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## Role of slow relaxation processes in the formation of the Kapitza jump on the boundary between a superconductor and a dielectric

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The effect of slow relaxation of excitons in a superconductor on the thermal Kapitza resistance between a superconductor and a dielectric is considered. It is shown that the presence of a hierarchy of relaxation length in the superconductor leads to the existence near the boundary of a region that is in strong disequilibrium. The observed Kapitza jump depends on the size of the energy gap. The results of the theory agree in order of magnitude with experiment.

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When heat passes through a boundary between two media, a temperature jump is produced, called a Kapitza jump. The theory of this phenomenon on the boundary between superfluid He II and a solid was constructed by Khalatnikov.<sup>1,2</sup> Little<sup>3</sup> extended this theory to the case of a contact between two bodies with different acoustic properties (densities and sound velocities).

The conclusions of the theory<sup>1-3</sup> reduce to the following. Only phonons transport the energy through the boundary. By virtue of the difference between the acoustic properties of the two media, the phonons have a definite probability of being reflected from the boundary. As a result, the energy flux  $I_c$  at the boundaries is connected with the temperatures in the two media on the boundary by the relation

$$I_c = A(T_0^4 - T_1^4) \quad (1)$$

Here  $A$  is a coefficient proportional to the probability of the passage of the phonons through the boundary. In the derivation of (1) it is assumed also that the phonon distribution functions in both media remain in equilibrium up to the boundary, with temperatures  $T_0$  and  $T_1$ .

There exist, however, experimental facts which have so far not been explained within the framework of the theory of acoustic mismatch.<sup>1-3</sup> One of them is that the Kapitza jump between a superconductor and a dielectric

depends on the energy gap in the superconductor, which is varied in the experiments with the aid of a magnetic field (see, e.g., Refs. 4 and 5). In the superconducting state the Kapitza jump turns out to be larger than in the normal metal at the same temperature. The change in the Kapitza jump fluctuates in various experiments from a fraction of several hundredths to a factor of several times.

This fact does not agree with the theory,<sup>1-3</sup> since neither the sound velocity nor the coefficient of the phonon reflection from the boundary is dependent in practice on the energy gap in the superconductor. Andreev<sup>6</sup> and Little<sup>7</sup> considered the influence of the conduction electrons on the phonon transmission coefficient through the boundary. However, allowance for this mechanism does not lead to a substantial improvement of the agreement between experiment and theory.

On the other hand, the assumption that the distribution functions retain their equilibrium form all the way to the boundary in generally speaking incorrect. Equilibrium distributions set in only at distances from the boundary that are larger than the characteristic relaxation lengths. In the immediate vicinity of the boundary the distribution functions of the excitations are not in equilibrium, and their form depends on the relaxation mechanisms.

The case when the medium contains only one type of excitation—phonons, whose relaxation is characterized by only one relaxation time  $\tau$ , was considered by Levinson.<sup>8</sup> The phonon distribution function depends in this case only on the dimensionless quantity  $z/w\tau$  ( $z$  is the distance from the boundary and  $w$  is the speed of sound), while at the immediate vicinity of the boundary, at  $z=0$ , the distribution of the phonon is described by a universal function that does not depend on  $\tau$  and is determined only by the structure of the collision integral in the phonon kinetic equation. Solution of this equation for the Kapitza jump yields an expression similar to (1), with a coefficient  $A$  that depends on the character of the reflection of the phonons from the boundary and on the structure of the collision integral, but agrees in order of magnitude with the expression obtained in Refs. 1–3.

Most media, however, are characterized by the presence of several types of excitations and of several relaxation times. In that case the form of the distribution function of the excitations at the boundary is not universal and depends on the ratio of the relaxation times. In this paper we show that the presence of a hierarchy of relaxation lengths influences substantially the value of the Kapitza jump. We consider the contact between a dielectric and a superconductor at low temperatures, when the low concentration of the quasiparticles in the superconductor gives rise to a hierarchy of the relaxation lengths of the quasiparticles and of the phonons. As a result, an “additional” temperature jump is produced in the volume of the superconductor, and its magnitude depends on the size of the energy gap.

Two types of excitation exist in a superconductor: quasiparticles and phonons. The kinetics of the excitations in superconductors is described by the quasiparticle and phonon kinetic equations

$$\frac{\partial}{\partial \mathbf{r}} \left( D \frac{|\xi|}{\varepsilon} \frac{\partial n_{\varepsilon}}{\partial \mathbf{r}} \right) = -J_p^* - I \quad (2)$$

$$w \frac{\partial N_q}{\partial \mathbf{r}} = -J_{ph}^* - J_{ph}^r \quad (3)$$

Here  $n_{\varepsilon}$  and  $N_q$  are the distribution functions of the quasiparticles and phonons, respectively:  $\varepsilon = (\xi^2 + \Delta^2)^{1/2}$ ;  $\xi = \mathbf{p}^2/2m - \eta$ ;  $\mathbf{p}$  and  $\mathbf{q}$  are the momenta of the quasiparticles and of the phonons;  $\eta$  is the chemical potential of the normal metal;  $\Delta$  is the half-width of the energy gap in the superconductor;  $D = v_F^2 \tau_{im}/3$  is the diffusion coefficient of the electrons in the normal metal (we consider only the case when the momentum relaxation time  $\tau_{im}$  is less than the remaining relaxation times);  $v_F$  is the Fermi velocity;  $J_p^r, J_{ph}^r$  are operators describing the recombination (generation) of the quasiparticles with emission (absorption) of high-frequency phonons;  $J_p^s, J_{ph}^s$  are operators describing the scattering of quasiparticles by phonons. Since we are considering only the case  $T \ll \Delta, x \ll 1$ , the corrections to the energy gap can be neglected.

The mean free paths of the high-frequency ( $\omega > 2\Delta$ )  $l_{\omega}$  and low-frequency ( $\omega < 2\Delta$ )  $l_{\omega}$  phonons in superconductors at low temperatures ( $T < \Delta$ ) are connected by the rela-

$$l_{\omega} \approx l_{\omega} / x \gg l_{\omega} \quad (4)$$

Here

$$x = \frac{1}{4N(0)\Delta} \sum_{\mathbf{p}\sigma} n_{\mathbf{p}\sigma} = \left( \frac{\pi T}{2\Delta} \right)^{1/2} e^{-\Delta/T}$$

is the dimensionless concentration of the quasiparticles, and  $N(0) = m p_F / \pi^2$  is the density of states of the Fermi surface in the normal metal ( $\sigma$  is the spin).

In addition, at low temperatures there exists in superconductors a hierarchy of quasiparticle relaxation times:

$$\tau_s \ll \tau_r \ll \tau_i^* \quad (5)$$

Here  $\tau_s \sim (\Theta_D^2/T^3)(\Delta/T)^{1/2}$  is the time of scattering of the quasiparticles by the phonons,  $\tau_r \sim \Theta_D^2/\Delta^3 x$  is the quasiparticle recombination time,  $\tau_i^* \sim \Theta_D^2/\Delta^3 x^2$  is the time of establishment of the equilibrium number of quasiparticles,<sup>9</sup> and  $\Theta_D = w p_F$  is the Debye energy. This hierarchy of relaxation times is connected with a hierarchy of diffusion lengths:

$$L_s = (D\tau_s)^{1/2} \ll L_r = (D\tau_r)^{1/2} \ll L_i^* = (D\tau_i^*)^{1/2} \quad (6)$$

The energy is transported through the boundary by the phonons. However, energy can be transported in the volume of the superconductor both by phonons and by quasiparticles, and the ratio of these two fluxes depends on the temperature.<sup>10</sup>

We consider first the case of not too low temperatures, when the energy transport in the superconductor is by the quasiparticles (as exact criterion will be written down later on), but assuming that  $T < \Delta$ . The presence of relations (4)–(6) leads to a distribution function of the form

$$n_{\varepsilon} = e^{(\nu - \varepsilon)/T} \quad (7)$$

and

$$N_{\omega} = e^{(2\nu - \omega)/T}, \quad \omega > 2\Delta, \quad (8a)$$

$$N_{\omega} = (e^{\omega/T} - 1)^{-1}, \quad \omega < 2\Delta. \quad (8b)$$

The functions (7) and (8) cause all the collision intervals, both quasiparticle and phonon, to vanish in (2) and (3), with the exception of the terms that describe the absorption of the high-frequency phonons by quasiparticles and contain an additional small factor—the concentration  $x \ll 1$ . Physically this is connected with the fact that the quasiparticles, becoming rapidly scattered by the low-frequency phonons, establish a quasiequilibrium distribution function with a chemical potential  $\nu$  and a temperature  $T$ .

Quasiparticle recombination gives rise to high-frequency phonons, which are absorbed over a small length  $l_{\omega}$ , again producing two quasiparticles each. These processes establish the equilibrium between the high-frequency phonons and the quasiparticles without changing the number of quasiparticles. The number of quasiparticles changes only when the quasiparticles absorb high-energy phonons, each of which is equivalent to a pair of quasiparticles.<sup>9</sup> On the whole, the situation is perfectly analogous with that considered in Ref. 9, the only difference being that in this case  $\nu$  and  $T$  are functions of the coordinates. In our case, when the energy in the

superconductor is transported mainly by quasiparticles, the relations  $l_c \ll L_s$  and  $l_s \ll L_s$  are satisfied. Therefore the phonons at each point adjust themselves to the local distribution of the quasiparticles. The quasiparticles establish a quasiequilibrium distribution over a length  $L_s$ , while  $\nu$  and  $T$  vary over a length  $L_s^>$  with  $\nu(\infty)=0$ .

To determine  $\nu(z)$  and  $G(z)$  we must use the conservation laws for the energy and for the number of quasiparticles. Multiplying (2) and (3) by  $\epsilon_p$  and  $\omega_q$  and summing respectively over  $p$  and  $q$ , we obtain the energy conservation law:

$$\sum_p \epsilon \frac{\partial}{\partial \mathbf{r}} \left( D \frac{|\xi|}{\epsilon} \frac{\partial}{\partial \mathbf{r}} n_s \right) = 0. \quad (9)$$

Summing (2) over  $p$ , we obtain the quasiparticle-number conservation law:

$$\sum_p \frac{\partial}{\partial \mathbf{r}} \left( D \frac{|\xi|}{\epsilon} \frac{\partial}{\partial \mathbf{r}} n_s \right) = 2 \sum_{q^i, \omega > 2\Delta} J_{ph^i}. \quad (10)$$

Here  $i$  is the polarization of the phonons.

In the derivation of (9) and (10) from (2) and (3) we have used the exact relations

$$\sum_p \epsilon (J_{p^+} + J_{p^-}) = - \sum_{q^i} \omega_q (J_{ph^i} + J_{ph^i}'), \quad \sum_p J_{p^+} = 0, \quad (11)$$

$$\sum_{q^i} J_{ph^i} = - \frac{1}{2} \sum_p J_{p^+}, \quad (12)$$

$$w \frac{\partial N_q}{\partial \mathbf{r}} = - J_{ph^i} \quad (\omega < 2\Delta), \quad (13)$$

which express the conservation laws for the energy and for the quasiparticle number in the collisions (11), the fact that one high-frequency phonon is produced when two quasiparticles recombine (12) and the fact that at  $\omega < 2\Delta$  the phonons can participate only in the scattering of the quasiparticles and make no contribution to the recombination-generation processes (12). In addition, we have neglected in the derivation of (9) and (10) the terms that describe the contribution of the phonons to the energy transport and to the quasiparticle number.

Substituting (7) and (8) in (9) and (10) and using the explicit form of  $J_{ph^i}$

$$J_{ph^i} = \sum_p |V_q|^2 \left( 1 + \frac{\xi_p \xi_{p+q} - \Delta^2}{\epsilon_p \epsilon_{p+q}} \right) \delta(\epsilon_p - \epsilon_{p+q} + \omega_q) \cdot [n_p(1-n_{p+q})N_q - n_{p+q}(1-n_p)(1+N_q)] \quad (14)$$

(where  $V_q$  is the matrix element of the electron-phonon interaction), we obtain in the approximation linear in  $\nu$  and in  $T$  two equations for  $\nu(z)$  and  $T(z)$

$$\frac{\partial^2 \nu}{\partial z^2} + \left( \frac{\Delta}{T} + 1 \right) \frac{\partial^2 T}{\partial z^2} = \Gamma \nu, \quad (15)$$

$$\frac{\partial^2 \nu}{\partial z^2} + \frac{\Delta^2 + 2\Delta T + 2T^2}{\Delta T + T^2} \frac{\partial^2 T}{\partial z^2} = 0, \quad (16)$$

where

$$\Gamma = \frac{2\Delta\sqrt{2}\Delta^3}{DN(0)l_s\pi^2\omega^2T} e^{\Delta/T} x^2. \quad (17)$$

Here and below we express all the relaxation lengths in terms of the experimentally measured  $l_s$ .

Equations (15) and (16) require four boundary conditions. Two of them can be obtained by specifying on the boundary the fluxes of the energy and of the number of quasiparticles:

$$I_s = -D \sum_p |\xi| \frac{\partial n_s}{\partial z} = -2DN(0) \left[ (\Delta + T) \frac{\partial \nu}{\partial z} + \frac{\Delta^2 + 2\Delta T + 2T^2}{T} \frac{\partial T}{\partial z} \right] e^{-\Delta/T}, \quad (18)$$

$$I_p \approx -D \sum_p \frac{|\xi|}{\epsilon} \frac{\partial n_s}{\partial z} = -2DN(0) \left[ \frac{\partial \nu}{\partial z} + \left( \frac{\Delta}{T} + 1 \right) \frac{\partial T}{\partial z} \right] e^{-\Delta/T}. \quad (19)$$

The energy flux through the boundary is determined mainly by the thermal ( $\omega \sim T$ ) phonons, while the quasiparticle flux is determined by the high frequency ( $\omega > 2\Delta$ ) phonons, the number of which is exponentially small. These phonons, being absorbed over a length  $l_s$  produce quasiparticle pairs. Therefore the fluxes of the energy and of the number of quasiparticles on the boundary are connected by the relation

$$I_p \approx \frac{I_s \Delta^2}{T^3} e^{-2\Delta/T}. \quad (20)$$

As a result, accurate to  $e^{-2\Delta/T} \ll 1$ , we can put  $I_p(0) = 0$ .

The third boundary condition reduces to the requirement that the solutions be bounded at  $z \rightarrow \infty$ , and finally, the fourth connects the temperatures in the dielectric and in the superconductor at the boundary.

In the immediate vicinity of the boundary, accurate to

$$e^{-2\Delta/T} \ll 1, \quad l_s/L_s \ll 1$$

expression (1) remains valid for the temperature jump. To verify this, we can solve the phonon equation (3), which is linear for the specified quasiparticle distribution function (7), and calculate the energy flux carried by the phonon through the boundary.

The solutions of (15) and (16) at the given boundary conditions are

$$\nu = \nu_0 e^{-z/\lambda}, \quad (21)$$

and

$$T = C_1 + C_2 z - \frac{\Delta T + T^2}{\Delta^2 + 2\Delta T + 2T^2} \nu_0 e^{-z/\lambda}, \quad (22)$$

where

$$\nu_0 = \frac{I_s e^{\Delta/T} \lambda}{2T^2 DN(0)}, \quad \lambda = \Gamma^{-1/2} \frac{T}{\Delta}, \quad C_2 = \frac{I_s}{\nu_s},$$

$\nu_s = 2DN(0)(\Delta^2/T)e^{-\Delta/T}$  is the electronic thermal conductivity of the superconductor, and  $C_1$  is determined from (1).

It is seen from (22) that the usual thermal conductivity regime with  $\nu=0$  and with linear variation of the temperature is realized only over lengths larger than  $\lambda$ . Near the boundary there is a nonzero chemical potential, and the temperature of the quasiparticles varies nonlinearly (see Fig. 1). If the Kapitza jump is taken to mean the temperature jump obtained as a result of linear extrapolation of the temperature dependence to infinity, as shown dashed in Fig. 1, then near the bound-

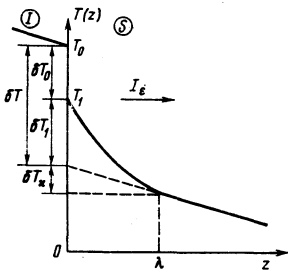


FIG. 1. Dependence of the temperature  $T$  on the coordinate  $z$  when heat flows through a boundary between a superconductor (S) and an insulator (I).

ary, over length of order  $\lambda$ , an "additional" temperature jump is produced:

$$\delta T_1 = I_e \frac{e^{\Delta/T} \lambda}{2DN(0)T}. \quad (23)$$

It is important that this additional jump increases with increasing ratio  $\Delta/T$ , in qualitative agreement with the results of the experiments.<sup>5</sup>

It is of interest to calculate the ratio of the additional Kapitza jump  $\delta T_1$  to the usual jump  $\delta T_0$  defined by relation (1). (The total jump  $\delta T$  is the sum of the two (see Fig. 1).) We have

$$\delta T_0 = \frac{I_e}{3AT^2}, \quad A = \frac{\pi^2 t}{30w^2}. \quad (24)$$

Here  $T$  is the coefficient of transmission of the phonons through the boundary,

$$\frac{\delta T_1}{\delta T_0} = \frac{\pi^2 t}{40} \left( \frac{T}{\Delta} \right)^{3/4} e^{2\Delta/T} \frac{\Delta}{\Theta_D} \left( \frac{l_{im}}{l_{>}} \right)^{1/2}, \quad (25)$$

$$l_{im} = \tau_{im} v_F.$$

This ratio increases exponentially with increasing ratio  $\Delta/T$ .

We consider now a criterion for the applicability of the results, i.e., for the smallness of the phonon energy flux compared with the quasiparticle flux. Neglecting the high-frequency phonons, the phonon energy flux takes the standard form  $I_e^{ph} = -\kappa_{ph} \nabla T$ . The temperature gradient is maximal at the boundary (see Fig. 1) and therefore, using (22), we get

$$(I_e^{ph})_{max} = \kappa_{ph} \frac{I_e e^{\Delta/T}}{2DN(0)T}, \quad \kappa_{ph} \sim \frac{2\pi^2}{15} \frac{l_{>} T^3}{w^2 x}, \quad (26)$$

where  $\kappa_{ph}$  is the phonon thermal conductivity of the superconductor, obtained in the relaxation-time approximation.

The condition  $I_e^{ph} \ll I_e$  takes at  $z=0$  the form

$$(\kappa_{ph}/\kappa_e) (\Delta/T)^2 \ll 1 \quad (27)$$

and is less stringent than the corresponding condition in the volume ( $\kappa_{ph} \ll \kappa_e$ ). This fact is connected with the strong disequilibrium of the distribution functions of the excitations at the boundary.

Substituting in (27) the expressions for  $\kappa_{ph}, \kappa_e$  we obtain ultimately

$$e^{2\Delta/T} < \frac{1}{15} \frac{l_{im}}{l_{>}} \left( \frac{\Theta_D}{\Delta} \right)^2 \left( \frac{\Delta}{T} \right)^{3/2}. \quad (28)$$

Using the condition (28), we obtain the upper bound

$$\frac{\delta T_1}{\delta T_0} < \frac{1}{10} t \left[ \left( \frac{\Theta_D}{\Delta} \right)^2 \frac{l_{im}}{l_{>}} \right]^{1/2} \sim \frac{t}{10} \left( \frac{\kappa_e^n}{\kappa_{ph}^n} \Big|_{T=\Delta} \right)^{1/2}. \quad (29)$$

Actually the expression in the square brackets in (29) is of the order of the ratio of the electronic  $\kappa_e^n$  and phonon  $\kappa_{ph}^n$  thermal conductivities in the normal metal at the temperature  $T=\Delta$ . Therefore the expression in the right-hand side of (29) is usually larger than unity (and in pure substances it can reach  $10^2$ ), while at sufficiently low temperatures the additional jump can become larger than the regular one.

The result can be explained qualitatively in the following manner. When (28) is satisfied and  $T \ll \Delta$ , the fluxes of the energy and of the number of quasiparticles in the interior of the superconductor are connected by the relation  $I_e \approx I_p \Delta$ . On the other hand, on the boundary these fluxes are connected by relation (20), i.e., near the boundary the flux of the quasiparticles is exponentially smaller than the value that should obtain in the thermal-conductivity regime at a specified energy flux. Therefore near the boundary, at distances shorter than the characteristic length of establishment of the equilibrium number of quasiparticles, the concentration of the quasiparticles deviates from the equilibrium value and this results in a nonzero chemical potential. The reverse energy flux, proportional to the gradient of the chemical potential, leads to an increase of the temperature gradient in the region next to the boundary at a given energy flux. The nonlinear change of the temperature in the region next to the boundary is in fact the cause of the additional temperature jump (Fig. 1).

Finally, let us compare the additional jump with the temperature "surge"  $\delta T_*$  over the length  $\lambda$  in the regime of ordinary electronic thermal conductivity:

$$\delta T_* = I_e \lambda / \kappa_e, \quad \delta T_1 / \delta T_* = (\Delta/T)^2 \gg 1. \quad (30)$$

Thus, the additional jump  $\delta T_1$  is substantially larger than  $\delta T_*$ .

We consider now the region of very low temperatures, when an inequality inverse to (27) is satisfied, and the entire energy transport in the superconductor is due to phonons. We assume that the fastest process is elastic scattering of phonons by the impurities, and the energy relaxation of the phonons is characterized by a single relaxation time. In this case it is easy to show that no additional temperature jump takes place in the volume of the superconductor.

In the case  $\Delta=0$ , when all the phonon mean free paths are of the same order  $l$ , and all the diffusion lengths are also of the same order  $L$ , there is likewise no additional jump, accurate to  $1/L \ll 1$ . As a result, the dependence of  $\delta T_1 / \delta T_0$  on  $\Delta/T$ , which can be measured by varying the temperature of the thermostat and the magnetic field strength, has a maximum at the point where the condition  $\kappa_{ph} \sim \kappa_e$  is satisfied (see Fig. 2).

In a magnetic field stronger than critical, when the superconductivity is completely destroyed, we have  $\delta T_1 = 0$  and the total jump is  $\delta T = \delta T_0$ . When the magnetic field is turned off, the measured jump increases by an amount equal to the additional jump  $\delta T_1$ .

In conclusion, we present an expression for the addi-

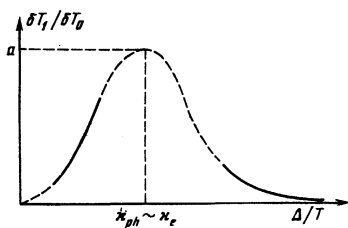


FIG. 2. Dependence of  $\delta T_1 / \delta T_0$  on  $\Delta / T$ ;

$$a = \frac{1}{10} \left( \frac{\kappa_e^n}{\kappa_{ph}^n} \Big|_{\tau=\Delta} \right)^{1/4}.$$

tional Kapitza jump in the case of a superconductor with finite thickness  $d$  assuming identical boundary conditions on both boundaries

$$\delta T_1(d) / \delta T_1(\infty) = 2 \text{th}(d/2\lambda). \quad (31)$$

The coefficient 2 is connected with the fact that we have calculated the additional jump on both boundaries. With decreasing sample thickness, the additional jump tends to zero. To observe this phenomenon it is necessary to have samples with thickness  $d < \lambda$ .

In order of magnitude,  $\lambda$  is equal to  $L_s^2 \sim e^{\Delta/T} l_{im}^{1/2}$  (this can be verified by recognizing that the electron-phonon and phonon-electron relaxation times are proportional to the same electron-phonon interaction constant and differ only in the final state densities), and can vary in a wide range, depending on the purity of the sample (i.e., on  $l_{im}$ ) and on the temperature. For example, at  $l_{im} = 10^{-6}$  cm and  $\Delta/T = 2$ ,  $l_s = 10^{-5}$  cm we have  $\lambda \approx 10^{-3}$  cm, while at  $l_{im} = 10^{-2}$  cm and  $\Delta/T = 5$  we have  $\lambda \approx 1$  cm.

The sign of the additional jump and its order of magnitude can be reconciled with the experimentally observed values. For a more detailed comparison of ex-

periment with theory, however, measurements in a larger temperature interval are needed.

A phenomenon similar to that considered above can arise also in He II, where there is likewise a hierarchy of phonon and roton relaxation lengths,<sup>2</sup> and according to estimates<sup>2,11</sup> the energy is transported through the boundary, in the region  $T \geq 1$  K, the rotons make the main contribution to all the thermodynamic functions and it is precisely they which transport the energy. This should produce in He II a relaxation region similar to that considered in the superconductor. We shall not, however, discuss this possibility in greater detail.

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## Effect of electron interaction on the Peierls instability

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The dependence of the energy of the ground state of a one-dimensional metal on the amplitude of a periodic deformation is considered in the Hubbard model for different values of the interelectron-interaction parameter. It is shown that a Mott one-dimensional dielectric is unstable to the Peierls deformation at all values of the interaction parameter. Relations are obtained between the responses to the perturbations with wave numbers  $Q = 2k_F$  and  $4k_F$  in the limiting case of strong and weak electron interaction.

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### INTRODUCTION

The instability of a one-dimensional metal to lattice deformation leads to the onset of a Peierls-Frölich state.<sup>1-5</sup> The investigation of various characteristics of

this state is of interest because of the advent of new quasi-one-dimensional conductors.<sup>3-8</sup>

The conclusion that a one-dimensional metal is unstable to lattice deformation was deduced by Peierls on