cay" after the light is turned off takes place as a result of a process having a lower activation energy: 0.2 eV—the motion of dislocations. As follows from results obtained in Ref. 8, if the dislocation charge is larger than the stationary value the dislocation will rapidly give off the excess electrons to the trapping centers that it encounters.

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Acoustic oscillations of superfluid solutions in narrow channels

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The linearized hydrodynamic equations for He^3–He^4 solutions are considered in the phonon temperature region $T < 0.7$ K under conditions of complete stopping of only the phonon part of the normal component. The velocities of two types of sound oscillations are calculated.

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A solution of He^3 in superfluid He^4 is an assembly of elementary excitations—phonons, rotons, and impurity quasiparticles—contained in a superfluid background. At temperatures $T \leqslant 0.7$ K the contribution of the rotons is exponentially small, and will hereafter be neglected. Under ordinary conditions the phonon and impurity subsystems are strongly coupled with each other, manifest, in particular, by the fact that the macroscopic motion of either the phonon gas or of the impurity-quasiparticle gas is characterized by the same local-equilibrium velocity of the normal motion. Under certain conditions, however, which will be spelled out below, these two systems can be decoupled and the contribution of the phonon part of the normal component can thus be separated.

Interactions of excitations in a solution are completely described by the corresponding scattering cross sections, which were calculated by Landau and Khalatnikov[1] and by Khalatnikov and Zharkov,[2] and are of the following order of magnitude:

$$\sigma_{i-i} \approx 2.8\cdot 10^{-15}\,\mathrm{cm}^2, \quad \sigma_{ph-ph} \approx 6\cdot 10^{-19}(xT)^4,$$
$$\sigma_{i-ph} \approx 4.7\cdot 10^{-20}(yT)(xT)^4\delta',$$

respectively for impurity-impurity scattering, for phonon-phonon interaction, and for scattering of a phonon by an impurity quasiparticle. Here x is the phonon in units of T/s (s is the speed of sound), y is the impurity energy in units of $3T/2$, and δ' is a function of the order of unity.

Knowing the interaction cross sections it is easy to verify that there exists a considerable region $c \gg 10^{-2}T^2$ (where c is the concentration and T is the temperature in degrees Kelvin) where all the collisions are determined mainly by the impurities, and the phonon mean free path l_{ph} strongly exceeds the free path l_i of the He^3 quasiparticles: $\lambda_{\text{ph}} \gg \lambda_i$. The number of hard phonons and impurities (with $x \gg 1$ and $y \gg 1$), for which this inequality is not satisfied, is exponentially small. If we now place the solution in a capillary of diamater d such that $l_{\text{ph}} \gg d \gg l_i$, then under conditions of diffuse reflection from the capillary walls only the phonon part of the normal component will be stopped. The influence of the capillary walls on the motion of the normal impurity part can in this case be neglected. The degree of diffuseness of sound reflection from a solid surface is proportional, as is well known, to $(\xi/\lambda_T)^2$, where ξ is the characteristic dimension of the roughness of the capillary wall, and λ_T is the thermal wavelength of the phonon. We consider next the case when the phonon part of the normal component is completely stopped, corresponding to satisfaction of the condition $\xi \gg \lambda_T$.

The macroscopic motion of the phonon gas is accompanied by transport of some He^3 and He^4 masses,[3] whose determination reduces to calculation of the corresponding fluxes $\mathbf{j}_3$ and $\mathbf{j}_4$ which enter in the continuity equation of the Khalatnikov two-speed hydrodynamics of solutions[4] (for an unbounded liquid) at $T=0$:

$$\mathbf{j}_3=\int \rho_3\mathbf{v}_n\,dV, \quad \mathbf{j}=\mathbf{j}_3+\mathbf{j}_4=\int (\rho_{n3}\mathbf{v}_n+\rho_{s3}\mathbf{v}_s)\,dV \tag{1}$$

where ρ_i are the densities of the particles of each species ($i=3,4$), $\mathbf{v}_n$ and $\mathbf{v}_s$ are the respective velocites of the normal and superfluid motions, ρ_{n3} is the impurity normal density, and $\rho_{n3}+\rho_{s3}=\rho_3+\rho_4=\rho$ (ρ_{s3} is thus the superfluid density of the solution at $T=0$).

As the ground state of the liquid we consider a solution at rest at $T=0$. The weakly excited state of the solution will be described with the aid of a localized wave packet[5]

$$\begin{gathered}\delta\rho_i=V^{-1/2}\sum_{\mathbf{k}}\rho_{ik}\exp[i(\mathbf{kr}-\omega_k t)]+\text{c.c.},\\ \mathbf{v}_n=V^{-1/2}\sum_{\mathbf{k}}\mathbf{v}_{nk}\exp[i(\mathbf{kr}-\omega_k t)]+\text{c.c.},\\ \mathbf{v}_s=V^{-1/2}\sum_{\mathbf{k}}\mathbf{v}_{sk}\exp[i(\mathbf{kr}-\omega_k t)]+\text{c.c.},\end{gathered} \tag{2}$$

where $\omega_k=uk$, u is the speed of first sound in the solution at $T=0$. The connection between the quantities ρ_{ik}, $\mathbf{v}_{nk}$, and $\mathbf{v}_{sk}$ is determined with the aid of the ordinary equations of the hydrodynamics of solutions[4] at $T=0$. Without dwelling on the straightforward calculations, we present the result:

$$\begin{gathered}\mathbf{v}_{nk}=u\frac{\mathbf{k}}{k}\frac{\rho_{3k}}{\rho_3}, \quad \rho_{4k}=-\frac{u^2-A_3}{u^2-A_4}\rho_{3k},\\ \mathbf{v}_{sk}=u\frac{\mathbf{k}}{k}\left(F-\frac{\rho_{n3}}{\rho_3}\right)\frac{\rho_{3k}}{\rho_{s3}}.\end{gathered} \tag{3}$$

Here $A_i=\rho_3\partial\mu_3/\partial\rho_i+\rho_4\partial\mu_4/\partial\rho_i$; μ_i is the chemical potential per unit mass for each species of particle, and $F=(A_3-A_4)/(u^2-A_4)$. We expand expressions (1) in powers of $\delta\rho_i$, $\mathbf{v}_n$, and $\mathbf{v}_s$. The linear term in this expansion vanishes because the wave packet (2) that describes the localized sound excitation in the solution decreases rapidly with distance.[6]

In second-order approximation, using formulas (2) and (3), we obtain expressions for the fluxes $\mathbf{j}_3$ and $\mathbf{j}$:

$$\mathbf{j}_3=2u\sum_{\mathbf{k}}\frac{|\rho_{3k}|^2}{\rho_3}\frac{\mathbf{k}}{k}, \quad \mathbf{j}=Q\mathbf{j}_3; \tag{4}$$

here

$$Q=\frac{1}{\rho_{s3}}\left\{(\rho-\rho_3F)\left[\frac{\partial\rho_{n3}}{\partial\rho_3}-\frac{\partial\rho_{n3}}{\partial\rho_4}(1-F)\right]+F(\rho_3F-\rho_{n3})\right\}.$$

On the other hand, the acoustic momentum $\mathbf{j}$ (per unit volume) can be represented in the form[3] $\mathbf{j}=\rho_{\text{ph}}\mathbf{v}$, where $\rho_{\text{ph}}=2\pi^2T^4/45\hbar^3u^5$ and $\mathbf{v}$ is the velocity of the macroscopic motion of the phonon gas.

In the case of complete stopping of the phonon normal component, the acoustic excitations do not participitate in the normal motion at all. Since the velocities enter in the expressions for the hydrodynamic fluxes linearly, it follows that to obtain the linearized hydrodynamic equations $l_{\text{ph}} \gg d \gg l_i$ and $\xi \gg \lambda_T$, without allowance for the dissipative terms, to gather in the mass and entropy fluxes the terms corresponding to the phonon-gas motion. The system of hydrodynamic equations in the variables ρ, c, σ ($\sigma=S/\rho$, S) is the total entropy of the solution takes then the form

$$\begin{gathered}\dot\rho+\rho_s\operatorname{div}\mathbf{v}_s+\rho_{n3}\operatorname{div}\mathbf{v}_n=0, \quad \dot{\mathbf{v}}_s+\nabla(\mu-cZ/\rho)=0,\\ \sigma\dot\rho+\rho\dot\sigma+\rho\sigma_3\operatorname{div}\mathbf{w}=0, \quad c\dot\rho+\rho\dot c+\rho c\operatorname{div}\mathbf{w}=0,\\ \rho_{n3}\dot{\mathbf{v}}_n+\rho_s\dot{\mathbf{v}}_s+\nabla P-S_{ph}\nabla T-\rho_{ph}\nabla(\mu-cZ/\rho)=0.\end{gathered} \tag{5}$$

Here $\mathbf{w}=(1-\rho_{\text{ph}}/\rho_3Q)\mathbf{v}_n$, $\sigma_3=S_3/\rho$, S_3 is the impurity entropy, $\rho_n=\rho_{n3}+\rho_{\text{ph}}$, $\rho_s=\rho-\rho_n$, P is the pressure, and the thermodynamic potentials μ and Z are connected with the chemical potentials of the solvent and of the dissolved matter by the relations[4]

$$\mu=c\mu_3+(1-c)\mu_4, \quad Z=\rho(\mu_3-\mu_4).$$

The last equation of (5) can be obtained in standard fashion with the aid of the equation of superfluid motion and the kinetic equation by going to the hydrodynamic equations. It must be recognized here that owing to the diffuse reflection from the capillary walls and to the satisfaction of the condition $\xi \gg \lambda_T$ the phonons are characterized by and equilibrium distribution function with a zero macroscopic velocity. Qualitatively, the term $-s_{\text{ph}}\nabla T$ corresponds to the partial contributions of the phonons $l_{\text{ph}} \gg d$ to the thermomechanical effect, while the term $-\rho_{\text{ph}}\nabla(\mu-cZ/\rho)=\rho_{\text{ph}}\dot{\mathbf{v}}_s$ reflects the fact that a definite mass of liquid, equal to ρ_{ph}, now no longer transports any momentum.

The acoustic solutions of the system (5) are determined with the aid of the usual procedure.[4,7] We assume that small perturbations of all the equilibrium quantities vary in the sound-wave field like $\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, and $s=\omega/k$ is the wave propagation speed. Using the hydrodynamic equation (5) and the thermodynamic identities, we readily obtain

$$\delta(\mu - cZ/\rho) = B_1\delta P - D_1\delta\sigma, \quad \delta T = B_2\delta P + D_2\delta\sigma,$$

$$B_1 = \frac{1}{\rho}\left(1+\beta_0-\gamma\frac{\partial\rho}{\partial P}\right), \quad B_2 = \frac{1}{\rho}\frac{\partial T}{\partial\sigma}\left(\alpha - c\frac{\partial\sigma}{\partial c}\frac{\sigma_{ph}}{\sigma_3}\frac{1}{c_1^2}\right),$$

$$D_1 = \frac{1}{\sigma_3}\left[\bar\sigma_1\bar\sigma_0\frac{\partial T}{\partial\sigma} + c^2\frac{\partial(Z/\rho)}{\partial c} + \beta_1\gamma\right],$$

$$D_2 = \frac{1}{\sigma_3}\frac{\partial T}{\partial\sigma}\left(\bar\sigma_1 - c\frac{\partial\sigma}{\partial c}\frac{\sigma_{ph}}{\sigma_3}\frac{1}{c_1^2}\frac{\partial P}{\partial\rho}\frac{\beta_1}{\sigma_3}\right);$$

$$\sigma_{ph} = \frac{S_{ph}}{\rho} = \frac{2\pi^2}{45\rho}\left(\frac{T}{\hbar u}\right)^3, \tag{6}$$

$$c_1^2 = \frac{\partial P}{\partial\rho}\left[1 - \frac{\sigma_{ph}}{\sigma_3}\left(\alpha\frac{\partial T}{\partial\sigma}\sigma_3 + \beta_1\right)\right], \quad \alpha = \frac{1}{V}\frac{\partial V}{\partial T};$$

$$\bar\sigma_0 = \sigma - c\frac{\partial\sigma}{\partial c}, \quad \bar\sigma_1 = \sigma_3 - c\frac{\partial\sigma}{\partial c},$$

$$\beta_i = \frac{c}{\rho}\frac{\partial\rho}{\partial c} - \alpha\frac{\partial T}{\partial\sigma}\bar\sigma_i \quad (i=0,1);$$

$$\gamma = \frac{1}{c_1^2}\frac{\sigma_{ph}}{\sigma_3}\frac{\partial P}{\partial\rho}\left[c^2\frac{\partial(Z/\rho)}{\partial c} - c\frac{\partial T}{\partial\sigma}\frac{\partial\sigma}{\partial c}\bar\sigma_0\right].$$

The system (5) is then equivalent to two coupled wave equations

$$\frac{\partial^2 P}{\partial t^2} - c_1^2(1 - S_{ph}B_2 - \rho_{ph}B_1)\Delta P + \frac{\partial P}{\partial\rho}\frac{\rho}{\sigma_3}\beta_1\frac{\partial^2\sigma}{\partial t^2} + c_1^2(S_{ph}D_2 - \rho_{ph}D_1)\Delta\sigma = 0,$$

$$\frac{\partial^2\sigma}{\partial t^2} - (D_1\Gamma_1 + S_{ph}D_2\Gamma_2)\Delta\sigma + [B_1\Gamma_1 + (1 - S_{ph}B_2)\Gamma_2]\Delta P = 0, \tag{7}$$

where

$$\rho_s\frac{\sigma}{\rho} - (\rho_s + \rho_{ph})\Gamma_2 = \Gamma_1, \quad \Gamma_2 = \frac{\sigma}{\rho} - \frac{\sigma_3}{\rho_{n3}}\left(1 - \frac{\rho_{ph}}{\rho_3 Q}\right).$$

The condition that Eqs. (7) be compatible leads to a biquadratic dispersion equation whose solutions are in the general shape quite cumbersome and will not be written out here.

Thus, with decreasing diameter of a sufficiently rough capillary and when the condition $l_{ph} \gg d \gg l_i$ is satisfied, the velocities of the first and second sounds vary in accord with Eqs. (5) and (7). Their difference from the first- and second-sound velocities in an unbounded liquid is greater the larger the relative contribution of the phonons to a-1 the thermodynamic quantities, i.e., at possibly smaller concentrations and larger temperatures ($T \approx 0.7$ K, $c \gtrsim 0.5\%$).

At low concentrations and neglecting the thermal-expansion coefficient α, expressions (6) and (7) are greatly simplified. Furthermore, by virtue of the condition $c \gg 10^{-2}T^2$, the number of the phonons is still much lower than the number of impurity atoms, and Eqs. (7) can be expanded in powers of ρ_{ph}/ρ_3 and σ_{ph}/σ_3. In first-order approximation in the small contribution, the system (7) splits into two independent equations that determine the velocities of the first (s_1) and second (s_2) sounds in a narrow channel ($s_1 \gg s_2$):

$$s_1^2 = \frac{\partial P}{\partial\rho}\left(1 - \frac{\rho_{ph}}{\rho}\right), \quad s_2^2 = c_2^2(1 + R_{ph}) - 2\frac{\rho_s}{\rho_n}\sigma_{ph}\bar\sigma_0\frac{\partial T}{\partial\sigma}, \tag{8}$$

where c_2 is the velocity of second sound in an unbounded liquid[4,7]:

$$c_2^2 = \frac{\rho_s}{\rho_n}\left[\bar\sigma_0^2\frac{\partial T}{\partial\sigma} + c^2\frac{\partial(Z/\rho)}{\partial c}\right],$$

and the function R_{ph} is defined as

$$R_{ph} = \frac{\rho_{ph}}{\rho}\left[1 + \frac{\rho}{\rho_n} - \frac{\rho}{\rho_3 Q}\right].$$

In the case of low concentrations, the function Q in the expression for R_{ph} can be calculated in explicit form with the aid of the hydrodynamic equations for a degenerate solution at $T=0$. Substituting in (4) the values of the chemical potentials μ_3 and μ_4 and of the first-sound velocity u at $T=0$ for a degenerate solution,[8] retaining only the terms of first order in the concentration, we get

$$Q = \left(\frac{M}{m_4}\right)^2\left(\varkappa + \frac{\delta m}{m_4}\right)^{-2}\frac{1}{c}.$$

Here m_3 and m_4 are respectively the masses of the atoms He^3 and He^4; $M \approx 2.3m_3$ is the effective mass of the solitary impurity quasiparticle, so that

$$\rho_{n3} = \rho_3 M/m_3, \quad \delta m = M - m_3, \quad \varkappa = (\rho_4/m_4 s_0^2)\,\partial(-\Delta)/\partial\rho_4 \approx 1.28;$$

s_0 is the speed of sound in pure He^4, and $(-\Delta)$ is the binding energy of the impurity He^3 atom in superfluid He^4.

The function cQ depends, as it should, on the quantities that describe the interaction of the impurity quasiparticle with the superfluid background, and is of the order of unity. The function R_{ph} then takes the form

$$R_{ph} = \frac{\rho_{ph}}{\rho_s}\left[1 - \left(\frac{m_4}{M}\right)^2\left(\varkappa + \frac{\delta m}{m_4}\right)^2 + \frac{1}{c}\frac{m_3}{M}\right] \approx \frac{m_3}{M}\frac{\rho_{ph}}{\rho_3}.$$

Within the limits of accuracy, in all the terms of (8) that contain the phonon factors σ_{ph}/σ_3 or ρ_{ph}/ρ_3, it suffices to consider only the contribution of the He^3 impurity atoms.

The relative change of the first- and second-sound velocities is thus of the order of σ_{ph}/σ and ρ_{ph}/ρ, and for a solution with concentration 0.5% and temperature 0.7 K it reaches 1 – 3% (for second sound) and can be observed in experiment. The condition on the capillary diameter then takes the form $0.9 \gg d \gg 0.3 \times 10^{-5}$ cm. With further decrease of the channel dimension and if the conditions $l_{ph} \gg d$ and $l_i \gg d$ are satisfied, only on acoustic mode remains in the solution—fourth sound.[9]

In all the arguments presented above we have neglected the presence of second-sound phonons in the solution. In the absence of a capillary, when the condition $c \gg 10^{-2}T^2$ is satisfied, the contribution of the second-sound phonons to all the thermodynamic quantities can be neglected since it is cut off by the very stringent requirment $\lambda \gg l_{ph} \gg l$ under which the hydrodynamic approach is valid. On the other hand in the presence of a capillary with $l_{ph} \gg d \gg l$, when the phonons do not take part in the hydrodynamic motion, the cutoff occurs at much shorter wavelengths, $\lambda \gg l_i \sim a/c$, where a is the mean distance between the He^4 atoms, and at larger solution concentrations the contribution of the second-sound phonons can become substantial. However, the effect in question is most noticeable precisely at low concentrations, when there is no need to take the second-sound phonons into account.

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Collective excitations of electrons and holes in a strong electromagnetic field and absorption of light in a semiconductor under parametric resonance conditions

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The Bogolyubov–Mitropol'skiĭ averaging method is used to solve the two-band equation in the Kane model of a semiconductor and to investigate the quasienergy spectrum of electron–hole excitations in the field of a strong circularly polarized electromagnetic wave whose frequency ω is closed to the band gap ε_g of the semiconductor. The main feature of the spectrum is that the electromagnetic wave lifts the spin degeneracy of the levels due to free electrons and holes. One of the split dispersion curves is shifted only slightly compared with the curve for a free particle and the other exhibits—under certain conditions—a discontinuity of the order of $\rho\varepsilon_g$ (ρ is a parameter proportional to the electric field of wave). The gap in the spectrum appears for $0<\omega-\varepsilon_g<\rho\varepsilon_g$; the overlap of the quasienergy bands closes the gap for $\omega-\epsilon_g>\rho\varepsilon_g$ but discontinuities of the dispersion curves remain. The absorption coefficient of a weak electromagnetic wave of frequency ω_1 is calculated. It is shown that creation of electron–hole excitations at levels without a discontinuity in a strong field results in considerable absorption near the frequency $\omega_1=\omega$.

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1. INTRODUCTION

A strong electromagnetic field acting on a semiconductor not only heats the carriers but can also cause more fundamental dynamic changes in the electron–hole subsystem, deforming and modifying its energy spectrum.[1-3] Distortions of the spectrum are particularly large in various resonance situations, for example, in the case of cyclotron resonance,[4,5] when a semiconductor is subjected to electromagnetic radiation of frequency close to the width of the band gap (forbidden band)[6] or of the gap between two conduction bands.[7] It is important to note that under resonance conditions even a weak interaction may alter radically the energy spectrum of the system (see, for example, Oleĭnik's paper).[4] Physically, a change in the energy spectrum of a particle under the action of an electromagnetic wave implies the appearance of collective excitations of the "particle (electron or hole) + photons" type, which are states of electrons and holes strongly coupled to the electromagnetic wave field. In electromagnetic fields causing radical changes in the energy spectrum and, consequently, altering the very nature of the electron and hole motion, the behavior of the system could be described entirely in terms of these collective excitations.

We shall use the Bogolyubov–Mitropol'skiĭ averaging method[8] to solve the two-band equation in the Kane model of a semiconductor (Sec. 2) and we shall investigate the quasienergy spectrum of electron–hole excitations in the field of a strong circularly polarized electromagnetic wave described by the potential[1)]

$$\mathbf{A}=a(\cos kx,\ \sin kx,\ 0), \tag{1}$$

where $a=\text{const}>0$ is the amplitude of the potential; $k=(k_0/c,0,0,k_z)$ is the wave four vector of a photon; $k_z=c^{-1}\varepsilon^{1/2}k_0$; $\varepsilon=\text{const}$ is the permittivity of the medium. We shall consider the case of a parametric resonance when the frequency of an electromagnetic wave $\omega\equiv k_0$ is related to the band gap ε_g of a semiconductor by

$$\omega=\varepsilon_g+\Delta,\qquad |\Delta|\ll\varepsilon_g. \tag{2}$$

We shall define a strong electromagnetic field by the inequality

$$\Omega\gg\omega_{c\tau}, \tag{3}$$

where Ω is the frequency of transitions between the

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