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[2)] Films of different widths (0.01-0.8 mm) and lengths (0.01-15 mm) were deposited on glass substrates with gold contacts (planar configuration of the Au–CuCl–Au bridge type).

[3)] The authors are grateful to S. D. Vangengeĭm and V. A. Sil'chenko for considerable help in these analyses.

[4)] The resistance scale for $dU/dI-U$ can be used to estimate quite easily the changes in the amplitudes of the phonon singularities.

[5)] The distortion of the $dI/dU-U$ characteristic near $U=\Delta_{N/S}\pm\Delta_{Al}$ for $d_N\approx 200$ Å is possibly due to the appearance of states inside the gap.[1]

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Effect of local flattenings of the Fermi surface on the absorption and dispersion of sound

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It is shown that the zero-curvature points and lines result in a sharp dependence of the absorption and velocity dispersion of sound on the direction of propagation. The nature of this dependence is determined by the shape of the local flattening and its orientation with respect to the principal directions. In contrast to ordinary absorption and dispersion, the contribution of the zero-curvature points and lines depends at $kl\gg 1$ on the frequency and the temperature. The presence of zero-curvature lines on the Fermi surface also leads to a change in the angular dependence of the absorption and dispersion in a strong magnetic field and to anisotropy of the tilt effect.

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As is well known,[1,2] a transition from viscous low-frequency absorption $\gamma/\omega\sim\omega/\nu$ to collisionless absorption $\gamma/\omega\sim s/v$ takes place in pure metals at low temperatures with increase in the sound frequency. In collisionless absorption, only the electrons of a narrow "belt" $\mathbf{k}\cdot\mathbf{v}=\omega$ take part; these electrons are in synchronism with the sound wave; therefore the absorption is usually little sensitive to the geometry of the Fermi surface. However, as has been pointed out previously, the absorption on flat and cylindrical parts of the Fermi surface[1)] differs significantly from described absorption in view of the fact that, when they propagate along the surface or the axis of a cylinder, all the electrons of these portions turn out to be simultaneously in synchronism with the wave.[4] The number of resonant electrons does not decrease with increase in frequency (as occurs on the belt) and the frequency-dependent increase continues up to the collision frequency (renormalized to the fraction of the flat or cylindrical portion of the Fermi surface). Correspondingly, a strong angular dependence exists, i.e., a dependence on the direction of **k**. This situation is similar to that occuring in a strong magnetic field when $\mathbf{k}\perp\mathbf{H}$ and for closed sections of the Fermi surface,[5] while the strong dependences of the absorption and dispersion on the direction of propagation are related to the tilt effect.[6]

In real metals, however, finite portions of sufficiently small curvature are very rare. At the same time, as noted by a number of authors,[7,8] similar effects can be observed even on local flattenings, i.e., in the neighborhoods of points and lines in which the Gaussian curvature of the Fermi surface vanishes. There are such points in virtually all metals with anisotropic Fermi surfaces. Figure 1 shows possible types of points of zero curvature: flattening points, at which both principal curvatures of the surface are equal to zero, and parabolic points, at which only one principal curvature is equal to zero. The associated singularities in the angular dependences of the absorption and dispersion of the sound velocity are also given. Generally speaking, the Fermi surfaces have zero-curvature lines consisting of flattening points or parabolic points. For example, flattening lines can occur on cylindrical Fermi surfaces (quasi two-dimensional metals) (Fig. 2a), parabolic lines on surfaces of revolution with generatrices that have inflection points (Fig. 2b). In the general case, the anomalous contribution to the absorption and dispersion of the sound velocity in a given direction of propagation is made by a single point of a zero-curvature line—the point of its intersection with the belt $\mathbf{k}\cdot\mathbf{v}=\omega$. The lines can also become tangent and, finally, overlap over a finite interval. For example, if the zero-curvature line is one of the generatrices of a cylindrical Fermi surface, then at some direction **k** it will be also a belt. In this case we have the strongest anisotropy of the sound absorption and dispersion.

The contribution of the zero-curvature points and also

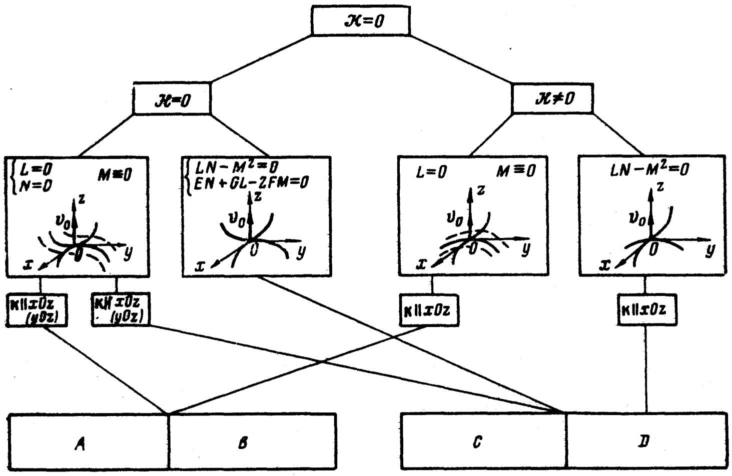

FIG. 1. Types of points and lines of zero curvature and the singularities in the sound absorption and velocity dispersion associated with them. Here

$$\mathcal{K}=\frac{LN-M^2}{EG-F^2},\quad \mathcal{H}=\frac{EN+GL-2FM}{2(EG-F^2)}$$

are the Gaussian and mean curvatures, E, G, F and L, M, N are the coefficients of the first and second quadratic form of the surface $p_z=f(p_x, p_y)$: $E=1+f_x^2$, $G=1+f_y^2$, $F=f_xf_y$; $L=f_{xx}$, $N=f_{yy}$, $M=f_{xy}$. The directions of the sound wave vector relative to the coordinate planes $x0z$ and $y0z$ are also shown. The conditions are:

A – for $\varepsilon_{xxx}(\mathbf{p}_0)<0$

$$\frac{\gamma}{\omega}\sim\left(\frac{s}{v}\right)^{1/2}\left(\frac{\omega}{\mathbf{k}\mathbf{v}_0-\omega}\right)^{1/2},\quad \frac{\Delta s}{s}\sim\left(\frac{s}{v}\right)^{1/2}\left(\frac{\omega}{\omega-\mathbf{k}\mathbf{v}_0}\right)^{1/2};$$

B – for $\varepsilon_{xxx}(\mathbf{p}_0)>0$

$$\frac{\gamma}{\omega}\sim\left(\frac{s}{v}\right)^{1/2}\left(\frac{\omega}{\omega-\mathbf{k}\mathbf{v}_0}\right)^{1/2},\quad \frac{\Delta s}{s}\sim-\left(\frac{s}{v}\right)^{1/2}\left(\frac{\omega}{\mathbf{k}\mathbf{v}_0-\omega}\right)^{1/2};$$

C – for $\varepsilon_{xxx}(\mathbf{p}_0)\varepsilon_{yyy}(\mathbf{p}_0)>0$

$$\frac{\gamma}{\omega}\sim\frac{s}{v}\theta\left(\frac{\omega-\mathbf{k}\mathbf{v}_0}{\omega}\right),\quad \frac{\Delta s}{s}\sim\frac{s}{v}\ln\left|\frac{\mathbf{k}\mathbf{v}_0-\omega}{\omega}\right|;$$

D – for $\varepsilon_{xxx}(\mathbf{p}_0)\varepsilon_{yyy}(\mathbf{p}_0)<0$

$$\frac{\gamma}{\omega}\sim\frac{s}{v}\ln\left|\frac{\omega}{\mathbf{k}\mathbf{v}_0-\omega}\right|,\quad \frac{\Delta s}{s}\sim\frac{s}{v}\theta\left(\frac{\omega-\mathbf{k}\mathbf{v}_0}{\omega}\right).$$

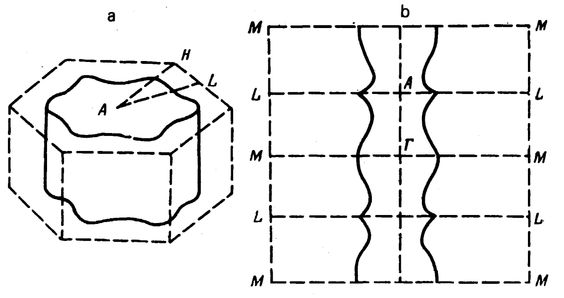

FIG. 2. Elements of the Fermi surface of a dichalgogenide of type $2H$ (a), and element of the Fermi surface of gadolinium (b).

of the finite zero-curvature sections, in contrast with the usual high-frequency absorption, depends significantly on the mean free path. In the limit as $l \to \infty$ the absorption and dispersion at the zero-curvature points have singularities. Their character gives an idea of the angular dependence at finite l and is pointed out below. In order to estimate the magnitude of the effect and to investigate its frequency dependence, the finiteness of the collision frequency must be taken into account.

For the description of the contribution of the points and lines of zero curvature to the damping and to the sound velocity, we make use of the equations of elasticity in metals.[9] For simplicity, we limit ourselves to consideration of a wave of a single polarization and only shain interaction. Then the dispersion equation has the form

$$\left(\frac{\omega}{\omega_0}\right)^2-1=-\frac{2i\omega}{h^3\rho s^2}\int\frac{dS}{v}\frac{\Lambda^2}{i(\mathbf{kv}-\omega)+\nu}, \tag{1}$$

where $\Lambda\equiv\Lambda_{j\varkappa}$ is the component of the strain potential, j is the index of polarization, $\varkappa=\mathbf{k}/k$, $\omega_0=ks$ is the unperturbed sound frequency, s is the sound velocity, ρ is the density, and ν is the relaxation time.[2)] In the integral (1) over the entire Fermi surface, we separate the contribution of the neighborhood of the zero-curvature points (lines)

$$\left(\frac{\Delta\omega}{\omega}\right)_{\text{sing}}\approx-g(\mathbf{p}_0)\int\frac{dS}{\mathbf{kv}-\omega-i\nu}, \tag{2}$$

where $g(\mathbf{p}_0)=2\omega\Lambda^2(\mathbf{p}_0)/h^3\rho s^2 v(\mathbf{p}_0)$; the slowly changing functions $\Lambda^2(\mathbf{p})$ and $v(\mathbf{p})$ are taken outside the integral sign at the point of zero curvature p_0.

The character of the singularity $(\Delta\omega/\omega)_{\text{sing}}$ is determined, first by the conditions for the curvature at the zero curvature point of the given type, and second by the direction of the vector $\mathbf{k}$ relative to the principal directions at the zero-curvature point.

The contribution of the flattening line is considered in Sec. 1. It is shown that the angular dependences of the sound absorption and velocity have in this case root singularities[8] (Fig. 1).

The contribution of the flattening point is investigated in Sec. 2. Since there are no preferred directions at an isolated flattening point, any direction $\mathbf{k}$ in the tangent plane (more accurately, in the place inclined at an angle s/v to it) is a resonant one. The absorption at the resonance $\mathbf{k}\cdot\mathbf{v}(\mathbf{p}_0)=\omega$ has a logarithmic singularity, while the sound velocity experiences a discontinuity (a point of the hyperbolic type) or, on the contrary, the absorption experiences a discontinuity while the sound velocity has a logarithmic singularity (a point of the elliptic type).[7,8] The preferred direction, however, appears if the flattening point belongs to a parabolic line. Then in the case of $\mathbf{k}$ parallel to its contacting plane or perpendicular to it, the absorption and dispersion have a square-root singularity, as in the case of the flattening line (Sec. 1).

In contrast to the flattening point, at a parabolic point there is one principal direction corresponding to an infinite radius of curvature. As shown in Sec. 3, the absorption and dispersion connected with the parabolic point have singularities only if $\mathbf{k}$ lies in the principal plane. The absorption has in this case a logarithmic singularity, and the change in the velocity of sound experiences a discontinuity as in the case of flattening point of the hyperbolic type.

In Sec. 4 we investigate how the zero-curvature lines appear in sound propagation in a strong magnetic field ($kr\ll 1$, r is the turning radius of the electron in the magnetic field). It is shown that the presence on the Fermi surface of a line of zero curvature leads to a change in the frequency dependence of the absorption and dispersion in the strong magnetic field and is responsible for the increase in the amplitude of the tilt effect.[8]

1. CONTRIBUTION OF THE FLATTENING LINE TO THE ABSORPTION AND CHANGE IN THE VELOCITY OF SOUND

Flattening lines can be encountered on the Fermi surface, for example, of layered metals which have a quasi-two dimensional electron dispersion $\varepsilon(p_x, p_z)$. On a cylindrical Fermi surface, where one principal curvature is identically equal to zero, when the curvature $\mathscr{K}$ of the cross section vanishes at some point $\mathbf{p}_0(p_{x0}, p_{z0})$, the corresponding generatrix is a flattening line.

If the z axis is identical with the normal to the surface at the point p_0 (Fig. 3), then the curvature is

$$\mathscr{K}=\frac{\varepsilon_{xx}}{v},\quad \varepsilon_{ik}=\frac{\partial^2\varepsilon(\mathbf{p})}{\partial p_i\partial p_k}$$

and on the flattening line,

$$\varepsilon_{xx}(\mathbf{p}_0)=0. \tag{3}$$

The expansion of $\mathbf{k}\cdot\mathbf{v}(p_x, p_z(p_x))$ in its immediate vicinity $|p_{\max}-p_{x0}|\ll p_0$ with account of (3) begins with terms of second order:

$$\mathbf{kv}=k_z v_0+ax^2, \tag{4}$$

where

$$v_0=v(\mathbf{p}_0),\quad x=p_x-p_{x0},\quad a={}^1/_2 k_x/\varepsilon_{xxx}+{}^1/_2 k_z/\varepsilon_{zxx}.$$

According to (2) and (4), the contribution of the flattening line to the complex change of frequency has the form

$$\left(\frac{\Delta\omega}{\omega}\right)_{\text{sing}}\approx-\frac{C}{\eta}\left(\frac{\omega}{\mathbf{kv}_0-\omega-i\gamma}\right)^{1/2}\cdot\begin{cases} i, & \varepsilon_{xxx}(\mathbf{p}_0)<0,\\ 1, & \varepsilon_{xxx}(\mathbf{p}_0)>0,\end{cases} \tag{5}$$

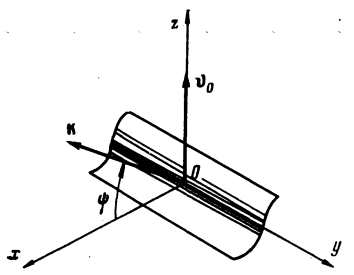

FIG. 3. Curvature lines and coordinates axes in the vicinity of a flattening line.

where (Δp_y is the length of the cylinder generatrix)

$$C=\frac{2\pi}{h^3\rho s^2}\frac{\Lambda^2(\mathbf{p}_0)\Delta p_y p_0}{v_0}\sim 1,\quad \eta=\left(\frac{p_0^2}{2s}|\varepsilon_{xxx}(\mathbf{p}_0)|\right)^{1/2}\sim\left(\frac{v}{s}\right)^{1/2}. \tag{6}$$

The expression (5) is applicable at $kl \gg 1$ and at directions of the wave vector of the sound k near resonance

$$|\psi-\psi_0|\ll 1,\quad \psi_0\approx s/v, \tag{7}$$

where $\pi/2-\psi$ is the angle between $\mathbf{k}$ and $\mathbf{v}$ (Fig. 3). The inequality (7) follows from the condition that the main contribution to the integral (2) is made by a pole.

It is seen from (5) and (1) that at $kl \gg 1$ the change in frequency connected with the flattening line $(\Delta\omega/\omega)_{\rm sing}$ significantly exceeds the contribution of the remainder of the Fermi surface and the latter contribution can be neglected in the dispersion equation (1). On the other hand, the constant C/η contains a smallness of the order $\sim(s/v)^{1/2}$. Therefore, right up to some frequency $\omega\sim\nu_{\rm eff}$ (for the definition of $\nu_{\rm eff}$ see below) $|\Delta\omega/\omega| \ll 1$ and $(\Delta\omega/\omega)_{\rm sing}$ can be regarded as a contribution to the unperturbed frequency ω_0. Then from (5) in the case $\varepsilon_{xxx}(p_0)$ we immediately obtain an expression for the absorption and the change in the sound velocity:

$$\frac{\gamma}{\omega},\frac{\Delta s}{s}\approx\frac{C}{\eta}\left[\frac{P\pm Q}{2P^2}\right]^{1/2}, \tag{8}$$

where

$$P=\left[\left(\frac{v_0}{s}\sin\psi-1\right)^2+\frac{\nu^2}{\omega^2}\right]^{1/2},\quad Q=\frac{v_0}{s}\sin\psi-1.$$

Here the upper sign corresponds to absorption $\gamma/\omega = -\mathrm{Im}(\Delta\omega/\omega)$ and the lower to dispersion $\Delta s/s = \mathrm{Re}(\Delta\omega/\omega)$.

As follows from (8), the absorption and the sound velocity has a narrow maximum in the angular dependence. Its location ψ_0 depends on the frequency: at $\omega\ll\nu$ we have $\psi_0\approx 1/kl$, for the absorption and $\psi_0\approx -1/kl$ for the velocity; at $\omega\geqslant\nu$, both for the absorption and for the dispersion, $\psi_0\approx s/v$. When account is taken of the symmetry of the Fermi surface relative to inversion transformation, γ/ω and $\Delta s/s$ have extrema also at $-\psi_0$.[7]

The absorption and the change in the sound velocity at the angle maximum increases with increase in frequency like

$$\frac{\gamma}{\omega}\bigg|_{\psi=\psi_0}\approx\frac{\Delta s}{s}\bigg|_{\psi=\psi_0}\approx\frac{C}{\eta}\left(\frac{\omega}{\nu}\right)^{1/2}. \tag{9}$$

However, at high frequencies, as will be seen below, the square-root growth of (9) gives way to a decrease in the absorption and to saturation in the dispersion. Formulas (8) and (9), which correspond to perturbation theory in $|\Delta\omega/\omega|\ll 1$, are inapplicable at high frequencies in the region of the angular resonance. In this case, the difference $\mathbf{k}\cdot\mathbf{v}-\omega$, which appears in the denominator of (2), cannot be regarded as given, but must be sought from the dispersion equation which, in the resonance approximation, has the form

$$(\omega-\omega_0)^2(kv_0\sin\psi-\omega-i\nu)=C^2\omega_0^3/4\eta^2. \tag{10}$$

The correction to the root $\omega=\omega_0$ at $C\leqslant 1$ and $\omega_0\gg\nu_{\rm eff}=\nu(\eta/C)^{2/3}$, is

$$\left(\frac{\Delta\omega}{\omega}\right)_{\psi=\psi_0}\approx\left(\frac{C}{\eta}\right)^{2/3}\left(1-i\frac{\nu_{\rm eff}}{\omega}\right) \tag{11}$$

as follows from (10). It is seen from (11) that at the frequency $\omega\sim\nu_{\rm eff}\sim\nu(v/s)^{1/3}$, the absorption and change in the sound velocity reach the maximum value $\sim(s/v)^{1/3}$; then, at $\omega\gg\nu_{\rm eff}$, the absorption falls off (Fig. 4), while the dispersion reaches saturation (Fig. 5). We note that these frequency dependences are similar to the dependences of γ/ω and $\Delta s/s$ for a finite cylindrical section where, however, the maximum values of γ/ω, $\Delta s/s\sim(S/S_F)^{2/3}$ and $\nu_{\rm eff}\sim\nu(S_F/S)^{2/3}$ were determined by the ratio of the area of the cylindrical portion S to the area S_F of the entire Fermi surface.[4] If the cylindrical section has also a flattening line, then the case in which the wave vector of the sound lies in it in a single normal plane is a special one: the flattening lines then make the same contribution to the absorption and dispersion as a finite flat section.[4]

At high frequencies, the dependence of the width of the angular resonance on the frequency also changes. At $\omega\ll\omega_{\rm eff}$ the maximum of both the absorption and the change in the sound velocity become narrower:

$$\Delta\psi\approx 1/kl. \tag{12}$$

At frequencies $\omega\gg\nu_{\rm eff}$ the width of the maxima of the dispersion does not depend on the frequency, and the width of the maxima of the absorption increases (Figs. 4 and 5):

$$\Delta\psi\approx(s/v)^2(\omega/\nu)^2 \tag{13a}$$

–absorption

$$\Delta\psi\approx(s/v)^{4/3} \tag{13b}$$

–dispersion.

As is seen from (5), in the case $\varepsilon_{xxx}(\mathbf{p}_0)>0$ the absorption and dispersion in the formulas (8)–(13) change places. The case of the vanishing of $\varepsilon_{\rm eff}(\mathbf{p}_0)$, and also of the higher derivatives, which corresponds to an inflection of $\mathbf{k}\cdot\mathbf{v}(\mathbf{p}_0)$, is not taken into account because of

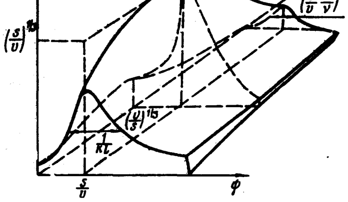

FIG. 4. Angular and frequency dependences of the sound absorption associated with the flattening line.

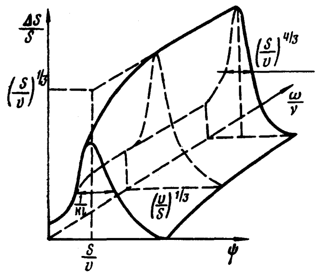

FIG. 5. Angular and frequency dependences of the dispersion connected with the flattening line.

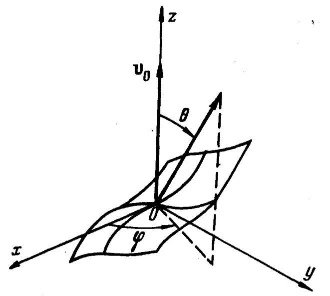

FIG. 6. Curvature lines and coordinates axes in the vicinity of a flattening point.

its lower probability. We note, however, that the singularity $(\Delta\omega/\omega)_{\rm sing}$ becomes stronger when the succeeding derivatives vanish.

2. FLATTENING POINTS

We now consider next the case in which a single zero-curvature point, a flattening point or a parabolic point, is located on the belt $\mathbf{k}\cdot\mathbf{v}=\omega$. The anisotropies of the absorption and the sound velocity dispersion turn out to be here weaker than in the case of tangency or coincidence of the zero-curvature line with the belt.

At the flattening point, we have not only zero Gaussian curvature

$$\mathcal{K}=\frac{\varepsilon_{xx}\varepsilon_{yy}-\varepsilon_{xy}^2}{v^2}, \tag{14}$$

but also zero average surface curvature

$$\mathcal{H}=\frac{1}{2v}(\varepsilon_{xx}+\varepsilon_{yy}). \tag{15}$$

(Here and below the z axis is directed along the normal to the surface at the zero-curvature point, see Fig. 6).

It follows from (14) and (15) that a flattening point is present on the surface if the conditions

$$\varepsilon_{xx}(\mathbf{p}_0)=\varepsilon_{yy}(\mathbf{p}_0)=\varepsilon_{xy}(\mathbf{p}_0)=0 \tag{16}$$

are compatible. A more realistic case is the case of a translation surface, i.e., a surface formed by a rigid-body translation of one curve along another, when the flattening point lies on a parabolic line. The equation of the translation surface does not contain terms with products of coordinates, a fact that can be expressed in the form

$$\varepsilon_{xy}=0. \tag{17}$$

With account taken of the conditions (16) and (17), the expansion of $\mathbf{k}\cdot\mathbf{v}$ in the vicinity of such a flattening point has the form

$$\mathbf{kv}=\mathbf{kv}_0+ax^2+cy^2, \tag{18}$$

where

$$a={}^1\!/_2k_x\varepsilon_{xxx}+{}^1\!/_2k_z\varepsilon_{zxx},\quad c={}^1\!/_2k_y\varepsilon_{yyy}+{}^1\!/_2k_z\varepsilon_{zyy},\quad k_x=k\sin\theta\cos\varphi,$$
$$k_y=k\sin\theta\sin\varphi,\quad k_z=k\cos\theta,\quad x=p_x-p_{x0},\quad y=p_y-p_{y0},$$

θ is the angle in the osculatory plane (Fig. 6), and φ is the angle reckoned from it in the tangent plane.

The contribution of the flattening point to the complex change in the frequency, as follows from (2) and (18), is

$$\left(\frac{\Delta\omega}{\omega}\right)_{\rm sing}\approx\frac{C}{\zeta}\ln\frac{(\mathbf{kv}_0-\omega-i\nu)^{1/2}}{(\omega d+\mathbf{kv}_0-\omega-i\nu)^{1/2}+(\omega d)^{1/2}}\cdot\begin{cases} i, & \varepsilon_{xxx}\varepsilon_{yyy}<0,\\ 1, & \varepsilon_{xxx}\varepsilon_{yyy}>0,\end{cases} \tag{19}$$

where

$$\zeta\approx(p_0^2/s)(|\varepsilon_{xxx}(\mathbf{p}_0)\varepsilon_{yyy}(\mathbf{p}_0)|)^{1/2},$$
$$C=\frac{4\pi}{h^3}\frac{\Lambda^2(\mathbf{p}_0)p_0^2}{v_0\rho s^2},\quad d\approx\frac{1}{2s}\varepsilon_{yyy}(\mathbf{p}_0)(p_{max}-p_{y0})^2. \tag{20}$$

This expression is valid in a small range of angles θ close to the resonance value $\theta_0\approx s/v$.

The constant C/ζ (20) is the order of s/v; therefore, the change in frequency connected with the flattening point, up to frequencies $\omega\sim v\nu/s$, is of the order of the contribution of the remainder of the Fermi surface. This allows us to limit ourselves in the dispersion equation (1) to the first approximation, namely, to assume $(\Delta\omega/\omega)_{\rm sing}$ to be a function of the unperturbed sound frequency.

We now consider the frequency and angular dependences of the absorption and change in the sound velocity in the case of a flattening point of the elliptical type. In the range of directions of $\mathbf{k}$ close to resonance

$$|\mathbf{kv}_0-\omega|\ll\omega d, \tag{21}$$

we have, for the absorption and dispersion, from (19) and (21),

$$\frac{\Delta s}{s}\approx\begin{cases} -\dfrac{C}{\zeta}\left(\dfrac{2\omega d}{\nu}\right)^{1/2} & \omega\ll\dfrac{\nu}{d},\\ -\dfrac{C}{2\zeta}\ln\dfrac{(4\omega d)^2}{(\mathbf{kv}_0-\omega)^2+\nu^2}, & \omega\gg\dfrac{\nu}{d},\end{cases} \tag{22}$$

$$\frac{\gamma}{\omega}\approx\frac{C}{\zeta}\,\mathrm{arctg}\left[\frac{((\mathbf{kv}_0-\omega)^2+\nu^2)^{1/2}-\mathbf{kv}_0+\omega}{((\mathbf{kv}_0-\omega)^2+\nu^2)^{1/2}+\mathbf{kv}_0-\omega}\right]^{1/2}$$
$$-\frac{C}{\zeta}\cdot\begin{cases}\dfrac{\pi}{4}-\left(\dfrac{\omega d}{2\nu}\right)^{1/2}, & \omega\ll\dfrac{\nu}{d},\\ \mathrm{arctg}\dfrac{((\mathbf{kv}_0-\omega)^2+\nu^2)^{1/2}}{\omega d}, & \omega\gg\dfrac{\nu}{d}.\end{cases} \tag{23}$$

We note that the constant d (20) is determined by the geometry of the Fermi surface and can be estimated in specific cases. We shall assume only that $v/s>d\gg1$.

As is seen from (22), the dispersion connected with a flattening point of the elliptical type at high frequencies $\omega\gg\nu/d$ has a minimum at $\theta_0\approx s/v$.[7,8] The frequency dependence of the minimum has the following form[8]:

$$\left.\frac{\Delta s}{s}\right|_{\theta=\theta_0}\approx-\frac{C}{\zeta}\ln 4d\frac{\omega}{\nu}. \tag{24}$$

At the same frequencies $\omega\gg\nu/d$ and directions of propagation k, the absorption experiences a discontinuity in the angular dependence by a finite amount C/ζ,[7,8] and depends on the frequency:

$$\frac{\gamma}{\omega}\Big|_{\theta\to\theta_0}\approx\frac{C}{2\zeta}\Big[\frac{\pi}{2}-\frac{v}{\omega d}\Big]. \qquad (25)$$

By virtue of the symmetry of the Fermi surface to inversion, the zero-curvature points as well as the zero-curvature lines occur in pairs; accordingly, the absorption and dispersion will have symmetric singularities at $\theta=-\theta_0$.[7] The region of low frequencies $\omega \ll v/d$ is less interesting, since the absorption and dispersion do not have sharp angular dependences in this region. Only the frequency dependence

$$\frac{\gamma}{\omega},\frac{\Delta s}{s}\Big|_{\theta\to\theta_0}\approx\pm\frac{C}{2\zeta}\Big(2d\frac{\omega}{v}\Big)^{1/2} \qquad (26)$$

is characteristic.

The case of points of hyperbolic type is not considered in detail, since the formulas for the frequency and angular dependences, as is seen from (18), can be obtained for them from (22)–(26) by the replacement of $-\Delta s/s$ by γ/ω and of γ/ω by $\Delta s/s$. Thus, the absorption in this case has a maximum and the dispersion experiences a discontinuity.

We note that if k lies in the osculating plane or in a place perpendicular to it ($\varphi=0, \pi/2$), the character of the singularity changes, since it is determined here not by a flattening point but by a parabolic line. For such directions of k, the zero-curvature line is a belt and makes a significantly greater contribution than the flattening point at the other directions in the tangent plane.

In the more general case, in which the Fermi surface is not a translation surface, the flattening point will be an isolated one and the expansion of $\mathbf{k}\cdot\mathbf{v}$ in its vicinity contains a term with a product of the coordinates. However, by rotation of the system of coordinates, this case reduces to the already considered case, except that here there is no preferred direction and correspondingly there is no intensification of the anisotropy in any direction of $\mathbf{k}$.

3. PARABOLIC POINTS

A parabolic point, in contrast with a flattening point, makes an anomalous contribution to the absorption and dispersion of the sound velocity only when the wave vector of the sound lies in a principal cross section corresponding to zero curvature. At a parabolic point, only the Gaussian curvature (14) vanishes, and, consequently,

$$\varepsilon_{xx}(\mathbf{p}_0)=\varepsilon_{xy}(\mathbf{p}_0)=0. \qquad (27)$$

The expansion of kv in this case contains a linear term:

$$\mathbf{kv}=\mathbf{kv}_0+Ax^2+Bxy+Dy, \qquad (28)$$

where

$$A={}^1/_2k_x\varepsilon_{xxx}+{}^1/_2k_y\varepsilon_{yxx}+{}^1/_2k_z\varepsilon_{zxx},$$
$$B=k_x\varepsilon_{xxy}+k_y\varepsilon_{yxy}+k_z\varepsilon_{zxy},\quad D=k_y\varepsilon_{yy}+k_z\varepsilon_{zy}.$$

The expansion (28) reduces by rotation through an angle $\Phi=-\frac{1}{2}\arctan(B/A)$ and by a shift of the origin of the coordinates to the form

$$\mathbf{kv}=\mathbf{kv}_0+ax^2+cy^2+(b_1^2a+b_2^2c)/4ac, \qquad (29)$$

where

$$a,c={}^1/_2[A\pm(A^2+B^2)^{1/2}],\quad b_1=D\sin\Phi,\quad b_2=D\cos\Phi.$$

At $\varphi=0$ ($D\approx0$) the expansion (29) differs from (18) only in that a and c always have different signs. It is clear that the singularities connected with the parabolic point at $\varphi=0$ reduce to the singularities considered in Sec. 2 for flattening points of the hyperbolic type, i.e., the absorption has a maximum and the dispersion has a discontinuity.

It remains to elucidate how the contributions to the absorption and sound velocity dispersion of an isolated zero-curvature point and zero-curvature line differ. For this purpose, we transform to curvilinear coordinates. If the coordinates lines $\xi=\text{const}$, $\eta=\text{const}$ are lines of curvature, then the principal curvatures of the surface are equal to

$$\mathscr{K}_1=\frac{L}{E},\quad \mathscr{K}_2=\frac{N}{G}, \qquad (30)$$

where

$$L=(\mathbf{p}_{\xi\xi},n),\quad N=(\mathbf{p}_{\eta\eta},n),\quad E=\mathbf{p}_\xi^2,\quad G=\mathbf{p}_\eta^2,$$

$\mathbf{n}$ is the normal to the surface $\mathbf{p}=\mathbf{p}(\xi,\eta)$, and $\mathbf{p}_{\xi\xi}=\partial^2\mathbf{p}/\partial\xi^2$. Since

$$L(\xi=0,\ \eta)=0 \qquad (31)$$

on the parabolic line, and consequently

$$\Big(\frac{\partial v_\xi}{\partial\xi}\Big)_{\xi=0}=0, \qquad (32)$$

the expansion in its vicinity has the form

$$\mathbf{kv}(\xi,\eta)=\mathbf{kv}(0,\eta)+{}^1/_2\mathbf{kv}_{\xi\xi}\xi^2. \qquad (33)$$

It is seen that if $\mathbf{k}\cdot\mathbf{v}(0,\eta)=\text{const}$, i.e., the normals to the surface along the parabolic line lie is one place (for example, if this line is a translation line), then it can coincide with a belt. The sound absorption and velocity dispersion will have here a square-root dependence on the frequency and on the direction of propagation, as in the case of a flattening line [see (5), (8)]. In the general case, the normals form a developable surface and there exists a cone of directions at which one of the points of the zero-curvature line falls into the belt. The expansion is carried out in the neighborhood of this line.

4. EFFECT OF POINTS OF ZERO CURVATURE ON THE TILT EFFECT IN A STRONG MAGNETIC FIELD

The points and lines of zero curvature change the character of many well known phenomena in metals. As has been noted by Avanesyan, Kaganov, and Lisovskiĭ,[7] they enhance the Kohn anomaly and change the frequencies and amplitudes of magnetoacoustic resonances. Local flattenings of the Fermi surface should manifest themselves also in the behavior of the sound absorption and velocity dispersion in a strong magnetic field ($kr \ll 1$), in particular, in the tilt effect of Reneker.[8]

For closed sections of the Fermi surface, the complex change in the sound frequency is expressed in terms of quantities averaged over the period of rotation in the magnetic field[11]:

$$\frac{\Delta\omega}{\omega}=-\frac{\omega}{\rho s^2}\frac{4\pi}{h^3}\int m^* dp_z \frac{\overline{\Lambda}^2}{k\bar{v}_z\cos\psi-\omega-i\nu}, \quad (34)$$

where p_z is the projection of the momentum of the electron on the direction of the magnetic field **H**:

$$\bar{\varphi}=\frac{1}{2\pi}\int_0^{2\pi}\varphi\,d\tau,$$

τ is the angle of rotation of the electron in the magnetic field, m^* is the cyclotron mass, and ψ is the angle between the vectors **k** and **H**.

If the field **H** is directed such that

$$\overline{(\partial v_z/\partial p_z)}_{p_z=p_{z0}}=0 \quad (35)$$

and the expansion of $\bar{v}_z$ has correspondingly the form

$$\bar{v}_z=\bar{v}_{z0}+{}^1/_2 q(p_z-p_{z0})^2, \quad q=\left(\overline{\frac{\partial^2 v_z}{\partial p_z^2}}\right)_{p_{z0}}, \quad (36)$$

where $\bar{v}_{z0}=\bar{v}_z(p_{z0})$; then the contribution of the zero-curvature line to $\Delta\omega/\omega$ will have the same strong anisotropy as when the belt coincides with the zero-curvature line in the absence of the field (Sec. 1)

$$\left(\frac{\Delta\omega}{\omega}\right)_{\rm sing}\approx\frac{C}{\eta_H}\left(\frac{\omega}{k\bar{v}_{z0}\cos\psi-\omega-i\nu}\right)^{1/2}\cdot\begin{cases} i, & q<0,\\ 1, & q>0,\end{cases} \quad (37)$$

where

$$C=\frac{4\pi}{h^3}\frac{\overline{\Lambda}^2(p_{z0})m^*(p_{z0})p_0}{\rho s^2}, \quad \eta_H=\left(\frac{p_0^2}{2s}\left(\overline{\frac{\partial^2 v_z}{\partial p_z^2}}\right)_{p_{z0}}\cos\psi\right)^{1/2},$$

$$|\cos\psi|\gg s/v.$$

The expression (37) is valid only near the pole of the integral (34), that is, at $k\bar{v}_{z0}\cos\psi\approx\omega$. This condition can be satisfied, first, if at some direction of **H** the Fermi surface has a section in which not only $\overline{\partial v_z/\partial p_z}$ but also $\bar{v}_z$ vanishes; second, at $v_z\neq 0$, because of the smallness of $\cos\psi$, i.e., of the relative orientation of **k** and **H**. The second situation, which corresponds to the tilt effect, is more realistic, since here the anisotropy connected with the local flattenings manifests itself upon satisfaction of the single condition (35) instead of two. Generally speaking, the connection between (35) and the curvature of the Fermi surface may not be simple, since the section $p_z=p_{z0}$ does not coincide with the zero-curvature line, and its curvature is equal to zero only at the points of intersection. However, it is clear from general considerations that since the zero-curvature line separates the regions of positive and negative Gaussian curvature, the average $\overline{\partial v_z/\partial p_z}$ can vanish on some section at certain directions of **H**. The simplest situation corresponds to a surface of revolution, where, in the case of **H** parallel to the axis, the sections $p_z=$ const and $\tau=$ const coincide with the curvature lines. In this case, if the section $p_z=p_{z0}$ is a line of zero curvature, then $(\partial v_z/\partial p_z)_{p_{z0}}=0$ at each point and thus (35) is satisfied automatically. In all these situations, the expansion has the form (36), and the angular and frequency dependences of the sound absorption and velocity are described by the formula (37). It is seen that the quantities γ/ω and $\Delta s/s$ at the angular maximum increase with frequency according to the square root law:

$$(\gamma/\omega)_{max}\approx(\Delta s/s)_{max}\approx C(\omega/\nu)^{1/2}, \quad (38)$$

while in the tilt effect for an isotropic Fermi surface[12] the dependences, as is well known are the following:

$$\left(\frac{\gamma}{\omega}\right)_{max}\approx C\,\mathrm{arctg}\,2\frac{\omega}{\nu},\qquad \left(\frac{\Delta s}{s}\right)_{max}\approx\frac{C}{2}\ln\left(4\frac{\omega^2}{\nu^2}+1\right). \quad (39)$$

Thus the presence of lines of zero curvature on the Fermi surface leads to a steeper frequency dependence of the absorption and dispersion and, consequently to an increase in the amplitude of the tilt effect in certain directions of propagation and of the field. Consequently, local flattenings can manifest themselves in the specific anisotropy of the tilt effect.

1)The role of flat and cylindrical portions of the Fermi surface in electron-phonon interaction, and in particular in the enhancement of the Kohn singularity, was first discussed by Afanas'ev and Kagan.[3]

2)We note that for finite sections of zero curvature, the introduction of the relaxation time is not justified.[10] However, it is possible here since the contribution to the interaction with sound is made only by a small region of p space—a belt, just as for the ordinary high-frequency absorption.

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