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Translated by R. T. Beyer

The effect of zero points in the modulation of light in a Fabry-Perot interferometer

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(Submitted 27 September 1978)
Zh. Eksp. Teor. Fiz. 76, 925–929 (March 1979)

The modulation of light by a homogeneous electric field in a Fabry-Perot interferometer filled with a nondispersive dielectric is considered and some features of the modulation are explained. A "zero point" effect is predicted which consists of the disappearance of the modulation of a light wave passing through the interferometer. The effect should occur for a certain number of frequencies p of the modulating field (irrespective of its amplitude) and for certain frequencies ω of the incident light wave, the derivatives of the light-wave modulation parameter with respect to the frequency p or ω being discontinuous at the zero points. It is pointed out that the effect can be used to measure the absolute optical length of the interferometer with an accuracy several orders of magnitude better than the light wavelength (and in some cases better than the thickness of the atomic layer of the substance). It can also be used to measure precisely the refractive-index deviations induced in matter by various external factors (for example, a light field, pressure, magnetic field, etc.).

PACS numbers: 07.60.Ly, 42.10.Jd

If a light wave passes through matter located in a strong, homogeneous electric field that modulates its index of refraction, in the general case the frequency and amplitude of the wave are modulated. Such a modulation was produced by Kaminow[1] and is widely used in condensed dielectrics[2,3] and gases.[4-8] The corresponding theory was proposed both for the case of condensed dielectrics[1-3,9] and for gases.[9-11]

The modulation of light in a Fabry-Perot interferometer filled with a condensed dielectric (located in an inhomogeneous electric field) was considered in Ref. 12. The consideration was limited to the case of spatial synchronism of the modulating microwave frequency with all the frequency components of the modulated light field. According to Gordon and Rigden,[12] because of this synchronism the modulation parameter of the

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light wave passing through the interferometer has a value larger by far than the value reached in the medium without the interferometer at the same length.

In the present study we consider the modulation of light in a Fabry-Perot interferometer filled with a nondispersive (nonabsorbing) dielectric placed in a homogeneous (time-varying) electric field. The time variation of the electric field is assumed to be rapid, that is, it is assumed that the condition

$$p \gtrsim \mu_0 \tag{1}$$

holds, where p is the frequency of the modulating field and μ_0 is the frequency halfwidth of the axial modes of the interferometer in the absence of modulation (the inequality (1) means that the modulation period is less than the lifetime of a photon in the interferometer).

The results obtained below show that the nature of the modulation in a homogeneous modulating field is qualitatively different from that described in Ref. 12. We point out the "zero point" effect, that is, the fact that the amplitude and frequency modulation of a light wave passing through the interferometer and reflected from it vanishes, irrespective of the value of the modulating field, if the relation

$$p = m\Delta\omega_{ax} \tag{2}$$

holds, where $m=2, 4, 6, \ldots$ and $\Delta\omega_{ax}=\pi c/Ln_0$ is the frequency interval between adjacent axial modes of the interferometer (L is the distance between its mirrors and $n_0=(\varepsilon_0)^{1/2}$ is the index of refraction of the matter in the absence of the modulating field). At $m=1, 3, 5, \ldots$ the gist of the effect manifests is that the amplitude modulation of the transmitted light wave vanishes and only the frequency modulation remains.

Below we also find the conditions under which the vanishing of the amplitude modulation with changing frequency p is abrupt [the derivative of the modulating parameter with respect to the frequency p undergoes a discontinuity with a change of sign on passing through the value (2)]. For a frequency p close but not equal to the value (2), the modulation of the transmitted and reflected light waves vanishes if the frequency ω of the incident light wave coincides exactly with the eigenfrequency ω_0 of one of the interferometer modes. The change of the modulation parameter with ω is then also abrupt and is accompanied by a discontinuity of the corresponding derivative on passing through the value $\omega=\omega_0$.

The observation of the zero points under these conditions can serve as a means of measuring the absolute value of the optical length L of the interferometer with an accuracy several orders of magnitude greater than the light wavelength.

The discussion below is based on the formalism of light eigenwaves in a time-modulated homogeneous medium which was developed by the author in earlier studies (Refs. 9, 11, and 13, for example). Let a plane, monochromatic light wave of frequency ω fall normally on one of the mirrors of an interferometer completely filled with a homogeneous dielectric with a time-modulated dielectric constant

$$\varepsilon=\varepsilon_0+\Delta\varepsilon\cos pt \tag{3}$$

($\Delta\varepsilon \ll \varepsilon_0, p \ll \omega$). Expanding the field of the incident wave in the light eigenwaves $\varphi_l(t)\exp[(\Delta_l)^{1/2}z - i\omega t]$ of the dielectric filling the interferometer and taking into account the boundary conditions at $z=0, L$ where the mirrors are located, we find the following general expressions for the complex amplitude E (with the coefficient $e^{-i\omega t}$) of the wave passing through the interferometer and the complex amplitude E^- of the wave reflected from it:

$$E=E^{(0)}\sum_l \frac{T_1T_2\langle S^{-1}\rangle_{l0}\varphi_l(t)}{\exp(-L\Lambda_l^{1/2})-R_1R_2\exp(L\Lambda_l^{1/2})},$$

$$E^-=E^{(0)}\sum_l\left\{\tilde{R}_1+\frac{T_1\tilde{T}_1R_2\exp(L\Lambda_l^{1/2})}{\exp(-L\Lambda_l^{1/2})-R_1R_2\exp(L\Lambda_l^{1/2})}\right\}\langle S^{-1}\rangle_{l0}\varphi_l(t). \tag{4}$$

Here T_j and R_j are the complex electric-field transmission and reflection coefficients for the j-th mirror when the wave is incident from a medium with a dielectric constant ε_0 on the surrounding medium, $\tilde{T}_j$ and $\tilde{R}_j$ are the same for the wave incident from the surrounding medium on the medium with a dielectric constant ε_0, and S is the matrix for the conversion (see Ref. 11) from monochromatic waves e^{-inpt} corresponding to the unmodulated medium to the light eigenwaves φ_l.

If the dielectric constant medium is modulated as in (3), a technique similar to that described earlier in Refs. 11 and 13 can be used to verify that the eigenwaves $\varphi_l(t)$ have the form

$$\varphi_l(t)=\exp(i\theta\sin pt - ilpt), \tag{5}$$

where $\theta=\omega\Delta\varepsilon/2p\varepsilon_0$, and the expressions for $\Lambda_l^{1/2}$ and $\langle S^{-1}\rangle_{l0}$ are written as

$$\Lambda_l^{1/2}=i\frac{\omega+lp}{c}\varepsilon_0^{1/2},\quad \langle S^{-1}\rangle_{l0}=J_l(\theta), \tag{6}$$

where $J_l(\theta)$ is the Bessel function. Equations (4)–(6) give the solution to our problem.[1)]

Let us first consider the properties of the solution for $p<\Delta\omega_{ax}$. For example, let $p\theta$, $|\omega-\omega_0|\ll\Delta\omega_{ax}$ and $\theta\gtrsim 1$. In this case, (4)–(6) show that the original interferometer bandwidth corresponding to a given mode splits into a number ($\sim 2\theta$) of components at spaces p apart. The fields of the transmitted and reflected waves are then modulated in frequency and in amplitude. At $p\gg\mu_0$ the field of the wave in the interferometer is primarily frequency-modulated and the field of the reflected wave is also amplitude-modulated. The total interval $\Delta\Omega$ of the frequency modulation of the transmitted wave is

$$\Delta\Omega=\omega\Delta\varepsilon/\varepsilon_0. \tag{7}$$

It is interesting that under these conditions this quantity does not depend on the interferometer length L and in general greatly exceeds the frequency modulation interval attained for the same length L in this medium without the interferometer. A feature of the amplitude modulation of the reflected wave is that, for example, at $\theta\gg1$ there arises a series of intensity pulsations of this wave at half the period (π/p) of the modulating field. The modulation parameter of the reflected wave

$$\eta^-=(|E^-|^2_{max}-|E^-|^2_{min})/|E^{(0)}|^2$$

and the "reflection coefficient" averaged over the period of the modulating field

$$r=\frac{p}{2\pi|E^{(0)}|^2}\int\limits_0^{2\pi/p}|E^-|^2\,dt$$

are then oscillatory functions of θ and ω, as seen from (4)–(6).

Let us now consider the solutions obtained at

$$|\theta x|\ll 1, \tag{8}$$

where

$$x=(p-m\Delta\omega_{ax})/\mu_0, \tag{9}$$

that is, in the vicinity of a zero point, and let us assume for simplicity that the mirrors are identical ($R_1=R_2=R$, $T_1=T_2=T$) and nonabsorbing ($T_1\tilde{T}_1=1-R^2$). When $|\omega-\omega_0|\ll\Delta\omega_{ax}$ (where the eigenfrequency ω_0 of one of the interferometer modes is given by $\omega_0=M_0\pi c/L\varepsilon_0^{1/2}$ with M_0 an integer) and $m=2, 4, \ldots$, we obtain from (4)–(6), taking (8) and (9) into account, the following expression for the complex amplitude of the field of the wave passing through the interferometer:

$$E=(-1)^{M_0+1}\frac{E^{(0)}}{1-iy}\left(1+\frac{i\theta x\cos pt}{1-iy}\right) \tag{10}$$

and at $m=1, 3, 5, \ldots$ we find

$$E=(-1)^{M_0+1}\frac{E^{(0)}}{1-iy}\exp[-2i\theta\sin(pt+\pi)]\left[1+\frac{i\theta x\cos(pt+\pi)}{1-iy}\right]. \tag{11}$$

Here

$$y=(\omega-\omega_0)/\mu_0. \tag{12}$$

According to (10) and (11), taking (8) into account, the intensity-modulation parameter of the transmitted wave

$$\eta=(|E|^2_{max}-|E|^2_{min})/|E^{(0)}|^2$$

for any $m=1, 2, 3, \ldots$ has the form

$$\eta=4\theta\frac{|xy|}{(1+y^2)^2}. \tag{13}$$

We see that for nonzero values of y (i.e., when the frequency ω of the incident light wave does not exactly equal the eigenfrequency ω_0 of the interferometer mode) the plot of η against x has a kink with a discontinuity of the derivative at $x=0$, meaning an abrupt change of the modulation parameter η with change of the frequency difference x. The modulation parameter becomes zero at the zero point $x=0$. The strongest dependence of η on x occurs at $y=\pm1/3^{1/2}$, in which case $\eta=(3^{3/2}/4)\theta|x|$. Furthermore, when the frequency p is not exactly equal to the value (2) (i.e., at $x\neq0$) the plot of η against y has a similar kink with a discontinuity of the derivative at $y=0$, which shows that the modulation parameter η depends strongly on the frequency difference y.

We note that expression (2), which determines the zero point, contains the modulation frequency p and the interferometer optical length n_0L. The modulation frequency p at a length $L\sim0.1$–10 cm, for example, can be found with accuracy better than 10^{-5}. Therefore, measurement of this frequency when observing a zero point (for $\omega\neq\omega_0$) can yield the absolute value of the interferometer optical length more accurately than the light wavelength. In addition, for deviations from (2) (i.e., at $x\neq0$) observation of a zero point with changing frequency ω of the incident wave can make it possible to register the coincidence of the frequency ω with the eigenfrequency ω_0 of an interferometer mode with accuracy an order of magnitude better than the frequency width of the interferometer modes. The absolute value of the interferometer optical length can thereby be determined with an overall accuracy several orders of magnitude better than the light wavelength (and even better than the thickness of the atomic layer of the substance). This, in particular, makes it possible to measure precisely refractive-index deviations induced in matter by various external factors (for example, a light field, pressure, magnetic field, etc.).

We also give the expression for the complex amplitude E^- of the wave reflected from the interferometer, which is valid for the condition (8) (and all $m=1, 2, 3, \ldots$):

$$E^-=RE^{(0)}\left(1-\frac{1}{1-iy}\right)-RE^{(0)}\frac{i\theta x\cos pt}{(1-iy)^2}. \tag{14}$$

As seen from this expression, at the zero points the modulation of the reflected wave also vanishes irrespective of the amplitude of the modulating field.

In conclusion we note that all the above effects can also be observed in an interferometer which is an optical waveguide bounded by mirrors. Here the electric field modulating the refractive index can be arbitrarily inhomogeneous in the cross section of the waveguide and periodically inhomogeneous along its length. In the latter case the scale of the longitudinal inhomogeneity must be much smaller than the interferometer length and must not be a multiple of the light wavelength. Our results remain valid, with the only modification that the refractive index [or the dielectric constant (3)] is in this case the corresponding effective refractive index for the guided waves. Observation of the zero points in this case can also permit the effective optical length of the interferometer to be determined with accuracy several orders of magnitude better than the light wavelength.

[1)] We note that corresponding expressions for the intensities of the transmitted and reflected waves are also obtained in the case when the incident wave itself is frequency-modulated. The zero-point effect can therefore also be observed for such (external) modulation.

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Translated by P. Millard