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Superconducting transition of an inhomogeneous bridge in a microwave field

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It is shown that, in agreement with the experimental results, the transition of a long inhomogeneous bridge to the superconducting state may decrease in width in a microwave field until it becomes a step. This happens because stimulation of the superconductivity varies with the value of T_c characterizing a given region. The influence of the edges increases the width of the transition at high microwave radiation powers. The dependence of the transition width on the microwave power is derived.

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Recent experimental investigations[1] of the superconducting transition in long bridges subjected to a microwave field revealed that an increase in the microwave power causes the superconducting transition to first decrease in width to a step and then to widen again to a finite width which increases with the microwave power. The length of such bridges L is much greater than the size of a pair ξ_0 and, therefore, they consist of regions whose critical temperatures T_c differ somewhat.[2] For this reason the superconducting transition of a long bridge has a finite width along the temperature axis. In a microwave field the electron energy distribution function $n(\varepsilon)$ differs from the Fermi form and this nonuniformity may result in stimulation of the superconductivity in a bridge.[3,4] However, stimulation varies from one region to another because of variation of T_c and, as shown below, this accounts for the observed behavior of the superconducting transition width on the microwave field power.

1. NARROWING OF A SUPERCONDUCTING TRANSITION AT HIGH RADIATION POWERS

The order parameter Δ of a bridge is found from the Ginzburg–Landau equation which, supplemented by the nonequilibrium term $\Phi(\Delta)$, is[4]

$$\frac{\pi D}{8T}\Delta''-\frac{I_s^2}{4\pi e^2\rho^2 DS^2\Delta^2 f(\Delta)}+\frac{T_c-T}{T_c}\Delta -\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^3}{T^2}+\Phi(\Delta)=0, \tag{1}$$

$$\Phi(\Delta)=\Delta\int_\Delta^\infty\left[f(\varepsilon)-\operatorname{th}\frac{\varepsilon}{2T}\right]\frac{d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}},$$

where $\overline{I_s^2}$ is the average value of the square of the superconducting current through a bridge proportional to the radiation power; the function $f(\varepsilon)$ gives the electron energy distribution $n(\varepsilon)=[1-f(\varepsilon)]/2$; $D=v_F l_{tr}/3$ is the coefficient of spatial diffusion of electrons; ρ is the density of states; S is the cross-sectional area of the bridge.

The electron energy distribution in a microwave field is found from the transport equation. The electron energy relaxation time τ_ε is long compared with the characteristic time of spatial diffusion of electrons in a bridge D/L^2 even if this bridge is long: $\xi_0 \ll L \ll \xi_0\sqrt{T\tau_\varepsilon}$. If we also assume that this relaxation time is long compared with the field period, we obtain the following equation for the function $f(\varepsilon)$ averaged over the coordinates and time

$$\frac{1}{\tau_\varepsilon}\left[f(\varepsilon)-\operatorname{th}\frac{\varepsilon}{2T}\right]\langle\overline{\varepsilon(\varepsilon^2-\Delta^2)^{-1/2}}\rangle=\frac{\partial}{\partial\varepsilon}\left(D_\varepsilon\frac{\partial f}{\partial\varepsilon}\right), \tag{2}$$

where the coefficient D_ε represents the electron energy diffusion due to the direct acceleration of electrons in the electric field[3] and oscillations of the gap characterizing the bridge in a microwave field[4]; the brackets $\langle\ldots\rangle$ denote averaging over the part of the bridge where $\Delta<\varepsilon$ and the bar represents the time averaging.

The transport equation (2) has to be supplemented by two boundary conditions. The distribution of electrons at the edges of a bridge retains its equilibrium form in a microwave field since the current density at the edges is low. Therefore, electrons with energies greater than the maximum value of the order parameter Δ_{max} may diffuse freely out of the bridge and they acquire an equilibrium energy distribution in contrast to the electrons whose energies are $\varepsilon<\Delta_{max}$ and which are "confined" to the bridge. This condition means

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$$f(\varepsilon)|_{\varepsilon=\Delta_{max}}=\operatorname{th}\frac{\varepsilon}{2T}\approx\frac{\Delta_{max}}{2T}. \quad (3)$$

On the other hand, the electron flux $D_\varepsilon \partial f/\partial\varepsilon$ should vanish at $\varepsilon=\Delta_{min}$ since there are no states with energies smaller than the minimum gap. This gives the second boundary condition:

$$\left.\frac{\partial f}{\partial\varepsilon}\right|_{\varepsilon=\Delta_{min}}=0. \quad (4)$$

When the radiation power is high, the left-hand side of the transport equation (2) can be assumed to be zero and, if allowance is made for the boundary conditions (3) and (4), the function $f(\varepsilon)$ becomes

$$f(\varepsilon)=\Delta_{max}/2T, \quad \Delta_{min}<\varepsilon<\Delta_{max}. \quad (5)$$

The nonequilibrium term $\Phi(\Delta)$ in the Ginzburg–Landau equation corresponding to this distribution function can be calculated from Eq. (1) and it has the form

$$\Phi(\Delta)=\Delta F_c(\Delta)=\frac{\Delta\Delta_{max}}{2T}\left(\ln\frac{1+(1-\Delta^2/\Delta^2_{max})^{1/2}}{\Delta/\Delta_{max}}-\left(1-\frac{\Delta^2}{\Delta^2_{max}}\right)^{1/2}\right). \quad (6)$$

We can see that the nonequilibrium term $\Phi(\Delta)$ given by Eq. (6) is large compared with the first term in the Ginzburg–Landau equation (1). Therefore, when the radiation power is high, the order parameter Δ_i of a bridge region of length a and with a critical temperature T_c^i depends weakly on the coordinates (significant changes occur only at the edges of this region at distances $\sim\sqrt{D/\Delta}\ll a$) and can be found from the system

$$\begin{gathered}\tau_i=F_P(\Delta_i)-F_C(\Delta_i),\\ F_P(\Delta)=\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^2}{T^2}+\frac{T^4}{\Delta^3\Delta_{max}}P,\\ \tau_i=\frac{T_c^i-T}{T_c},\end{gathered} \quad (7)$$

where $P=(2\pi e^2\rho^2DS^2T^3)^{-1}\bar{I}_s^2$ is the dimensionless parameter proportional to the radiation power and $F_c(\Delta)$ is found from Eq. (6). The maximum order parameter Δ_{max} is found from

$$\tau_{max}=\frac{T_c^{max}-T}{T_c^{max}}=F_P(\Delta_{max}), \quad (8)$$

since $F_c(\Delta_{max})=0$ (there is no stimulation for $\Delta=\Delta_{max}$). Simultaneous solution of the system (7) and of Eq. (8) gives the order parameters Δ_i for different parts of a bridge as a function of temperature and radiation power.

The solution of the system (7)–(8) is shown graphically in Fig. 1. We can see that, for a given radiation power, at a temperature

$$T_1=T_c^{max}(1-\tau_P), \quad \tau_P=3\left(\frac{\zeta(3)}{16\pi^2}\right)^{2/3}P^{1/3}, \quad (9)$$

the region with the maximum critical temperature becomes superconducting and its order parameter is

$$\Delta_P=[16\pi^2/7\zeta(3)]^{1/6}P^{1/6}T.$$

The nonequilibrium term results in stimulation of the superconductivity also in other regions; consequently,

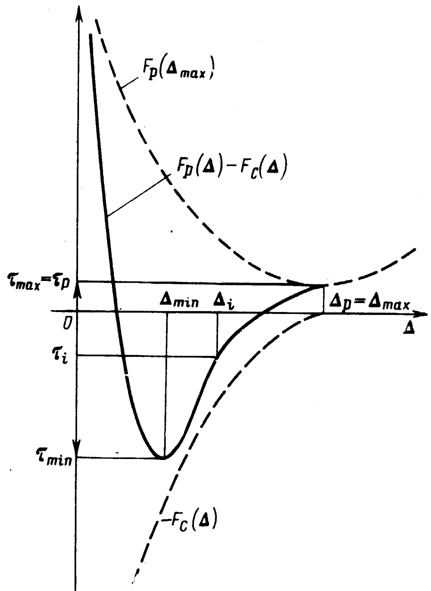

FIG 1. Graphical solution of the system (7)–(8). The results in the figure correspond to the radiation power P_2 in which the superconducting transition width vanishes.

the regions with the critical temperatures $T_c^i<T_c^{max}$ can also be in the superconducting state at the same temperature if the corresponding straight lines $\tau_i=(T_c^i-T)/T_c^i$ intersect the curve $F_P(\Delta)-F_C(\Delta)$ and the intersections are used to find the order parameters Δ_i (Fig. 1).

The absolute minimum of the function $F_P(\Delta)-F_C(\Delta)$ arises on increase of the radiation power because this increases the stimulation term F_C proportional to $\Delta_{max}=\Delta_P$ and the suppression term F_P rises more slowly [see Eqs. (6)–(8)]. Consequently, when the power becomes

$$P_2=\frac{28\zeta(3)}{\pi^2}\left(\frac{3\Delta\tau}{\ln\Delta\tau}\right)^6, \quad (10)$$

where $\Delta\tau=(T_c^{max}-T_c^{min})/T_c$, all the other parts of the bridge become superconducting at the transition temperature of the region with the maximum T_c (this case is shown in Fig. 1) so that the transition to the superconducting state is in the form of an abrupt step. The width of the super conducting transition along the temperature axis remains zero also in the power range $P>P_2$.

At lower powers the width of the superconducting transition ΔT_P is finite and it is given with logarithmic precision by

$$\begin{aligned}\frac{\Delta T_P}{T_c}=\frac{P}{(6\Delta\tau)^4}\ln^4\frac{(\Delta\tau)^5}{P}+\frac{63\zeta(3)}{2\pi^2}(\Delta\tau)^2\ln^{-2}\frac{(\Delta\tau)^5}{P}\\ -\frac{27}{64}\left(\frac{\zeta(3)}{2\pi^2}\right)^{2/3}P^{1/3}.\end{aligned} \quad (11)$$

The width of the transition has a maximum $\Delta T_P^{max}=36T_c(\Delta\tau/\ln\Delta\tau)^2$, where the power is $P=28\pi^{-2}\zeta(3)(6\Delta\tau)^6|\ln^{-9}\Delta\tau|$, which is clearly small compared with the width of the transition in the absence of radiation $\Delta T_0=\Delta\tau T_c$. Equation (10) ceases to be valid at low powers when we can no longer regard the left-hand side of the transport equation (2) as zero and we cannot use Eqs. (5) and (6) for the distribution function $f(\varepsilon)$ and the nonequilibrium term $\Phi(\Delta)$.

2. WIDTH OF A SUPERCONDUCTING TRANSITION AT LOW RADIATION POWERS

At low radiation powers the transition temperature of a region with the maximum T_c (beginning of the transition) is still given by Eq. (9) because the spatial diffusion of electrons makes the nonequilibrium term $\Phi(\Delta)$ vanish. However, the superconducting transition temperature of the region with the minimum T_c (end of the transition) can be found only if we know the nonequilibrium (stimulation) term at low radiation powers. The main contribution to the nonequilibrium term is made by electrons of energies ε close to $\Delta_{\min}$ and the order parameter of the region with the minimum T_c depends weakly on the coordinates at the transition temperature (the transition occurs when the nonequilibrium term has its maximum value $\sim \Delta^3_{\min}/T$, which is much greater than the gradient term $\sim \Delta^3_{\min}/T^2$ in the Ginzburg–Landau equation). On the other hand, at low radiation powers at the moment of the transition we have $\Delta_{\min} \ll \Delta_{\max}$, so that the spatial diffusion of electrons out of the bridge can be ignored in calculating the nonequilibrium term. These circumstances make it possible to find the nonequilibrium term employing the results obtained for a homogeneous superconductor[3]:

$$\Phi(\Delta)=\frac{5}{4\pi}\frac{DE^2\tau_\varepsilon}{T}\ln\frac{\Delta^2}{\tau_\varepsilon DE^2}, \tag{12}$$

where E is the electric field responsible for the energy diffusion of electrons.

The transition temperature of the region with the minimum T_c is found from

$$\tau_{\min}=\frac{T_c^{\min}-T}{T_c}=\max\left\{\frac{\Phi(\Delta)}{\Delta}\right\}, \tag{13}$$

because the nonequilibrium term in the Ginzburg–Landau equation (1) is large compared with all the other terms at low radiation powers.

Since the relationship between the field intensity E and the superconducting current I_s through a bridge is

$$\chi=\frac{2eV}{\omega}=\frac{2TI_s}{\pi e\rho DS}\int\frac{dx}{\Delta^2}, \tag{14}$$

where $V=EL$ is the voltage across the bridge and $\chi = \int \nabla\varphi dx$ is the phase difference φ between the order parameters of the edges (this integral is dominated by the region with $\Delta=\Delta_{\min}$), the width of a superconducting transition is

$$\frac{\Delta T_P}{T_c}=\Delta\tau-k\left(\frac{a}{L}\right)^{1/3}\left(\frac{\omega^2\tau_\varepsilon}{T}\right)^{1/6}P^{1/6}, \tag{15}$$

where a is the length of the region with the minimum T_c and k is a number of the order of unity.

Equation (15) is valid up to powers of the order of $P_1=(T/\omega^2\tau_\varepsilon)\cdot$ when the transition of a region with $T^{\min}$ to the superconducting state makes $\Delta_{\min}$

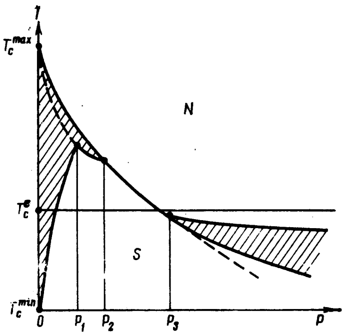

FIG. 2. State diagram of an inhomogeneous superconducting bridge in a microwave field (the shaded region is the intermediate state); P is the radiation power in dimensionless units.

$\sim T\Delta\tau(P/P_1)^{1/6}$ comparable with $\Delta_{\max}\sim T\Delta\tau$. If $P > P_1$, we find that Eq. (11) now applies. Throughout the power range $P<P_1$, a reduction in the order parameter by the current is samll compared with the stimulation effect because of the large value of τ_ε:

$$\frac{\omega^2\tau_\varepsilon}{T}\left(\frac{a}{L}\right)^2>\Delta\tau.$$

Equation (15) ceases to be valid at very low powers $P<(\omega/T\Delta\tau)^6P_1$, when the nonequilibrium term can no longer be described by Eq. (12) (the transition width is then close to $\Delta\tau$). Oscillations of the gap characterizing the bridge[4] then gives rise to an expression for $\Phi(\Delta)$ which is proportional to I_s^4 and, therefore, it is small for $P<P_1$.

The dependence of the transition width on the radiation power is shown in Fig. 2. It is interesting to note that for $P_1<P_0$ [the power P_0 corresponds to the maximum of Eq. (12)] the width of the superconducting transition increases nonmonotonically, in agreement with the experimental results.[1]

3. INFLUENCE OF EDGES ON THE SUPERCONDUCTING TRANSITION WIDTH AT HIGH RADIATION POWER

An increase in the radiation power enhances the current-induced reduction in the order parameter, whereas this parameter is hardly affected at the edges because the current density is small. Therefore, beginning from the power

$$P_3=\frac{2^8\pi^4}{49\zeta^2(3)}\left(\frac{\Delta\tau_i}{3}\right)^3,\quad \Delta\tau_i=\frac{T_c^{\max}-T_c^e}{T_c},$$

the edges of a bridge become superconducting first (it is assumed that the critical temperature of the edges T_c^e is of the order of the average value of T_c, because $\Delta\tau_i\sim\Delta\tau$).

The transition of a bridge to the superconducting state can begin at a temperature higher than the initial value T_i [Eq. (9)] because the nonequilibrium term of the region with the maximum T_c is finite if $\Delta_{\max}<\Delta_e$. In this situation the superconducting transition temperature of the region with T_c is found from the system

$$\begin{gathered}\tau=\frac{T_c-T}{T_c}=F_P(\Delta,\Delta_e)-F_c(\Delta,\Delta_e),\\ \tau_e=\frac{T_c^e-T}{T_c}=F_0(\Delta_e)=\frac{7\zeta(3)}{8\pi^2}\frac{\Delta_e^2}{T^2},\\ F_P'(\Delta,\Delta_e)-F_c'(\Delta,\Delta_e)=0,\end{gathered} \tag{16}$$

where the last equation corresponds to the stimulation

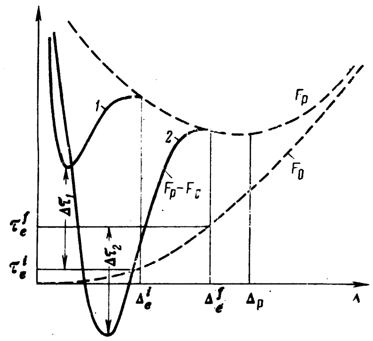

FIG. 3. Graphical solution of the system (16). Curve 1 corresponds to the beginning of the transition of a bridge to the superconducting state and curve 2 to the end of the transition.

maximum at the moment of the transition and the functions F_P and F_C are given by Eqs. (6) and (7), where Δ_{max} is replaced with Δ_e. Substituting in Eq. (16) the values $\tau = \tau_{max} = \tau_e + \Delta\tau_1$ and $\tau + \tau_{min} = \tau_e - \Delta\tau_2$ [where $\Delta\tau_2 = (T_c^e - T^{min})/T_c$], we can determine the initial (τ_e^i) and final (τ_e^f) values of τ_e corresponding to the beginning and end of the transition. The graphical solution of the system (16) is shown in Fig. 3. Bearing in mind that $F_P \gg F_0$ if $P > P_3$, we find that the width of the transition is now

$$\Delta T_P/T_c = 0.31 P^{1/3}\Delta\tau. \tag{17}$$

An increase in the power causes the width of the transition to increase. The formula (17) ceases to be valid for $P \sim 1$, when the temperature of the transition changes considerably ($\tau \sim 1$) and we can no longer use the Ginzburg–Landau theory.

4. DISCUSSION OF RESULTS

Figure 2 shows the normal and superconducting regions of a bridge as a function of temperature and radiation power. Shading is used to show the intermediate state in which a part of the bridge is in the superconducting state and another part is in the normal state. It is clear from Fig. 2 that the superconducting transition narrows down a step and then the transition width rises on increase of the power. This is in agreement with the experimental results.[1]

A step-like transition is observed experimentally at high radiation powers. This is evidently due to the smallness of the first term in the Ginzburg–Landau equation (1), describing the spatial change in the order parameter Δ, when the radiation power is high. Therefore, the order parameter cannot vary smoothly with the coordinate and the transitions of the separate regions to the superconducting state should be abrupt, which is why the transition of a bridge from the normal to the superconducting state is step-like. An increase in the radiation power makes a gradual transition impossible for inhomogeneities of decreasing size so that the number of steps incerases.

The reverse transition of a bridge from the superconducting to the normal state is always step-like because when the region with the minimum T_c goes over to the normal state the voltage across the bridege increases and, therefore, the current becomes greater so that the whole bridge goes over to the normal state. The considerable hysteresis found experimentally is evidently associated with the heating of a sample in the normal state by the microwave field (this is indicated by the difference between the results obtained on cooling in liquid and gaseous helium). Therefore, even when the radiation power is low, the beginning of the transition of a bridge from the normal (N) to the superconducting (S) state shifts toward temperatures below T_c^e, whereas in the reverse case the stimulation of the transition may cause it to appear at a temperature higher than T_c^e (Ref. 1).

We shall conclude by considering the resistance of a bridge before the transition from the superconducting to the normal state if the transition takes place at a temperature $T > T_c^e$ (Ref. 1). Clearly, this resistance is equal to the resistance of the parts of the bridge adjoining the edges where the temperature is below T_c^{max} since in the range $T > T_c^e$ the superconductivity is retained, as a result of stimulation, only between the two regions with $T_c^{max} > T_c^e$.

We should also mention that in a qualitative explanation of the effect we can ignore the restriction on the length of the bridge, $L \ll \sqrt{D\tau_\varepsilon}$, which allows us to assume that electrons with energies $\varepsilon > \Delta_{max}$ are in equilibrium and that the stimulation term is $\Phi(\Delta_{max}) = 0$. Narrowing of the transition must occur if the maximum nonequilibrium term stimulating superconductivity in the region with the maximum T_c is less than for other regions. This is always true because the stronger diffusion of electrons out of this region of the bridge makes the electron distribution function closer to the equilibrium form than the functions of the other regions.

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