Paramagnetic effect of the two-dimensional "mixed" state in type I superconductors

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The magnetic properties of the two-dimensional "mixed" state (the Tm state) that appears in multiply connected type I superconductors carrying a strong current are studied on the basis of the linear, time-dependent Ginzburg-Landau equations. It is shown that the destruction of the TM state in a hollow cylindrical specimen occurs near a critical current which depends on the external magnetic field. The TM state at the inner surface of the sample exhibits a paramagnetic effect. When the TM state is moved to the outer surface the paramagnetic response changes to a diamagnetic one.

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1. INTRODUCTION

In 1943, Steiner and Schöneck[1] first observed the paramagnetic effect in current-carrying superconductors. They established the fact that the mean magnetic permeability of a cylindrical sample exceeds unity if the superconductivity in the cylinder is partially destroyed because of current in the presence of a longitudinal magnetic field. In 1956, a similar phenomenon was observed in hollow cylinders by Meissner.[2] Special interest attaches to the investigation of a multiply connected sample in connection with the appearance in similar systems of the so-called two-dimensional "mixed" state. For an understanding of the significance of this new type of state, we consider various states that arise in a current-carrying hollow cylinder in the case of different values of the flowing current and of the magnetic field intensity.

The system is in the superconducting state if the current is less than the first critical current J_{c1}, which is determined from the condition that the sum of the inherent magnetic field on the external surface and the external magnetic field H_a is equal to the critical magnetic field H_c, i.e.,

$$J_{c1}=(r_2c_0H_c/2)(1-H_a^2/H_c^2)^{1/2},$$

where c_0 is the speed of sound, r_2 is the external radius of the cylinder.

If the flowing current becomes greater than J_{c1}, the superconductivity is partially destroyed. In this case, a normal layer surrounds the central region, which is in the so-called intermediate state, comprising a system of alternating normal and superconducting domains, the thickness of which amounts to about 10^{-12} cm.[3] Meissner[4] proposed a model of the intermediate state in the presence of a longitudinal field, and attributed the paramagnetic effect with the existence of helical currents. An attempt to explain these currents within the framework of the theory of the intermediate state of London-Andreev[3,5] encounters serious difficulties. Rothen[6] has shown that a stationary paramagnetic solution of the London-Andreev equations does not exist and that a generally periodic solution is unstable.

If the flowing current exceeds the value $J_t = J_{c1}(r_1^2 + r_2^2)/2r_1r_2$, the radius of the region of the intermediate state becomes smaller than the inner radius of the sample r_1, so that there is actually no region of the intermediate state. On the other hand, the system cannot be in a purely normal state, since the magnetic field created by the current near the inner surface of the sample in smaller than the critical value H_c. And, finally, a purely superconducting external layer cannot be stable, since the electric field must exist right up to the inner surface because of the finite conductivity of the sample and the continuity of the electric field at the boundary between the phases. Therefore, even back in 1938, L. D. Landau (private communication to D. Shoenberg, see Ref. 7) came to the conclusion that the inner surface of the hollow cylinder is covered by a layer which is in the "mixed" state, in which superconductivity and an electric field coexist.

Finally, if the current increases so that the thickness of the layer at the inner surface of the hollow cylinder, in which the magnetic field of the current is less than critical, becomes smaller than the superconducting coherence length, then the probability of the appearance of superconductivity becomes extremely small. The corresponding characteristic current $J_{c2}(H_a)$ is connected with the transition of the sample to the normal state.

A purely "mixed" state in the absence of an intermediate state, which is the subject of our research, was discovered experimentally and studied by I. Landau and Sharvin in 1969.[8,9] They called the "mixed" state two-dimensional (TM state), since the radial dimensions of the samples were much greater than the thickness of the layer in the "mixed" state. Moreover, in their experiments, these authors found the conditions under which the TM state arises, not only on the inner surface of the hollow cylinder, but also inside the sample and on the external surface.[9] In later studies, I. Landau[10,11] observed the paramagnetic effect in the system in the presence of an external longitudinal magnetic field (i.e., the magnetic field in the opening of the sample becomes greater than the applied field). Upon dis-

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placement of the TM state inside the sample, the paramagnetic effect decreases and finally disappears at some distance from the surface. Near the outer surface, the TM state becomes diamagnetic. This effect becomes very noticeable if the center of the TM state is located on the external surface of the cylinder.

The purpose of the present work is the explanation of the magnetic properties of the TM state within the framework of linear nonstationary theory of Ginzburg-Landau. In contrast to the situation described above with paramagnetism of the intermediate state, the problem of the paramagnetic effect of the TM state can be solved exactly. The model of Andreev *et al.*[12-14] serves as the basis of the present theory. In this model, the TM state is considered as a dynamical system of superconducting fluctuations localized near the inner surface, where the magnetic field is less than critical. The behavior of the fluctuations in time is determined by the electric field.

In Sec. 2, we formulate the considered model and obtain general expressions for the fluctuating currents. Section 3 is devoted to the calculation of the fluctuation spectrum and of the second critical current $J_{c2}(H_a)$. In Sec. 4, the longitudinal and azimuthal fluctuation currents which determine the conductivity and magnetic properties of the system are given as functions of the flowing current and of the external magnetic field for different locations of the TM state in the sample. It turns out that the TM state is paramagnetic on the inner surface and diamagnetic on the outer surface. It is shown that the linear theory is applicable only in a small vicinity of the critical current $J_{c2}(H_a)$. The width of this vicinity is smaller the purer the sample. Section 5 is devoted to a discussion of the results, in particular, to the possibility of the appearance of hysteresis of the paramagnetic effect.

2. THE GENERAL PREMISES

We consider a hollow cylindrical type I superconductor with an inner radius r_1 and outer radius r_2, along which passes an electric current J and to which is applied a longitudinal magnetic field H_a. In this case, the region in which the total magnetic field is less than the critical field is located close to the inner surface of the sample. In order to shift this region to the inside of the sample, I. Landau and Sharvin placed a wire in the opening of the cylinder, along which passed a current J_0 in the direction opposite that of the current of the sample. Then the region in which the total magnetic field of the two currents J_0 and J is less than H_c can be localized at any distance from the axis of the cylinder, depending on the current in the central wire.

Since we shall use the linear Ginzburg-Landau theory, we must limit our consideration to the case in which the fluctuation superconductivity current I_s of the layer, which is in the TM state, is much smaller than the normal current I_n in this layer:

$$I_s \ll I_n. \tag{1}$$

In this case, the magnetic field $H(r)$ in the sample is determined basically by the currents J and J_0:

$$H(r)=(2/c_0 r)\left[J(r^2-r_1^2)/(r_2^2-r_1^2)-J_0\right].$$

We denote by r_0 the radius of the cylindrical surface on which the magnetic field is equal to zero: $H(r_0)=0$. Then r_0 is determined by the relation

$$r_0=\left[r_1^2+J_0(r_2^2-r_1^2)/J\right]^{1/2}. \tag{2}$$

We further assume that the thickness of the TM state is much less than r_0. In this case it is convenient to use a local Cartesian coordinate system. The z axis here coincides with the axis of the sample, the y axis is parallel to the layer and the x axis is perpendicular to the layer:

$$x=r-r_0.$$

In the approximation linear in the small ratio x/r_0, the magnetic field intensity is equal to

$$H(x)=4Jx/c_0(r_2^2-r_1^2).$$

The vector potential in the presence of such a magnetic field and of a simultaneously applied electric field is given by the formulas

$$A_x=0, \quad A_y=H_a x, \quad A_z=-2Jx^2/c_0(r_2^2-r_1^2)-c_0 Et. \tag{3}$$

We introduce a thermal random force into the equation of motion for the superconducting order parameter[15,16]

$$\partial\psi/\partial t-\nu\{\psi+\xi^2(\nabla-2ie\mathbf{A}/c_0)^2\psi\}=f(\mathbf{r},t), \tag{4}$$

where $\xi=\xi(T)$ is the coherence length, $\nu=8(T_c-T)/\pi$, and $f(\mathbf{r},t)$ is a Gaussian random force which satisfies the condition

$$\langle f(\mathbf{r},t)f^+(\mathbf{r}',t')\rangle=4mT\nu\xi^2\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'); \tag{5}$$

here m is the mass of the electron.

The superconducting current density is determined by the second equation of the Ginzburg-Landau theory:

$$\mathbf{j}=(2e/m)\,\mathrm{Im}\left[\psi^+(\nabla-2ie\mathbf{A}/c_0)\psi\right]. \tag{6}$$

It is convenient to introduce the following dimensionless quantities:

$$\tilde{\mathbf{r}}=\frac{\mathbf{r}}{\xi}, \quad \tilde{t}=\nu t, \quad \tilde{f}=\frac{1}{2\nu}\left(\frac{\xi}{mT}\right)^{1/2}f, \quad \tilde{\psi}=\frac{1}{2}\left(\frac{\xi}{mT}\right)^{1/2}\psi.$$

Then Eqs. (4) and (5) take on the form

$$\partial\tilde{\psi}/\partial\tilde{t}-\tilde{\psi}+L\tilde{\psi}=\tilde{f}(\tilde{\mathbf{r}},\tilde{t}), \tag{7}$$

where $L=-\partial^2/\partial\tilde{x}^2-(\partial/\partial y-i\gamma\tilde{x})^2-(\partial/\partial\tilde{z}+i\varepsilon\tilde{t}=i\beta\tilde{x}^2)^2$,

$$\langle\tilde{f}(\tilde{\mathbf{r}},\tilde{t})f^+(\tilde{\mathbf{r}}',\tilde{t}')\rangle=\delta(\tilde{\mathbf{r}}-\tilde{\mathbf{r}}')\delta(\tilde{t}-\tilde{t}'), \tag{8}$$

$$\langle j_{\tilde{z}}\rangle=(8eT/\xi^2)\,\mathrm{Im}\,\langle\tilde{\psi}^+(\partial/\partial\tilde{z}+i\varepsilon\tilde{t}+i\beta\tilde{x}^2)\tilde{\psi}\rangle, \tag{9}$$

$$\langle j_{\tilde{y}}\rangle=(8eT/\xi^2)\,\mathrm{Im}\,\langle\tilde{\psi}^+(\partial/\partial\tilde{y}-i\gamma\tilde{x})\tilde{\psi}\rangle. \tag{10}$$

The dimensionless parameters $\varepsilon, \beta, \gamma$ are equal to

$$\varepsilon=2eE\xi/\nu, \quad \beta=4eJ\xi^3/(r_2^2-r_1^2)c_0^2, \quad \gamma=2e\xi^2H_a/c_0.$$

In what follows, we use dimensionless quantities only and omit the tilde everywhere. We shall seek a solution of Eq. (7) in the form

$$\psi=\sum_{k_y,k_z,n}a^{(n)}_{k_y k_z}(t)\,\psi^{(n)}_{k_y k_z}(\mathbf{r},t), \tag{11}$$

where

$$\psi^{(n)}_{k_y k_z}(t)=(1/l)\exp\{ik_y y+ik_z z\}\,\Phi^{(n)}_{k_y k}(x), \tag{12}$$

$k=k_z+\varepsilon t$, and is the normalized length.

The function $\psi^{(n)}_{k_y k_z}(t)$ is an eigenfunction of the operator L:

$$L\psi^{(n)}_{k_y k_z}(t)=\lambda^{(n)}_{k_y k}\psi^{(n)}_{k_y k_z}(t). \tag{13}$$

The functions $\Phi^{(n)}_{k_y k}(t)$ form an orthogonal set:

$$\int_{-\infty}^{+\infty}\Phi^{(n)}_{k_y k}(x)\Phi^{(m)}_{k_y k}(x)\,dx=\delta_{nm}. \tag{14}$$

The condition (13) is equivalent to the following equation for $\Phi^{(n)}_{k_y k}(x)$:

$$d^2\Phi^{(n)}_{k_y k}(x)/dx^2+[\lambda^{(n)}_{k_y k}-(k_y-\gamma x)^2-(k+\beta x^2)^2]\Phi^{(n)}_{k_y k}(x)=0. \tag{15}$$

We assume that the electrical energy $2eE\varepsilon$ is small in comparison with the relaxation frequency ν:

$$\varepsilon\ll 1. \tag{16}$$

After substitution of the formula (11) in Eq. (7) and a series of simple transformations, we obtain equations for the time dependent coefficients $a^{(n)}_{k_y k_z}(t)$, which we can easily solve.[12] The solution has the form

$$a^{(n)}_{k_y k_z}(t)=\int_{-\infty}^{t}\exp\left\{\int_{t'}^{t}(1-\lambda^{(n)}_{k_y k''})\,dt''\right\}$$
$$\times\frac{1}{l}\int dV\exp\{-ik_y y-ik_z z\}\Phi^{(n)}_{k_y k}(x)f(\mathbf{r},t')\,dt'. \tag{17}$$

The expressions for the current (9) and (10), with account of (11) and (17), take the form

$$\langle j_y\rangle=\frac{2eT}{\xi^2\pi^2\varepsilon}\sum_{n=0}^{\infty}\iint_{-\infty}^{+\infty}dk_y dk_z(k_y-\gamma x)\,|\Phi^{(n)}_{k_y k}(x)|^2$$
$$\times\int_0^\infty d\tau\exp\left\{\frac{2}{\varepsilon}\int_{k-\tau}^{k}(1-\lambda^{(n)}_{k_y k''})\,dk''\right\}, \tag{18}$$

$$\langle j_z\rangle=\frac{2eT}{\xi^2\pi^2\varepsilon}\sum_{n=0}^{\infty}\iint_{-\infty}^{+\infty}dk_y dk_z(k+\beta x^2)\,|\Phi^{(n)}_{k_y k}(x)|^2$$
$$\times\int_0^\infty d\tau\exp\left\{\frac{2}{\varepsilon}\int_{k-\tau}^{k}(1-\lambda^{(n)}_{k_y k''})\,dk''\right\}, \tag{19}$$

where $\tau=\varepsilon(t-t')$.

The complete fluctuation current of the TM state is obtained by integration of (18) and (19) with respect to ξdx:

$$I_y=\frac{eT}{\xi\pi^2\varepsilon}\sum_{n=0}^{\infty}\iint_{-\infty}^{\infty}dk_y dk\frac{\partial\lambda^{(n)}_{k_y k}}{\partial k_y}\int_0^\infty d\tau\exp\left\{\frac{2}{\varepsilon}\int_{k-\tau}^{k}(1-\lambda^{(n)}_{k_y k''})\,dk''\right\}, \tag{20}$$

$$I_z=\frac{eT}{\xi\pi^2\varepsilon}\sum_{n=0}^{\infty}\iint_{-\infty}^{\infty}dk_y dk\frac{\partial\lambda_{k_y k}}{\partial k}\int_0^\infty d\tau\exp\left\{\frac{2}{\varepsilon}\int_{k-\tau}^{k}(1-\lambda^{(n)}_{k_y k''})\,dk''\right\}. \tag{21}$$

3. SPECTRUM AND SECOND CRITICAL CURRENT

In order to calculate the fluctuation currents (20) and (21), it is first necessary to determine the spectrum of eigenvalues $\lambda^{(n)}_{k_y k}$. This can be done by generalization of the method described in Ref. 12 for the particular case $H_a=0$ and $J\approx J_{c2}$. The quantity J_{c2} is the so-called second critical current, first introduced and calculated by Andreev.[17] It is defined as the maximum current at which the superconducting fluctuations that increase with time still exist, i.e., at $J<J_{c2}$ there exist such n, k and k_y at which $\lambda^{(n)}_{k_y k}<1$. At $J>J_{c2}$ the opposite inequality $\lambda^{(n)}_{k_y k}>1$ is satisfied for arbitrary n, k and k_y. Physically, J_{c2} is the value of the current near which the sample becomes normal. The calculations carried out in Ref. 12 show that account of the eigenvalues $\lambda^{(n)}_{k_y k}$ with $n\neq 0$ leads only to an insignificant contribution to the current. It is quite evident that the destruction of the superconductivity in the presence of an additional longitudinal magnetic field H_a will take place at smaller currents. Thus J_{c2} should decrease with increase in H_a.

Assume that $(J_{c2}(H_{ac}),H_{ac})$ corresponds to some point on an as yet unknown critical curve. Then the minimum of $\lambda^{(0)}_{k_y k}$, which is reached at $k=k_m$ and $k_y=k_{ym}$, is equal to unity. Because of this, we can expand $\lambda^{(0)}_{k_y k}$ for the case $J\approx J_{c2}$, $H_a\approx H_{ac}$, $k\approx k_m$ and $k_y\approx k_{ym}$ in the following fashion:

$$\lambda^{(0)}_{kk_y}=1+a_1\frac{J-J_{c2}}{J_{c2}}+a_2\frac{H_a-H_{ac}}{H_{ac}}+\frac{b}{2}(k-k_m)^2$$
$$+c(k_y-k_{ym})(k-k_m)+\frac{d}{2}(k_y-k_{ym})^2. \tag{22}$$

We shall determine the coefficients a_1, a_2, b, c, d by a variational method. As the test function we can choose a simple expression that depends on whether the TM state is located

1) on the inner surface ($r_0=r_1$),

2) on the outer surface ($r_0=r_2$),

3) completely inside the sample ($|r_0-r_1|>D$, $|r_0-r_2|>D$) (D is the thickness of the TM state). This expression, which satisfies the boundary condition $d\Phi/dx=0$, has the form

$$\Phi^{(0)}_{kk_y}(x)=N(8\alpha/\pi)^{1/4}\exp(-\alpha x^2), \tag{23}$$

where $N=1$ for the cases 1) and 2) and $N=2^{-1/2}$ for the case 3). In correspondence with this expression, the thickness of the TM state can be determined as $D=\alpha^{-1/2}$, i.e., $D=\xi\alpha^{-1/2}$ in ordinary units.

Multiplying (15) by the quantity $\Phi^{(0)}_{kk_y}(x)$, which is specified by Eq. (24), and integrating the resultant equation with respect to x in the limits

I) from 0 to ∞, II) from $-\infty$ to 0, III) from $-\infty$ to ∞,

we obtain

$$\lambda^{(0)}_{kk_y}=k_y^2-2\mu\gamma k_y/(2\pi\alpha)^{1/2}+\gamma^2/4\alpha+\alpha+k^2+k\beta/2\alpha+3\beta^2/16\alpha^2, \tag{24}$$

where $\mu=1$ in the case I, $\mu=-1$ in case II and $\mu=0$ in case III. Minimizing $\lambda^{(0)}_{kk_y}$ with respect to α, k and k_y, we obtain

$$k_m=-\beta/4\alpha(k_m,k_{ym}),\quad k_{ym}=\mu\gamma/[2\pi\alpha(k_m,k_{ym})]^{1/2},$$

where $\alpha(k_m,k_y)$ is given by

$$\alpha^3(k_m,k_{ym})-A\gamma^2\alpha(k_m,k_{ym})-\beta^2/4=0,$$
$$A=1/4-\mu^2/2\pi.$$

The value of the parameter β corresponding to the second critical current is determined by the condition

$$\lambda^{(0)}_{k_m k_{ym}}=1, \tag{25}$$

whence we have

$$\beta_{c2}=2(^2/_3)^{3/4}[^1/_2-(^3/_2)^2A\gamma^2+^1/_2(1-3A\gamma^2)^{3/2}]^{1/2}. \tag{26}$$

The critical curve is given by the equation

$$J_{c2}(H_a)=\beta_{c2}(\gamma)c_0^2(r_2^2-r_1^2)/4e\xi^3, \tag{27}$$

where

$$\gamma=2e\xi^2H_a/c_0=H_a/2^{1/2}\varkappa H_c=H_a/H_{c2}. \tag{28}$$

The results of the numerical calculation of the critical curves (26) and (27) are shown in Fig. 1.

We now expand $\lambda^{(0)}_{kk_y}$ at some point on the critical curve in a series up to first order in $(\beta-\beta_{c2})/\beta_{c2}=(J-J_{c2})/J_{c2}$ and $(\gamma-\gamma_c)/\gamma_c=(H_a-H_{ac})/H_{ac}$ and up to second order in $k-k_m$ and k_y-k_{ym}, taking it into account that α is a function of k and k_y and is determined by minimizing $\lambda^{(0)}_{k_yk}$ only with respect to α. Equating the result with expression (23), we get

$$a_1=\beta_{c2}/4\alpha_m^2,\quad a_2=2A\gamma_c^2/\alpha_m, \tag{29}$$

$$b=2-\frac{2}{7}\left[1+\alpha_m\left(\frac{\gamma_c}{\beta_{c2}}\right)^2\left(\frac{4}{7}-\frac{6\mu^2}{7\pi}\right)\right]^{-1}, \tag{30}$$

$$c=\mu\gamma_c\beta_{c2}\left(\frac{\alpha_m}{2\pi}\right)^{1/2}\left[\frac{7}{4}\beta_{c2}^2+\alpha_m\gamma_c^2\left(1-\frac{3}{2\pi}\right)\right]^{-1}, \tag{31}$$

$$d=2-\mu^2\left[7\alpha_m\left(\frac{\pi}{2}-1+\frac{21\pi}{8}\left(\frac{\beta_{c2}}{\gamma_c}\right)^2\right)+2-\frac{3\pi}{4}\right]^{-1}, \tag{32}$$

where

$$\alpha_m=\alpha(k_m,k_{ym}),\ \beta_{c2}(\gamma_c)=A\gamma_c^2+(A^2\gamma_c^4+3\beta_{c2}^2/8)^{1/2},\quad A=1/4-\mu^2/2\pi. \tag{33}$$

4. RESULTS AND LIMITS OF APPLICABILITY

By determining the spectrum of the eigenvalues $\lambda^{(0)}_{kk_y}$ we can proceed to the calculation of the fluctuation currents. Substituting (22) in (20) and (21), we obtain, after simple integration with respect to k'', k and k_y:

$$I_y=\frac{c}{2\pi(db-c^2)^{1/2}}\frac{eT}{\xi}\int_0^\infty d\tau\exp\left\{\frac{2\Delta\tau}{\varepsilon}-\frac{b\tau^3}{12\varepsilon}\right\}, \tag{34}$$

$$I_z=\frac{b}{2\pi(db-c^2)^{1/2}}\frac{eT}{\xi}\int_0^\infty d\tau\exp\left\{\frac{2\Delta\tau}{\varepsilon}-\frac{b\tau^3}{12\varepsilon}\right\}, \tag{35}$$

where

$$\Delta=a_1(J_{c2}-J)/J_{c2}+a_2(H_{ac}-H_a)/H_{ac}. \tag{36}$$

The expansion coefficients a_1 and a_2 are shown in Fig. 2 for all points of the critical curve.

We note that the relation between the densities of the azimuthal and longitudinal fluctuation currents is given

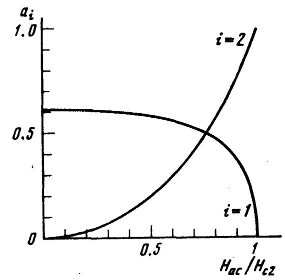

FIG. 2. Expansion coefficients a_1 and a_2 of the spectrum of the inner TM state. The corresponding coefficients for the surface TM states are obtained by shifting the abscissa by an amount $(1-2/\pi)^{1/2}H_{ac}/H_{c2}\equiv H_{ac}/H_{c3}$.

by the simple relation

$$bI_y=cI_z. \tag{37}$$

In correspondence with Eq. (31), the ratio I_y/I_z is equal to zero in the case of an internal ($\mu=0$) TM state. In this case the azimuthal current is absent. The ratio I_y/I_z has equal absolute values but opposite signs for the TM states on the outer ($\mu=1$) and inner ($\mu=-1$) surfaces. Figure 3 shows the dependence of I_y/I_z on the external field for the case $\mu=1$:

By virtue of (37) it suffices to obtain the explicit expression only for I_z. The value of the integral in (35) depends to a significant degree on the relative values of Δ and ε and on the sign of Δ. At $\Delta\gg\varepsilon^{2/3}$; the integral can be calculated by the saddle-point method; we get

$$I_z=\frac{b}{\pi^{1/2}(bd-c^2)^{1/2}(8b)^{1/4}}\frac{\varepsilon^{1/2}eT}{\xi\Delta^{1/4}}\exp\left\{\frac{8}{3}\left(\frac{2}{b}\right)^{1/2}\frac{\Delta^{3/2}}{\varepsilon}\right\}. \tag{38}$$

The numerical calculation of the coefficient of the exponential shows that it is approximately constant along the entire critical curve:

$$b/\pi^{1/2}(bd-c^2)^{1/2}(8b)^{1/4}=0.30\pm0.03.$$

The function $\eta_\mu=(8/3)(2/b)^{1/2}$ in the exponential is shown in Fig. 4.

At $|\Delta|\ll\varepsilon^{2/3}$, we can neglect the first term in the argument of the exponential in (35). We obtain

$$I_z=\frac{b^{2/3}}{(bd-c^2)^{1/2}}\frac{\Gamma(1/3)(3/2)^{1/6}}{3\pi}\frac{eT}{\xi}\varepsilon^{1/3}, \tag{39}$$

where

$$[b^{2/3}/(bd-c^2)^{1/2}][\Gamma(1/3)(3/2)^{1/6}/3\pi]=0.28\pm0.03$$

on the critical curve.

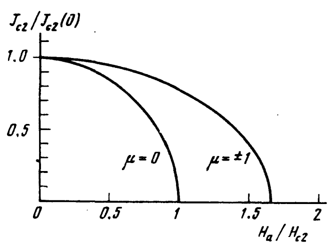

FIG. 1. Second critical current of the TM state on the inner surface ($\mu=1$), on the outer surface ($\mu=-1$) and inside the sample ($\mu=0$).

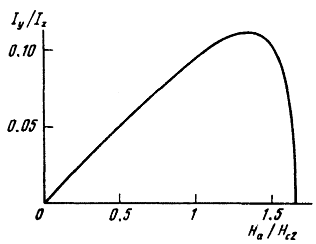

FIG. 3. Ratio of the density of the azimuthal current to the density of the longitudinal current for the TM state on the inner surface as a function of the external magnetic field.

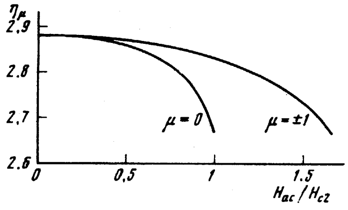

FIG. 4. Fluctuation current (with exponential accuracy) below the critical curve for surface TM states ($\mu=\pm1$) and the interior TM state ($\mu=0$).

At $-\Delta \gg \varepsilon^{2/3}$, we can neglect the second term in the exponential of the expression (35). In this case, the fluctuation current is given by the formula

$$I_z=\frac{b}{4\pi(bd-c^2)^{1/2}}\frac{eT}{\xi}\frac{\varepsilon}{\Delta}, \tag{40}$$

where on the critical curve we have

$$b/4\pi(bd-c^2)^{1/2}=0.083\pm0.009.$$

The physical characteristics measured experimentally are the effective resistance of the sample R and the magnetic field intensity $H_i=H_a+H_s$ in the opening of the hollow cylinder. The quantities R and H_s are connected with the fluctuation currents in the following way:

$$R=V/J=V/(J_n+2\pi r_0 I_z)\cong R_n(1-2\pi r_0 I_z/J), \tag{41}$$

$$H_s=(4\pi/c_0)I_\varphi=(4\pi c/bc_0)I_z, \tag{42}$$

where $R=V/J_n$ is the normal resistance of the sample.

The limits of applicability of the results are limited by the condition (1), i.e., the linear theory is applicable only in the case in which the normal current I_n in the TM layer is greater than the longitudinal fluctuation current I_s. Near the second critical current, the normal current is equal, in order of magnitude, to

$$I_n\sim\frac{2J_{c2}r_0D}{r_2^2-r_1^2}\sim\frac{J_{c2}}{J_{c2}(0)}\frac{r_0c_0^2}{\xi^2 e}. \tag{43}$$

The fluctuation current in the immediate vicinity of the critical curve is given by Eqs. (39) and (41). Substituting (39), (41), (43) in (1), we obtain the limits of applicability of our theory:

$$J_{c2}/J_{c2}(0)\gg(\xi/\xi_0)^{3/2}(a_0/\lambda_0)^3\varkappa(\xi_0/l)^{1/2}, \tag{44}$$

where Δ_0, λ_0 are the energy gap and the London penetration depth for $T=0$; $\xi_0=\hbar v_f/\pi\Delta_0$; $a_0\sim\hbar/p_f$ is the interatomic distance; l is the mean free path of the electrons (in our units, $\hbar=1$).

In the derivation of Eq. (44), we have used the relations

$$c_0/e\xi^2=2^{3/2}\varkappa H_c,\qquad eT/\xi=1.2c_0H_c\varkappa(\xi/\xi_0)(a_0/\lambda_0)^2$$

and the fact that ε can be expressed in terms of β and of the normal conductivity of the metal σ in the following way:

$$\varepsilon=\beta c^2/16\sigma(T_c-T)\xi^2. \tag{45}$$

We note that the condition (44) is satisfied along the entire critical curve, with the exception of a small region near the H_a axis, which is smaller the purer the sample. The region of applicability inside the critical curve can be determined in the following way. On the boundary of the region of applicability of the theory, the quantities I_n and I_s are of the same order:

$$I_n\sim I_s. \tag{46}$$

Substituting (43) and the expressions (38) and (41) for the current near the critical curve ($\Delta\gg\varepsilon^{2/3}$) in (46), we obtain

$$\Delta\cong2\varkappa^{4/3}\left(\frac{\xi_0}{l}\right)^{2/3}\left(\frac{J_{c2}}{J_{c2}(0)}\right)^{2/3}\times\left[\ln\left(\frac{1}{\varkappa}\left(\frac{l}{\xi_0}\right)^{1/2}\frac{\xi_0}{\xi}\left(\frac{\lambda_0}{a_0}\right)^2\left(\frac{J_{c2}}{J_{c2}(0)}\right)^{1/2}\right)\right]^{2/3}, \tag{47}$$

i.e., the width of the transition Δ is smaller the purer the sample and the larger the external field H_a.

5. CONCLUSION

The calculations that have been carried out show that the presence of an external magnetic field changes the properties of the TM state in several respects. The first rather obvious consequence is the existence of a critical curve that reflects the decrease in the second critical current as a function of the applied magnetic field (Fig. 1). It was noted that the second critical current of the internal TM state ($\mu=0$) is less than the current of the TM state on the surface ($\mu=\pm1$). In the limit, as the current approaches zero, the critical magnetic fields are determined by the relations $H_{ac}=2^{1/2}\varkappa H_c=H_{c2}$ for the inner TM state and $H_{ac}=[2/(1-2\pi)]^{1/2}\varkappa H_c=H_{c3}$ for the surface TM states; here H_{c2} and H_{c3} are the so-called second and third critical magnetic fields known from the theory of type-II superconductors. It must be noted, however, that the inner TM state cannot be achieved under this condition for type-I superconductors, for the reasons discussed in Sec. 4.

Another interesting manifestation of the external field is the formation of helical fluctuation currents. The sign of the rotation of the total helical current depends on the location of the TM state in the sample is such that the TM state on the inner surface gives a paramagnetic effect ($\operatorname{sign}H_s=\operatorname{sign}c=\operatorname{sign}\mu=1$) and the diamagnetic effect ($\operatorname{sign}H_s=-1$) on the outer surface. [In a previous paper[14] the author calculated the paramagnetic effect of the TM state on the inner surface for type-I superconducting alloys ($\varkappa\sim1$, $l\sim\xi_0$) for the limiting case $H_a/H_c\ll J/J_{c2}<1$.] The inner TM state does not give a magnetic effect. In this respect the present theory confirms the following conclusion, made by I. Landau on the basis of his experiments: The TM state consists of two parts separated by a central bounding surface ($r=r_0$) on which the circular magnetic field $H(r)$ of the currents I and I_0 vanishes—$H(r)$ has opposite directions in these two parts. The sign of the magnetic effect relative to the external field is determined by the sign of $H(r)$, so that effects of the two halves of the TM state have opposite directions. Since we neglect the effect of curvature and consider the TM layer as plane, the effect is identical in magnitude for both parts of the layer. If the TM state is entirely concentrated inside the sample, then the effects of the two halves compensate one another, so that the magnetic effect is lacking. If one of the halves is partially or completely intersected by the surface, then the contributions to the magnetic effect from both halves no longer compensate each other, and the total magnetic effect is different from zero.

The features of the transition of the TM state from the superconducting to the normal state, such as, for example, the appearance of hysteresis in the volt-ampere characteristic, were discussed in Ref. 12 for the case $H_a=0$. These considerations can easily be extended to the case of a non-zero magnetic field, and therefore there is no necessity of repeating it here. A new type of instability in the presence of a magnetic field is the hysteresis in the paramagnetic effect. In the experiment, the field $H_i+H_a+H_s$ inside the hollow was measured as a function of the applied field H_a. It was observed[11] that the hysteresis sets in at $H_i \sim H_{ac}$. The hysteresis-induced decreasing portion of the curve $H_i(H_a)$ can be explained on the basis of our results.

We now show that the derivative $\partial H_i/\partial H_a$ becomes negative under certain conditions. In correspondence with (38) and (42), the quantity $\partial H_i/\partial H_a$ is given by

$$\frac{\partial H_i}{\partial H_a}=1-\frac{6\pi n_1 a_2}{\varepsilon^{2/3} c_0 H_{ac}}\left(\frac{\Delta}{\varepsilon^{2/3}}\right)^{1/2} I_y. \tag{48}$$

If we substitute formula (37) and the microscopic expression for ε in (48), we get

$$\frac{\partial H_i}{\partial H_a}=1-\left\{f(\gamma)\left(\frac{\Delta}{\varepsilon^{2/3}}\right)^{1/2}\left(\frac{l}{\varkappa^2\xi_0}\right)^{2/3}\right\}\frac{I_s}{I_n},$$

where

$$f(\gamma)=0.732(a_2 n_1/\alpha^{1/2}\gamma)(I_y/I_z)(J_{c2}/J_{c2}(0)),$$
$$I_s=2\pi r_0 I_z.$$

The coefficient $(l/\varkappa^2\xi_0)^{2/3}$ in the curly brackets is much greater than the corresponding coefficient for pure samples, and $(\Delta/\varepsilon^{2/3})^{1/2}$ is also large. Numerical calculations of $f(\gamma)$ show that this function is equal to zero for $\gamma=0$ and $\gamma=H_{c3}/H_{c2}$ and is of the order of 10^{-1} for the middle portion of the critical curve ($f(\gamma)>0.1$ at $0.8<\gamma<1.4$). In this region, the expression in the curly brackets can be appreciably greater than unity. If we decrease the applied magnetic field, beginning with values greater than H_{ac}, the derivative $\partial H_i/\partial H_a$ becomes negative, when the ratio I_s/I_n exceeds the reciprocal of the expression in curly brackets. This phenomenon manifests itself more rapidly and in more explicit form the purer the sample.

Quantitative comparison of our results with experimental data requires a more detailed experimental study of the magnetic effect in the immediate vicinity of the second critical current. In his experiments, I. Landau[11] spanned the entire range of values of the external magnetic field H_a at a fixed current J; however, these experiments did not give sufficiently detailed information for a quantitative comparison in the small transition region where the linear theory is applicable. The paramagnetic response of the TM state on the inner surface and the diamagnetic response on the outer surface agree qualitatively with the results of I. Landau. Typical phenomena such as, for example, the decreasing portion, due to hysteresis, of the curve $H_i(H_a)$ have received excellent theoretical confirmation.

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