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Spontaneous bremsstrahlung of an electron in the field of an intense electromagnetic wave

R. V. Karapetyan and M. V. Fedorov

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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The spectral intensity and total power are found for the spontaneous bremsstrahlung of an electron in scattering by ions in the field of an intense electromagnetic wave (of frequency ω_0). A resonance structure of the radiation is observed at frequencies close to $n\omega_0$, where $n = 1,2,3,\ldots$. The width of the resonance curve, the locations of the peaks, the number of resonance peaks, and the values of spectral intensity at the peaks are determined. The maximum number of quanta of the intense field which can be absorbed in spontaneous bremsstrahlung is found. It is shown that the maximum energy of the spontaneously radiated photon is determined by the energy of the oscillations of the electron in the field of the wave. The nonresonance part of the spectral radiation intensity, which falls off with increase of the pumping wave intensity E_0 as E_0^{-1}, is determined.

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1. INTRODUCTION

In recent years definite attention in the literature has been devoted to the theoretical study of induced bremsstrahlung of electrons in scattering by ions in the presence of an intense electromagnetic wave[1–3] (see also the references cited in our earlier article[2]). The interest in phenomena of this type is apparently due primarily to the development of the physics of laser plasmas. The theory of induced bremsstrahlung has treated both induced multiphoton radiation and absorption at the frequency of the intense wave and induced bremsstrahlung at other frequencies $\omega \neq \omega_0$ which arises on passage of a second, weak (probing) wave. It should be noted that the strong field of the pumping wave affects not only induced bremsstrahlung processes, but also the spontaneous bremsstrahlung of the electron, which may present independent interest. Borisov and Zhukovskiĭ[4] discuss the effect of an external electromagnetic wave on the spontaneous bremsstrahlung of ultrarelativistic electrons. With regard to a laser plasma, however, greater interest is presented by the case of nonrelativistic electrons, which is considered in the present work.

In discussing the pumping wave, we have in mind those values of field strength E_0 at which the velocity of the electron's oscillations in the wave v_E exceeds the velocity of its translational (or thermal) motion v:

$$v_E = eE_0/m\omega_0 \gg v. \tag{1}$$

Plasma states of this type are nonequilibrium and can exist only for a finite length of time less than the time of heating of the electrons to a temperature corresponding the velocity v_E. The processes of induced and spontaneous bremsstrahlung in this case are multiphoton processes in the field of a strong pumping wave. For example, in spontaneous radiation the electron can absorb or radiate n quanta $\hbar\omega_0$ and radiate one proton $\hbar\omega$, where n is an arbitrary integer. In view of these circumstances there is no direct and known beforehand relation between the induced and spontaneous bremsstrahlung processes such as exists between the Einstein coefficients in the absence of a pumping wave. Therefore in spite of the fact that the solution of the problem of induced bremsstrahlung at arbitrary frequencies ω for any values of E_0 is known,[2,3] the study of spontaneous bremsstrahlung for $v_E \gg v$ is an independent problem, to solution of which the present article is devoted.

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We shall discuss the interaction of the electron with the Coulomb potential of the ion in the first Born approximation, the usual criterion for applicability of which $Ze^2/\hbar v \ll 1$ in the asymptotic case of a strong field (1) obviously is replaced by the condition

$$Ze^2/\hbar v_E \ll 1. \tag{2}$$

We shall assume that the energy of the pumping-wave quantum $\hbar\omega_0$ is significantly less than the kinetic energy of translational motion of the electron,

$$\xi = 2m\hbar\omega_0/p^2 \ll 1. \tag{3}$$

In regard to the frequency of the spontaneously radiated photon, in the asymptotic case of a strong field (1) the quantum energy $\hbar\omega$ is not limited to a value $p^2/2m$.

2. SPECTRAL DENSITY OF RADIATION

On the basis of the nonrelativistic nature of the electron motion: $v,\ v_E \ll c$, we shall use the dipole approximation and discuss an electron in the field of a wave with intensity $\mathbf{E} = \mathbf{E}_0 \cos\omega_0 t$. The problem formulated above consists of describing the spontaneous bremsstrahlung of an electron in a field $\mathbf{E}$ in scattering by a potential $V(r) = Ze^2/r$. For solution of this problem, following Ref. 1, we shall use as the zeroth approximation the known wave functions of an electron in a field $\mathbf{E}$, after which we shall take into account by means of perturbation theory both the potential $V(r)$ and the operator of interaction with a quantized field. It is also possible to use another standard semiclassical approach to this problem.[5] Namely, we can use the known expressions for the probabilities of electron transitions in scattering by a potential $V(r)$ in the field of two waves $\mathbf{E}$ and $\mathbf{E}' = \mathbf{E}_0' \cos\omega t$, one of which $\mathbf{E}$ is strong, and the other $\mathbf{E}'$ is weak (Eq. (3) of Ref. 2). After this the transition to spontaneous radiation is accomplished by means of the standard substitution[5]

$$\frac{E_0'^2}{8\pi} \to \frac{\hbar\omega^3 d\omega d\Omega}{8\pi^3 c^3},$$

where $d\Omega$ is the element of solid angle in the direction of the spontaneously emitted photon. Both of these methods naturally lead to the same results and permit one to obtain the following expression for the energy emitted by an electron per unit time per unit solid angle and per unit frequency interval in radiation of a photon of frequency ω with a polarization vector $\mathbf{e}$:

$$\frac{d\mathcal{E}}{dt\,d\omega\,d\Omega} = \frac{Z^2e^6n_i}{\pi^2m^2c^3}\sum_n\int d\mathbf{p}'\frac{[\mathbf{e}(\mathbf{p}'-\mathbf{p})]^2}{(\mathbf{p}'-\mathbf{p})^4}J_n^2\left[\frac{e\mathbf{E}_0(\mathbf{p}'-\mathbf{p})}{m\hbar\omega_0^2}\right] \times\delta\left(\frac{p'^2-p^2}{2m}+\hbar\omega-n\hbar\omega_0\right), \tag{4}$$

where n are positive or negative integers (or zero), n_i is the concentration of ions, and $\mathbf{p}$ and $\mathbf{p}'$ are the electron momenta before and after scattering.

We note that in terms of the approximations used in Eq. (4) and everywhere below, there are no resonance singularities due to the possibility of the electron's Green's function reaching the mass shell.[6,7] This is explained by the fact that in the dipole approximation these resonance singularities cancel: Here the numerator and resonance denominator in the transition probabilities simultaneously vanish under the conditions of the resonance, which leads everywhere to finite results. Generally speaking, use of the dipole approximation in the vicinity of resonances of this type and the role of such resonances in the presence of Coulomb collisions require independent investigation. In the present work, however, we shall not go into details of this question, assuming that the applicability of the dipole approximation is justified by the ordinary conditions $v,\ v_E \ll c$. We note also that we will observe below a resonance behavior of the spectral density of radiation in the asymptotic case of a strong field. These resonance singularities have nothing in common with resonances arising from arrival of the Green's function at the mass shell.[6,7] In particular, in contrast to Refs. 6 and 7, in the case considered a resonance structure arises only in the asymptotic case of a strong field.

Summation over the polarizations $\mathbf{e}$ and integration over the directions of propagation of the photon in Eq. (4) are carried out in a standard manner and permit conversion to the spectral density of the radiated energy

$$\frac{d\mathcal{E}}{dt\,d\omega} = \frac{8Z^2e^6n_i}{3\pi m^2c^3}\sum_n\int d\mathbf{p}'\frac{1}{(\mathbf{p}'-\mathbf{p})^2}J_n^2\left[\frac{e\mathbf{E}_0(\mathbf{p}'-\mathbf{p})}{m\hbar\omega_0^2}\right] \times\delta\left(\frac{p'^2-p^2}{2m}+\hbar\omega-n\hbar\omega_0\right). \tag{5}$$

In what follows we shall consider not only a directed motion of the electrons, but also a distribution of such motions isotropic in the translational motion (i.e., in the momentum $\mathbf{p}$). Averaging over the directions of $\mathbf{p}$ decreases the number of free parameters on which the spectral density of radiated energy depends. In Eq. (5) in this case it is convenient to go over to integration over the momentum transfer $\mathbf{q} = \mathbf{p}' - \mathbf{p}$, which permits the result of averaging to be represented in the form

$$\left\langle\frac{d\mathcal{E}}{dt\,d\omega}\right\rangle = \frac{4Z^2e^6n_i}{3\pi m^2c^3v}\sum_n\int\frac{d\mathbf{q}}{q^3}J_n^2\left[\frac{e\mathbf{E}_0\mathbf{q}}{m\hbar\omega_0^2}\right], \tag{6}$$

where $q_{\min} \leq |\mathbf{q}| \leq q_{\max}$, and

$$q_{\min} = p\,|1-[1+\xi(n-\omega/\omega_0)]^{1/2}|,$$
$$q_{\max} = p(1+(1+\xi(n-\omega/\omega_0))^{1/2}), \tag{7}$$
$$1+\xi(n-\omega/\omega_0) \geq 0.$$

Before turning to investigation of the asymptotic case of a strong field E_0, we point out that in the opposite limit on turning off the pumping wave ($E_0 \to 0$) Eqs. (4)-(6), of course, go over to the known expressions describing the spontaneous bremsstrahlung of a free nonrelativistic electron.[8] For example, Eq. (5) in the limit $E_0 \to 0$ takes the form

$$\frac{d\mathscr{E}}{dt d\omega}=\frac{16Z^2e^6n_i}{3m^2vc^3}\ln\frac{p+p'}{p-p'}, \tag{8}$$

where in the present case $p'=(p^2-2m\hbar\omega)^{1/2}$. The total energy radiated by a free electron in the bremsstrahlung process is determined by the well known expression

$$\frac{d\mathscr{E}}{dt}=\frac{16Z^2e^6n_iv}{3mc^3\hbar} \tag{9}$$

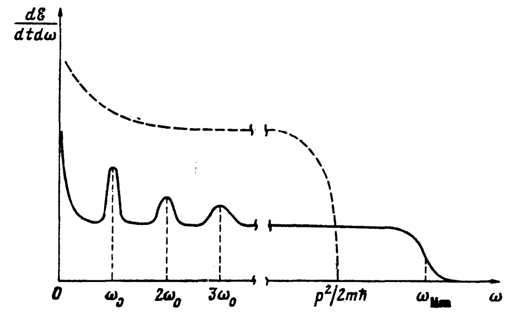

FIG. 1. Spectral intensity of spontaneous bremsstrahlung in the field of an intense external wave (of frequency ω_0). The dashed curve shows the intensity of radiation of a free electron (without a wave) in the first Born approximation.

3. RESONANCE RADIATION

We consider the asymptotic behavior of the spectral density of radiation $d\mathscr{E}/dtd\omega$ under the conditions determined by the inequality (1). The general method of calculation of integrals containing squared Bessel functions J_n^2 with a large argument ($\sim E_0 \to \infty$) has been described by us previously.[2] The general formulas obtained in that work can be considered as the basis of the following simpler method of calculation (which has been used previously in Refs. 1 and 3 and which leads to correct results). For large E_0 [inequality (1)] the argument of the Bessel functions is large over a wide region of variation of the variables of integration. This permits use of the asymptotic representation of $J_n(u)$ for $|u| \gtrsim (1, |n|)$, further averaged over the fast oscillations,

$$J_n^2(u)\approx 1/\pi|u|. \tag{10}$$

If, however, at some point of the region of integration $u=0$, then in the immediate vicinity of this point the asymptotic representation (10) is inapplicable. However, as follows from estimates and from the results of Ref. 2, this small region does not contribute substantially to the integral. Therefore the asymptotic calculation of the integrals which contain $J_n^2(u)$ can be carried out by means of the representation (10), and the condition ($|u| \gtrsim (1, |n|)$) must be discussed as an additional limitation of the region of integration.

Applying the method described to Eqs. (5) and (6) and carrying out the integrations, it is easy to see that the spectral density of radiation in both cases has the form of the sum of resonance and nonresonance terms (Fig. 1):

$$\frac{d\mathscr{E}}{dtd\omega}=\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{\rm res}+\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{\rm nonres}. \tag{11}$$

The resonance part $(d\mathscr{E}/dtd\omega)_{\rm res}$ as a function of frequency has sharp peaks at $\omega \approx n\omega_0$, where $n=1, 2, \ldots$, while the nonresonance term $(d\mathscr{E}/dtd\omega)_{\rm nonres}$ depends smoothly on the frequency. We emphasize that the appearance of resonances is a specific feature of the asymptotic case of a strong field. It is well known[2,3] that a similar result occurs also in induced bremsstrahlung at frequencies $\omega \approx n\omega_0$. Let us consider the form of the resonance term $(d\mathscr{E}/dtd\omega)_{\rm res}$ in several special cases.

1. *Electron momentum perpendicular to the polarization vector:* $\mathbf{p}\perp\mathbf{E}_0$. Calculation of the resonance part of $d\mathscr{E}/dtd\omega$ in the vicinity of the point $\omega=n\omega_0$ by the method described above gives

$$\left(\frac{d\mathscr{E}}{dt\,d\omega}\right)_{\rm res}=\frac{16Z^2e^5n_i\omega_0}{3\pi mc^3E_0|n-\omega/\omega_0|}\ln\frac{|n-\omega/\omega_0|+[(n-\omega/\omega_0)^2+(n\delta)^2]^{1/2}}{n\delta}, \tag{12}$$

where $\delta\equiv v/v_E \ll 1$.

The width of the resonance is $\Delta\omega\approx\omega_0 n\delta=\omega_0 n(v/v_E)$. Far from the maximum (at $|\omega-n\omega_0|\gg\Delta\omega$, but $\omega_0 \gg|\omega-n\omega_0|$) we have

$$\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{\rm res}=\frac{16Z^2e^5n_i\omega_0}{3\pi mc^3E_0|n-\omega/\omega_0|}\ln\frac{2|n-\omega/\omega_0|}{n\delta}. \tag{13}$$

The value of the spectral intensity at the maximum, found from Eq. (12), is achieved at $\omega=n\omega_0$ and is determined by the quantity

$$\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{\rm res\text{-}m}\approx\frac{16Z^2e^5n_i\omega_0}{3\pi mc^3E_0n\delta}=\frac{16Z^2e^6n_i}{3\pi m^2c^3vn}, \tag{14}$$

which with accuracy to a logarithmic factor and a numerical coefficient $1/\pi n$ coincides with the spectral density of radiation of a free electron (8). With increase of n the height of the peaks falls off as $1/n$ and the width of each maximum is significantly less than the distance between them, i.e., for $n<\delta^{-1}$. Consequently the parameter $v_E/v=\delta^{-1}\gg 1$ determines the number of resonance peaks.

We note that the calculation method used permits the calculation to be carried out with logarithmic accuracy. This means, in particular, that the behavior $(d\mathscr{E}/dtd\omega)_{\rm res}$ at the wings of the resonance is described by Eq. (13) with accuracy to small corrections not containing the large logarithm $\ln(|n-\omega/\omega_0|/n\delta)$. At the same time the value of $(d\mathscr{E}/dtd\omega)_{\rm res}$ at the maximum is determined by Eq. (14) only with accuracy to a coefficient of the order of unity, since it no longer contains the large logarithmic factor. Nevertheless the use of the resonance formula (12) is justified, since it is applicable over a wide range of frequencies $|\omega - n\omega_0 \lessgtr \Delta\omega$ and gives a readily interpreted picture of the dependence of the spectral density of radiation on frequency near resonance.

An important feature of the case considered is the symmetric nature of the resonance peak (Fig. 2a): $(d\mathscr{E}/dtd\omega)_{res}$ (12) depends only on $|\omega - n\omega_0|$. This result is preserved over a wide range of angles between the vectors **p** and $\mathbf{E}_0$. In the general case the formulas for $(d\mathscr{E}/dtd\omega)_{res}$ are more complicated than the expression (12). We do not give them here, especially since the results of the calculations are qualitatively similar to those formulated above. Important differences, which are discussed in the next section, appear only in the case when the directions of the vectors **p** and $\mathbf{E}_0$ are very close together.

2. *Electron momentum parallel to the polarization vector*: $\mathbf{p} \parallel \mathbf{E}_0$. The shape of the resonance peak in this case is not symmetric and is not described by a single simple formula similar to Eq. (12). The width of the resonance as before is $\Delta\omega \approx \omega_0 n\delta$. In the region $\omega/\omega_0 - n \leqslant -n\delta(1 + \frac{1}{4}\xi n\delta)$, calculation of the resonance part of the spectral density of radiation gives

$$\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{res} = \frac{8Z^2e^5n_i\omega_0}{3\pi mc^3E_0(n-\omega/\omega_0)} \times \ln 4\frac{(n-\omega/\omega_0)^2-(n\delta)^2}{\xi(n\delta)^2(n-\omega/\omega_0)}. \tag{15}$$

Far from the maximum (on the left wing of the resonance curve, Fig. 2b) Eq. (15) goes over to the expression

$$\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{res} = \frac{8Z^2e^5n_i\omega_0}{3\pi mc^3E_0(n-\omega/\omega_0)} \times \ln\frac{4(n-\omega/\omega_0)}{\xi(n\delta)^2}. \tag{16}$$

In the frequency region $-n\delta(1 + 1/4\,\xi n\delta) \leqslant \omega/\omega_0 - n \leqslant n\delta(1 - 1/4\,\xi n\delta)$ the result of calculation of $(d\xi/dtd\omega)_{res}$ has the form

$$\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{res} = \frac{8Z^2e^5n_i\omega_0}{3\pi mc^3E_0(n-\omega/\omega_0)} \times \ln\left(1+\frac{n-\omega/\omega_0}{n\delta}\right). \tag{17}$$

Finally, in the region $\omega/\omega_0 - n \geqslant n\delta(1 - 1/4\,\xi n\delta)$ the form of the resonance curve is determined by the expression

$$\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{res} = \frac{8Z^2e^5n_i\omega_0}{3\pi mc^3E_0(\omega/\omega_0-n)} \ln\frac{4}{\xi(\omega/\omega_0-n)}. \tag{18}$$

The maximum of the resonance curve is reached at $\omega/\omega_0 - n = n\delta(1 - 1/4\,\xi n\delta)$. The behavior of the spectral density of radiation in the immediate vicinity of the maximum is described by Eqs. (17) and (18). The value of the function at the maximum is determined by the expression

$$\left(\frac{d\mathscr{E}}{dtd\omega}\right)_{res\text{-}m} \approx \frac{8Z^2e^6n_i}{3\pi m^2c^3vn} \ln\frac{4}{\xi n\delta}. \tag{19}$$

Thus, in contrast to the preceding case, the resonance curve is asymmetric (Fig. 2b), the maximum is shifted to the right in frequency (relative to the value $\omega = n\omega_0$) by an amount of the order of the width of the resonance $\Delta\omega$, and the value of the spectral density of radiation at the maximum differs from the case $\mathbf{p} \perp \mathbf{E}_0$ [Eq. (14)] by a large logarithmic factor.

Similar results are obtained not only for parallel **p** and $\mathbf{E}_0$ vectors but also in a certain small range of angles $\Delta\theta \lesssim \theta_0$ between them, $\theta_0 \sim (n\xi\delta)^{1/2}$. The observed singularities of $(d\xi/dtd\omega)_{res}$ can be investigated in scattering of beams of electrons with an angular divergence not exceeding θ_0. In the opposite case, for $\Delta\theta \gg \theta_0$, and in particular for an isotropic electron distribution, the region of directions of the momentum **p** close to the direction of the vector $\mathbf{E}_0$ does not contribute substantially to the averaged values. Therefore, in particular for an isotropic distribution of electrons as will be discussed below, the resonance maxima are symmetric relative to values $\omega = n\omega_0$.

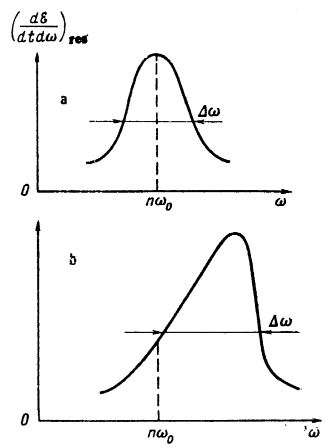

FIG. 2. Structure of the radiation spectrum in the vicinity of a resonance peak ($\Delta\omega = \omega_0 n\delta$ is the width of the resonance, $\Delta\omega \ll \omega_0$): a)—in motion of the electrons in the plane perpendicular to the polarization vector of the intense wave, b)—in motion along the polarization vector.

4. ISOTROPIC DISTRIBUTION OF ELECTRONS

Integration over the momentum transfer **q** in Eq. (6) is carried out by means of the method described in Section 3, and in the general case it is possible to represent $\langle d\mathscr{E}/dtd\omega\rangle$ in the form

$$\left\langle\frac{d\mathscr{E}}{dtd\omega}\right\rangle = \frac{16Z^2e^5n_i\hbar\omega_0^2}{3\pi mc^3vE_0}\sum_n \Phi_n, \tag{20}$$

$$\Phi_n = \frac{1}{\tilde{q}_{min}}\left(\ln\frac{eE_0\tilde{q}_{min}}{\nu m\hbar\omega_0^2}+1\right) - \frac{1}{q_{max}}\left(\ln\frac{eE_0q_{max}}{\nu m\hbar\omega_0^2}+1\right), \tag{21}$$

where

$$\tilde{q}_{min} = \max\left\{q_{min}, \frac{\nu m\hbar\omega_0^2}{eE_\nu}\right\}, \quad \nu = \max(1, |n|). \tag{22}$$

The lower limit of the summation over n in Eq. (20), n_{min}, is determined from the obvious condition that the electron energy after scattering is positive:

$$n_{min} \sim -\left[\frac{p^2}{2m\hbar\omega_0} - \frac{\omega}{\omega_0}\right], \quad (23)$$

where $[x]$ is the integral part of the number x.

In regard to large values of n, the sum (20) is formally extended to ∞. However, an important contribution to this sum is actually made only by terms with a number n less than a limiting value n_{max}. The value n_{max} can be found from the condition $q_{max} > \tilde{q}_{min}$, which is the criterion of applicability of the asymptotic case of a strong field to the limiting case of multiphoton processes. As can easily be seen

$$n_{max} \sim e^2 E_0^2 / m\hbar\omega_0^3. \quad (24)$$

Consequently, the maximum number of quanta of the strong field which can be absorbed in the spontaneous bremsstrahlung process is determined by the ratio of the oscillation energy of the electron to the quantum energy of the pumping wave. In view of the conditions (1) and (3) we have $n_{max} \gg 1$. For $n > n_{max} \gg 1$ the order of the Bessel functions in Eqs. (3) and (4) exceeds the value of the argument. The contribution of these terms to the spectral density of radiation is very small and they can be neglected. From these considerations it also follows that the value of the bremsstrahlung spectral density itself is not small for the condition that $|n_{min}| \ll n_{max}$. This condition obviously is satisfied in the region of comparatively small frequencies $\omega < p^2/2m\hbar$, which follows from the inequality (1). The requirement $|n_{min}| < n_{max}$ can be violated at large values of the frequency ω. The condition $|n_{min}| \sim n_{max}$ is actually determined by the limiting frequency which can be radiated in the bremsstrahlung process in the field of a strong wave. It is equal to

$$\omega_{lim} \sim \omega_0 n_{max} \sim m v_E^2/\hbar, \quad (25)$$

i.e., the energy of the corresponding quantum is equal to the energy of oscillations of the electron in the wave.

The general expression (20) contains both the resonance and the nonresonance parts of the spectral density of radiation. The resonance properties can appear in the vicinity of the frequencies $\omega \approx n\omega_0$, $n = 1, 2, \ldots$. Here $\tilde{q}_{min} \ll q_{max}$ and in the sum over n one term, a resonance one, is the principal one.

The width of the resonance is determined by the condition $q_{min} \sim nm\hbar\omega_0^2/eE_0$ and as before is equal to $\Delta\omega = \omega_0 n\delta$. Specific expressions for the spectral density of radiation in the wings of the resonance curve ($|n\omega_0 - \omega| > \omega_0 n\delta$) and in the immediate vicinity of the maximum ($|n\omega_0 - \omega| < \omega_0 n\delta$) are determined respectively by Eqs. (13) and (14).

The nonresonance part of the spectral density of radiation is determined by a large number of terms in the sum over n in Eq. (20). We consider first the region of comparatively small frequencies ω. In this case the region of summation over n can be broken down into two intervals: from $-|n_{min}|$ to $+|_{min}|$ and from $+|_{min}|$ to n_{max}. The function Φ_n (21) in these subregions can be approximated by the respective expressions

$$\frac{v}{\hbar|n\omega_0 - \omega|}\ln\frac{v_E}{v}, \quad \frac{v}{2n\hbar\omega_0}\ln\frac{n_{max}}{n}. \quad (26)$$

Going over from summation over n to integration, it is easy to see that the nonresonance part of the spectral density of radiation can as a result be represented in the form

$$\left\langle\frac{d\mathscr{E}}{dtd\omega}\right\rangle_{nonres} = \frac{16Z^2e^5 n_i\omega_0}{3\pi mc^3 E_0}\ln\frac{v_E}{v}\ln\frac{m^2v^3v_E}{\hbar^2\omega_0^2}. \quad (27)$$

In Eq. (27) we have correctly taken into account the squares and products of logarithms of the two large parameters v_E/v and $p^2/2m\hbar\omega_0$. In a strictly asymptotic sense $\ln(v_E/v) \gg \ln(p^2/2m\hbar\omega_0)$ as $E_0 \to \infty$ and it would appear that the product of the logarithms in Eq. (25) can be replaced by $\ln^2(v_E/v)$. However, in reality the ratio of the large parameters v_E/v and $p^2/em\hbar\omega_0$ can be different and the contribution of the corresponding logarithms in $\langle d\mathscr{E}/dtd\omega\rangle_{nonres}$ can be comparable in magnitude. In Eq. (27) we have dropped linear logarithmic corrections and also all logarithm-free corrections, which are relatively small.

We note that the nature of the principal dependences in Eq. (27) (without the logarithmic factors) can be found from expression (8) for the spectral density of spontaneous radiation of a free electron by means of the substitution $v \to v_E$.

Equation (27) correctly describes the spectral density of radiation in the region of frequencies $\omega < \omega_0 v_E/v$, $p^2/2m\hbar$. Without dwelling in detail on analysis of all features associated with increase of ω, let us consider the opposite limiting case of high frequencies ($evE_0/\hbar\omega_0) < \omega < \omega_{lim}$. The sum over n in Eq. (20) receives important contributions from the region where the asymptotic case of a strong field works satisfactorily and where $q_{max} > \tilde{q}_{min} \gg nm\hbar\omega_0^2/eE_0$. This condition in the frequency range considered is satisfied for $n - \omega/\omega_0 \gg \omega^2/\omega_0^2 n_{max}$, $n \ll n_{max}$. The function Φ_n in this case can be approximated by the latter of the expressions (26). Summation over n in these limits gives

$$\left\langle\frac{d\mathscr{E}}{dtd\omega}\right\rangle_{nonres} = \frac{4Z^2e^5 n_i\omega_0}{3\pi mc^3 E_0}\ln^2\frac{\omega_{lim}}{\omega}. \quad (28)$$

Equation (28) describes the decrease of spectral intensity of the radiation as the frequency ω approaches its limiting possible value ω_{lim}. Knowledge of the asymptotic behavior of $\langle d\mathscr{E}/dtd\omega\rangle_{nonres}$ at large ω permits determination also of the total radiated power by integration of (28) over the frequency ω from some comparatively small value ($\ll \omega_{lim}$) up to ω_{lim}:

$$\left\langle\frac{d\mathscr{E}}{dt}\right\rangle = \frac{8Z^2e^7 n_i E_0}{3\pi m^2c^3\hbar\omega_0}. \quad (29)$$

The qualitative nature of the principal dependences in Eq. (29) follows from Eq. (9) if in the latter we replace the velocity v by the velocity v_E of the oscillations. This result is quite natural, although it is not universal: Not all physical quantities in the asymptotic case of a strong field (1) can be obtained from the corresponding quantities in the absence of a field by means of the substitution[2,3] $v \to v_E$. We note that since, as the result of integration of Eq. (24) over ω, the large logarithmic factors in Eq. (29) cancel, the accuracy of this formula is reduced in comparison with Eqs. (27) and

(28) for the spectral density of radiation. Equation (29) is actually determined with accuracy to a coefficient of the order of unity.

In particular, the resonance part of the spectral density of radiation can also make a contribution to the total radiation power comparable with Eq. (29).

5. ENERGY LOSS OF THE ELECTRON

The rate of change of the electron energy in the field of a strong wave during the bremsstrahlung process in the general case can be determined as

$$\frac{dW}{dt}=\frac{8Z^2e^6n_i}{3\pi m^2c^3}\sum_n\int d\omega\left(n\frac{\omega_0}{\omega}-1\right)\int d\mathbf{p}'\frac{1}{(\mathbf{p}'-\mathbf{p})^2}\times J_n^2\left[\frac{e\mathbf{E}_0(\mathbf{p}'-\mathbf{p})}{m\hbar\omega_0^2}\right]\delta\left(\frac{p'^2-p^2}{2m}+\hbar\omega-n\hbar\omega_0\right). \quad (30)$$

However, the integral (30) over the frequency ω diverges as $\omega\to 0$. This divergence is a manifestation of the well known and well studied infrared catastrophe.[8] A feature of the process being studied is that, in contrast to ordinary bremsstrahlung, in our case the infrared divergence appears not only in the total probability (or cross section) but also in the energy loss. Usually the $1/\omega$ divergence in the energy loss is compensated by the energy of the radiated photons $\hbar\omega$ and the expression for the energy loss as $\omega\to 0$ does not diverge. In the presence of an external electromagnetic field this is not the case. A change of the electron energy in the n photon process is $n\hbar\omega_0-\hbar\omega$. This quantity remains finite as $\omega\to 0$, $n\neq 0$, and does not compensate the divergence in Eq. (30). As usual,[8] the region of integration over ω can be broken down into two intervals: $[0,\omega_{\min}]$ (region A) and $[\omega_{\min},\infty]$ (region B); $\omega_{\min}\ll\omega_0$. In the region of soft radiated photons in the integrand of Eq. (30) it is possible to separate a factor

$$dw_{\mathbf{pp}'}^{(1)}=\frac{2e^2(\mathbf{p}'-\mathbf{p})^2}{3\pi m^2c^3\hbar}\frac{d\omega}{\omega}, \quad (31)$$

which has the meaning of the probability of radiation of one soft photon in a frequency interval $d\omega$ in an arbitrary direction for a fixed value of electron momentum.[8] The expression (31) determines the probability of radiation of a photon in the first order of perturbation theory. It is inapplicable in integration over a finite frequency $\Delta\omega$ at sufficiently small ω. Going outside the framework of perturbation theory, as usual,[8] permits determination of the probability of emission of an arbitrary number N of soft photons $w^{(N)}$ and summation of these probabilities $\sum w^{(N)}=1$. This procedure leads to an obvious result: The rate of change of the electron energy in emission of an arbitrary number of soft photons $(dW/dt)^{(A)}$ is equal to the rate of change of its energy in the induced bremsstrahlung process $(dW/dt)_0$ (Ref. 1),

$$\left(\frac{dW}{dt}\right)=4Z^2e^4n_i\hbar\omega_0\sum_n n\int\frac{d\mathbf{p}'}{(\mathbf{p}'-\mathbf{p})^4}\times J_n^2\left[\frac{e\mathbf{E}_0(\mathbf{p}'-\mathbf{p})}{m\hbar\omega_0^2}\right]\delta\left(\frac{p'^2-p^2}{2m}-n\hbar\omega_0\right). \quad (32)$$

Only the rate of change of the energy in this combined process has physical meaning. The probability and rate of change of energy by themselves in the induced bremsstrahlung process without emission of soft photons are equal to zero. The relation of induced bremsstrahlung and bremsstrahlung in the field of an intense wave is completely analogous to the relation between elastic Coulomb scattering and ordinary bremsstrahlung.[8]

Thus, instead of the integral over the region A $[0,\omega_{\min}]$ in Eq. (30), as the result of taking into account processes of radiation of many soft photons we obtain the rate of change of the electron energy $(dW/dt)_0$ (32). In regard to the region B $[\omega_{\min},\infty]$, in this case calculation by means of perturbation theory are satisfactory and Eq. (30) correctly describes the contribution of this region to dW/dt. However, the quantity $(dW/dt)^{(B)}$ depends logarithmically on the lower limit $\omega_{\min}$, which is not completely defined. This dependence is unphysical and, as usual, must be compensated in inclusion of radiation corrections to the induced bremsstrahlung process. The problem of calculation of radiation corrections to induced bremsstrahlung in the field of an intense wave, as far as we know, has not been discussed up to the present time. Its solution presents undoubted interest but is beyond the scope of the present article. We present here only an estimate of the contribution of spontaneous bremsstrahlung in the field of an intense wave to the electron energy loss. It is evident that with inclusion of radiation corrections and the contribution to dW/dt from region B we have

$$\left(\frac{dW}{dt}\right)^{(R)}+\left(\frac{dW}{dt}\right)^{(B)}\sim\frac{d\mathscr{E}}{dt}\sim\frac{e^2}{\hbar c}\left(\frac{v_E}{c}\right)^2\left(\frac{dW}{dt}\right)_0, \quad (33)$$

where the evaluation has been carried out in the asymptotic case of a strong field (1). It is evident from this that, in the presence of a strong external field satisfying condition (1) and in terms of the nonrelativistic approximation used above, the contribution of spontaneous bremsstrahlung to the electron energy balance is small in comparison with the rate of change of the energy as the result of induced bremsstrahlung $(dW/dt)_0$. Consequently, from the point of view of analysis of the electron energy change, spontaneous bremsstrahlung can be neglected in this case. Nevertheless, the investigation carried out above (Sections 1–4) may have independent value, since it refers to physical quantities which do not depend on dW/dt—the spectral density and energy of spontaneous bremsstrahlung in the field of an intense wave.

In this connection we note that in addition to spontaneous bremsstrahlung, Compton scattering of photons in the pumping field can also make a definite contribution to the energy radiated by the electron. In terms of the nonrelativistic approximation v, $v_E\ll c$, the field $\mathbf{E}$ can be assumed weak in comparison with Compton scattering, and this process itself can be discussed as Thomson scattering of light by a stationary electron. The total energy scattered by the electron per unit time has the form

$$\left(\frac{d\mathscr{E}}{dt}\right)_T=\frac{1}{3}\frac{e^4E_0^2}{m^2c^3}. \quad (34)$$

This quantity, generally speaking, is not necessarily small in comparison with the bremsstrahlung power (9) and (29). However, bremsstrahlung and Thomas scattering differ greatly in their spectral properties. The spectral intensity of Thomson scattering $d\mathscr{E}/dtd\omega)_T$ is concentrated in the vicinity of the frequency $\omega=\omega_0$ and has a distribution width equal to the spectral width of the pumping, $\Delta\omega$, i.e., in principle it can be made arbitrarily small. As was shown above, resonance maxima of bremsstrahlung arise in the vicinity of many pumping harmonics and have a finite width determined by the magnitude of the field strength. The average value of the spectral intensity of bremsstrahlung between resonances is also different from zero (Fig. 1).

The observation of spontaneous bremsstrahlung can be accomplished, for example, by study of the luminescence of a laser plasma. Another situation in which the effects discussed can in principle occur is the photoemission of electrons from a metal surface under the action of a strong external field. This phenomenon is usually accompanied by radiation of harmonics of the light wave.

One of the possible mechanisms of generation of harmonics is the spontaneous bremsstrahlung discussed above. In traversing the near-surface region, the electrons interact both with the field of the intense wave and with the ions of the crystal lattice, which may be the cause of extremely intense bremsstrahlung having a resonance nature.

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Polarization effects in the photoionization of atoms

N. A. Cherepkov

A. I. Ioffe Physicotechnical Institute, USSR Academy of Sciences, Leningrad

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The atoms of Ar and Xe are used as examples in considering the various possibilities of obtaining polarized electrons by absorption of circularly polarized, linearly polarized, and unpolarized light by unpolarized atoms. The results are given of a calculation of the degree of electron polarization in the random phase approximation with exchange.

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1. Improvements in the experimental techniques and the use of the coincidence method have made it possible to ensure continuing increase in the detailed information available on collisions of particles with atoms. However, the simplest case of atomic collisions—photoionization—has been considered theoretically so far without allowance for the photoelectron spin orientation. A full quantum-mechanical description of the photoionization of an atom is as follows. Light of known polarization is incident on unpolarized atoms (we shall consider only this case). It is necessary to find that the probability of emission of photoelectrons along a given direction $\varkappa$, with spin directed along some vector $\mathbf{s}$, where $\varkappa$ and $\mathbf{s}$ are unit vectors. The polarization of photoelectrons generated by absorption of circularly polarized light in alkali atoms was first considered by Fano.[1] He showed that near a Cooper minimum the total photoelectron flux is polarized in the direction of the spin vector of the photons because of the spin–orbit interaction in the continuous spectral state. The angular distribution of photoelectrons with this spin orientation was considered by Heinzmann *et al.*[2] It was found that as a consequence of the influence of the spin–orbit interaction the asymmetry coefficient β of the angular distribution undergoes a sudden change near a Cooper minimum, whereas in the *LS* coupling approximation for the *s* subshells it is equal to 2, irrespective of the photon energy. Finally, the general formula for the angular distribution with an arbitrary spin orientation in the Fano effect was obtained by Brehm.[3]

The Fano effect also appears in the photoionization of the s^2 subshells by circularly polarized light if the photoionization cross section has a Cooper minimum.[4] This condition is satisfied, in particular, by the outer s^2 subshells of alkaline-earth atoms and atoms of the inert gases, beginning from Ca and Ar, respectively.[5]

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