm) while for the registration of the rf field bunches it is limited by the resolution of the P5-19 and P5-20 receivers (about 1 μs which for a flow velocity of the plasma of $v = 0.4 \times 10^6$ cm/s is ~0.4 cm when converted to a spatial scale). This means that the actual width of the solitons may be even less than the above-mentioned values.

9) In some (rather common) experimental conditions there is no such detailed correlation as is shown in the figures considered above between the rf field bunches and the plasma density wells although the general nature of the field oscillations and that of the density are qualitatively approximately identical—see, for instance, Fig. 3d. Cases of this kind are apparently examples of developed soliton turbulence[8,11-14] or sometimes (e.g., as in the case of Fig. 3c) are due to an insufficiently precise choice of the frequency of the receiver of the oscillations.

One can presumably explain the latter as follows. The transverse size of the diagnostic oscillator is larger than the size of the plasma and the oscillator thus measures the density well averaged over the cross section of the plasma filament. Separate sections of the filament may possibly, due to its radial inhomogeneity, oscillate at the lower frequency but not completely synchronously. At the same time the width which is in the indicated sense in step with the frequency band of the oscillations in the soliton is as is shown below around 15 MHz (there are possibly several of such bands corresponding to different sections of the plasma filament cross section). The degree of the observed agreement of the (averaged) density wells and rf field bunches depends apparently appreciably on the tuning of the receiver measuring the rf probe signal.

10) The deviations of the plasma density from the average value have the same shape as the wells described above and also as the plasma bunches. The pulse 2 in Fig. 3b and the group of pulses in Fig. 3d are examples of such bunches. The value of $\delta n/n$ in the plasma bunches can be, for instance, 0.1 to 0.3 as in the wells.

11) The modulational instability of the Langmuir waves described here—with sharp field bunches and the density wells corresponding to them which are clearly distinguished above the background of the eigen-noise of the plasma—is observed starting from some threshold in the energy of the plasma oscillations. This threshold has a clear tendency to decrease when the beam electron energy W_1 increases.

We can see an example of this behavior in Figs. 6a, b where the envelopes of the amplitudes of the Langmuir oscillations are compared with the density wells for different W_1. Figure 6a refers to an energy of the beam electrons $W_1 = 400$ eV. In that case one can already observe the rf field bunches but the wells connected with them cannot yet be seen: they are not larger than the level of the plasma eigen-noise. Figure 6b refers to an energy of the beam electrons $W_1 = 800$ eV and the density wells 1 and 2 which agree with rf field bunches are already clearly observed; it is clear that when we want to "synchronize" the wells with the wave bunches of the lower oscillogram of Fig. 6b we must shift them, as before, to the left by approximately 5 μs.

The frequency spectrum of the oscillations in the Langmuir soliton is clearly rather complicated because the plasma density is not constant over the cross section of the plasma filament. In the present work we restricted the measurement of the width to that spectral band within which the low-frequency oscillations of the envelopes of the amplitudes of the rf waves can be assumed to be approximately synchronous. In those measurements the signal of the rf probe was simultaneously measured by two receivers (P5-19, P5-20) which are tuned to different frequencies and connected in parallel. These measurements show that if at the observed frequency (which is closest to the Langmuir frequency) $f = 500$ MHz[2)] the difference Δf of the tuning frequencies of the receivers is less than 5 MHz, the oscillograms of the (rectified) signals from their outputs are practically the same—Fig. 8a (a certain difference in the shape of the pulses is caused by the fact that the resolving times of the receivers are not identical). When $\Delta f = 15$ MHz the pulses from the two receivers are the same in about one half of the cases (Fig. 8b), and for $\Delta f = 25$ MHz the agreement of the pulses turns out to be relatively rare—Fig. 8c. To complete the picture we note also that we sometimes observe cases where the oscillograms turn out to be very similar for $\Delta f = 50$ MHz. This kind of measurement enabled us to conclude that the width of the frequency spectrum of the non-linear Langmuir waves which are approximately synchronous with the lower frequency is under conditions when solitons are formed: $\Delta f_{\text{nonlin}} \approx 15$ MHz for a basic frequency $f \approx f_p = 500$ MHz.

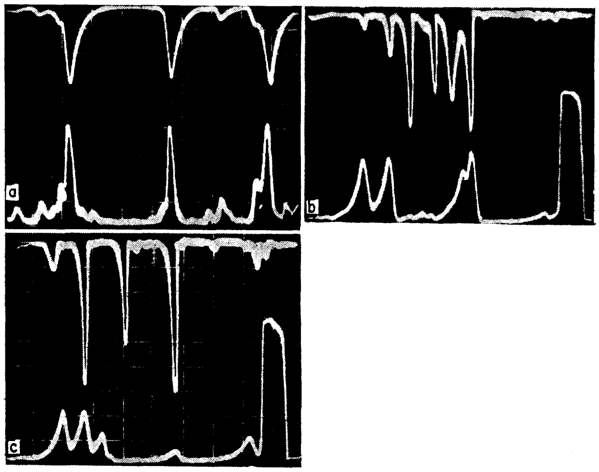

FIG. 8. Oscillograms of the rf pulses from the same probe at two frequencies close to the electron Langmuir frequency: $f_1 = 500$ MHz and $f_2 = f_1 + \Delta f$ for three values of the frequency shift Δf: a—5 MHz, b—15 MHz, c—25 MHz. The time base is 5 μs/division. The pulse at the end of the oscillograms is connected with the switching-off of the beam.

§3. DISCUSSION OF THE EXPERIMENTAL DATA

We carry out a comparison of the observed properties of the Langmuir solitons and the theoretical predictions in the following directions: 1) the connection of the longitudinal size (width) Δ of the soliton and the density well depth, 2) the connection of the characteristic scale of the modulational instability of the Langmuir waves (modulational wavelength λ_M) and the beam and plasma parameters, 3) the threshold of the observed modulational instability and its dependence on the energy of the beam electrons, 4) the frequency spectrum of the oscillations in a Langmuir soliton. We consider now the experimental data given above in the order indicated.

1) For a comparison of the depth and the width of the soliton we turn first of all to Figs. 3b, c. In the conditions of Fig. 3b (pulse 3) the soliton has a density well depth $\delta n/n = 0.1$ and the width (at a level of $1/e$ of the maximum) $\Delta = 0.25\ \text{cm} \approx 6 r_D$ where r_D is the electron Debye radices. In the conditions of Fig. 3c (pulse 1) the well depth $\delta n/n$ is also equal to 0.1 and the soliton width $\Delta = 0.35\ \text{cm} \approx 8.5 r_D$. Hence, the average value of the soliton width is experimentally $\Delta_{\text{exp}} = 7.5\ r_D$ for $\delta n/n = 0.1$. On the other hand, according to theory the soliton width is uniquely determined by the density well depth[1]:

$$\Delta_{\text{theor}} = r_D (24n/\delta n)^{1/2} = r_D (24nT/W)^{1/2} \qquad (2)$$

(nT is the thermal energy density of the plasma, $W = E_0^2/16\pi$ is the electric energy density of the Langmuir wave, and E_0 is the wave amplitude). According to this relation the soliton width in the cases considered of Figs. 3b, c must be $\Delta_{\text{theor}} \approx 15\, r_D$. Hence $\Delta_{\text{exp}}/\Delta_{\text{theor}} \approx \frac{1}{2}$. Such a ratio of the experimental to the theoretical values of the longitudinal size of the soliton must for the present level of the understanding of soliton physics apparently be considered rather to be a fair agreement than an appreciable discrepancy. The theoretically expected proportionality between the well depth and the field energy density also agrees qualitatively with experiments as can be seen, in particular, from Fig. 4a. In the conditions of Fig. 4a the relative depth of the deepest wells $\delta n/n = 0.3$ is three times larger and the plasma density $n = 1 \times 10^9$ cm^{-3}—three times smaller than in the conditions of Figs. 3b, c. At the same time the width of the most intensive solitons is about the same when expressed in cm, but in Debye radii smaller by a factor of about $\sqrt{3}$. This also agrees with Eq. (2). In absolute magnitude in the conditions of Fig. 4a the magnitude of $\Delta = 5$ to $6\ r_D$. For such a small soliton width its amplitude must according to the theory[12-14] be limited by dissipative processes.

2) According to the experimental result considered in §2 and, in particular, the one given in Fig. 7b the wavelength of the modulation of the nonlinear Langmuir waves $\lambda_M \approx \lambda$ or $\lambda_M \approx \frac{1}{2}\lambda$, where $\lambda = u/f_p$ [Eq. (1)]. This means that the initial (linear) Langmuir wave turns out to be the envelope of the field modulation with a shorter wavelength, which arises as the result of the modulational instability. Such a picture of the nonlinear waves also corresponds to the theory: it is characteristic for the theoretically predicted aperiodic instability (OTSI).[3-8,1] This variant of the modulational instability must develop when the energy density of the oscillations is sufficiently large:

$$W/nT \approx \delta n/n \approx (kr_D)^2, \tag{3}$$

where k is the wavenumber of the oscillations which are modulated by the initial Langmuir wave which has a wavelength λ (i.e., $k > 2\pi/\lambda$). If the energy of the oscillations is less than the value indicated there may occur another variant of the modulational instability for which the wavelength of the modulations of the oscillations is larger than the wavelength of the initial (linear) Langmuir wave. The solitons which then occurs are called envelope solitons.[5,9,1] In contrast to this case the situation corresponding to Figs. 4 to 6 can be characterized by the term "envelope of the solitons."

We have shown above that the observed Langmuir solitons are to a first approximation not moving relative to the plasma, i.e., in the frame of the moving plasma they are bunches of standing nonlinear waves. The picture of standing waves turns out to be the simplest one in that case (Fig. 4), when the spatial period of the wave modulation λ_M is equal to half the wavelength λ of the initial wave, i.e., is the same as the spatial intensity period $\frac{1}{2}\lambda$ of a wave. In that case the wave forms in the plasma a regular sequence of density wells with a period $\frac{1}{2}\lambda = u/2f_p$ and is localized in these wells. In particular Fig. 4 shows this self-compression of nonlinear Langmuir waves; from this figure it is clear that there is practically no rf electric field in the intervals between the narrow rf electric field wave bunches which are formed.

A more complicated picture is demonstrated in Figs. 5a, b: the nonlinear wave on a section of length $\frac{1}{2}\lambda$ is split up into several solitons of size $\Delta \approx 7$ to $8\ r_D$. We note that the process of the formation of a standing nonlinear wave, in particular with half the wavelength of the initial pump wave, was observed experimentally[15] (in conditions of a strongly collisional unmagnetized plasma) and in numerical experiments.[8,10] In Ref. 8 (see also Ref. 16) the picture of the subsequent turbulence of the standing wave was (numerically) calculated. Nonetheless the experimental data given here still require apparently a theoretical interpretation.

According to theory[12] nonlinear Langmuir waves which are strongly modulated in amplitude must be stable. This conclusion of the theory was not verified in the present work and one can only say that the available experimental data do not yet contradict it.

3) The threshold (critical) Langmuir wave energy density starting from which the aperiodic modulational instability occurs is according to theory determined by the relation[3-9,1]

$$(W/nT)_{\text{cr}} \approx (kr_D)^2. \tag{4}$$

Since, as was already indicated

$$k^2 > \left(\frac{2\pi}{\lambda}\right)^2 = \frac{4\pi^2 f_p^2}{u^2} = \frac{2\pi n e^2}{W_1},$$

$$r_D^2 = T_e/4\pi n e^2,$$

we have $(kr_D)^2 > T_e/2W_1$ and the threshold is thus

$$(W/nT)_{\text{cr}} \gtrsim T_e/2W_1. \tag{4$'$}$$

For instance, for $W_1 = 800$ eV and $T_e = 10$ eV the instability threshold is

$$(W/nT)_{\text{cr}} \gtrsim 0.6 \cdot 10^{-2}. \tag{4$''$}$$

Relation (4′) shows that the threshold for the aperiodic modulational instability to a large extent is determined by the energy of the electrons in the beam which excites the wave. The threshold can be high when the electron energy is relatively small. For instance, for $W_1 = 150$ eV and $T_e = 10$ eV (conditions of Ref. 1) we need for the occurrence of the modulational instability variant considered here an initial wave energy density $W/nT \gtrsim 3 \times 10^{-2}$ and, hence, in the wave bunches which are formed $W/nT > 0.1$. Such intense solitons were not observed in Ref. 1. In the conditions of Ref. 1 the wave energy was thus insufficient for the occurrence of the instability variant considered here.

In agreement with this, Fig. 6a shows that for a relatively small beam electron energy ($W_1 = 400$ eV) we do not see those density wells in which the observe Langmuir wave bunches are localized. This means that the modulational instability is comparatively close to the threshold for its occurrence and therefore does not form sufficiently deep wells which are clearly distinguishable above the background of the intrinsic noise of the plasma. Increasing the beam electron energy to 800 MeV must according to Eq. (4′) considerably lower the instability threshold and it is shown in Fig. 6b that clearly distinguishable wells are then, indeed, formed in which the field bunches which form them are localized. The depth of the wells in the case of Fig. 6b is 4 to 5×10^{-2}, i.e., it is larger than the threshold (4″) by about an order of magnitude. This enables us to assume that in that case the instability threshold was exceeded and the wells were substantially deepened. Solitons were then formed the width of which we can estimate from Fig. 6b to be approximately 0.8 cm $\approx 20\ r_D$. According to theory (see, e.g., Ref. 1) the characteristic wave number of these solitons is $k_0\approx 2/\Delta\approx 1/10\,r_D$, i.e., $(k_0r_D)^2\approx 10^{-2}$ and the field energy density and the relative well depth must now be $W/nT\approx\delta n/n=6(k_0r_D)^2\approx 6\times 10^{-2}$ whereas experimentally (see the legend of Fig. 6b) $\delta n/n\approx 4$ to 5×10^{-2}. This comparison shows that good agreement between the theoretical and experimental threshold values is observed.

Finally, we note yet one more fact pertaining to the threshold of the instability considered. It is clear from a comparison of the conditions of the experiments corresponding to Figs. 4 to 6 that the instability with a spatial period $\lambda_M=\frac{1}{2}\lambda$ developes for an appreciably larger wave energy than for the case $\lambda_M=\lambda$ (where $\lambda=u/f_p$). As the characteristic wavenumber of the modulated waves is larger in the first case than in the second (see the legends of the figures) this fact also corresponds to the theoretical Eq. (4).

The qualitative agreement between experiment and theory enables us to use the theoretical relation

$$\frac{E_0^2}{16\pi}\approx\frac{\delta n}{n}nT_e$$

to estimate the electric field strength in the observed Langmuir solitons. For $\delta n/n=3\times 10^{-1}$ and $n=1\times 10^9$ cm^{-3} (conditions of Fig. 4) or for $\delta n/n=1\times 10^{-1}$ and $n=3\times 10^9$ cm^{-3} (conditions of Fig. 3) and $T_e=10$ eV we have $E_0=150$ V/cm.

4) The frequency spectrum of the Langmuir waves, which are approximately synchronous with the lower frequency and observed when there is a modulational instability, turns out to be rather narrow. For instance, it follows from Fig. 8 that the characteristic band width of that spectrum $\Delta f_{\text{nonlin}}\approx 10$–15 MHz $\approx f_i$, where $f_i=f_p(m/M)^{1/2}\approx 10$ MHz is the Langmuir frequency of the molecular hydrogen ions for a plasma density $n=3\times 10^9$ cm^{-3}, typical for our experiments, and M is the ion mass. Such a width of the "synchronous" band considered of the frequency spectrum of the electron oscillations in the Langmuir soliton corresponds approximately to the theory developed in Ref. 12. We must bear in mind that, as we have already indicated above, the complete spectrum of the oscillations in the soliton, of course, must, due to the radial inhomogeneity of the plasma density, be wider and must apparently include at least several bands of the indicated width Δf_{nonlin}. This problem still requires a further experimental investigation.

We can thus in concluding this section note that to a first approximation there are not yet large discrepancies, either qualitatively, or at the level of quantitative estimates, between the experimental data obtained in the present work and theory.

As regards the problem of the temporal (spatial) evolution of the Langmuir solitons, in particular their stability, it was not studied in the present work and our information is so far restricted to the results of Ref. 1. We do also not yet know the transverse structure of the solitons. These problems must be the subject of subsequent experiments.

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[1)] It is important to take into account the fact that the velocity of motion of the plasma is strictly speaking not constant even within one oscillogram.

[2)] In the given set of measurements this frequency corresponds to the maximum of the amplitude of the electron oscillations.

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