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Magnetoplasma resonance in electron-hole drops in germanium

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A theory of magnetoplasma resonance (MPR) in electron-hole drops (EHD) in germanium is considered which takes account of the real quantum spectrum of the Ge carriers, as well as the shape of the drops in the magnetic field, **H**. The main laws governing MPH in EHD are analyzed. A number of principal parameters characterizing the electron-hole liquid in Ge are determined on the basis of a comparison of the theory with experiment. These are the effective carrier masses, the variation of the equilibrium particle concentration in the drops under the action of up to 40-kOe **H** ∥ [100] and **H** ∥ [111] fields, and the dependence of the carrier-momentum relaxation time on the photon frequency and the magnetic-field intensity. Various mechanisms of plasmon attenuation in EHD in Ge are analyzed.

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1. INTRODUCTION

As is well known, the condensation of excitons into electron-hole drops (EHD) of the metallic type is observed in a number of semiconductors at low temperatures and during intense optical generation of nonequilibrium carriers.[1] Of the wide range of qualitatively new phenomena connected with exciton condensation,[1-3] the plasma[4-6] and magnetoplasma[7-16] phenomena in EHD are some of the most interesting. Caused by the interaction of the EHD with the electromagnetic waves in the region of the plasma and cyclotron frequencies of the carriers, they have a strongly pronounced resonance character.

The investigations of the magnetoplasma phenomena in EHD are especially promising in connection with the study of the fundamental properties of the electron-hole liquid in semiconductors. In the first place this pertains to the determination of the equilibrium density of the liquid, as well as of the parameters of the energy spectrum of the elementary excitations of the liquid under different conditions. It is important to note that, as a result of the smallness (on the atomic scale) of the binding energy of the EHD, the application of a magnetic field not only allows a more thorough investigation of the properties of the electron-hole liquid, but also makes it possible to significantly change the ground state of the liquid under experimental conditions. This significantly broadens the potentialities of submillimeter, UHF and microwave spectroscopies of EHD in a magnetic field in comparison with ordinary metals and semiconductors. At the same time, the complex character of the very magnetoplasma phenomena in EHD makes the extraction of quantitative information from experimental data substantially difficult.

In the present paper we formulate for the magnetoplasma resonance (MPR) in EHD in Ge a theoretical model which takes into account the magnetic-field induced changes in the shape of the drops and in the energy spectrum of the Ge crystal, and allows a detailed quantitative comparison with the experimental data to be carried out. The shape of the drops in a magnetic field is analyzed with allowance for the main influencing factors. A procedure for numerical computations is presented with the aid of which we determine on the basis of a comparison of the theory and experiment a number of fundamental characteristics of the electron-hole liquid in Ge (part of the results of such a comparison has been published in the form of short communications[11,15]). The obtained data on the properties of EHD are discussed from the point of view of existing theories.

2. A THEORY OF MAGNETOPLASMA RESONANCE IN EHD

As shown in Refs. 5 and 6, Mie's general theory,[17] which describes the interaction of electromagnetic waves with spherical particles having arbitrary dimensions and characterized by a scalar permittivity $\epsilon(\omega)$, can be used to interpret the spectra of the plasma resonance (PR) in EHD. Since the dimensions of EHD in undeformed crystals at $T=1.5$ K usually do not exceed 1–2 μ,[4,6] the Rayleigh case k_0r, $|kr|\ll 1$ ($\mathbf{k}_0$ and $\mathbf{k}$

are the wave vectors of the electromagnetic radiation in the Ge crystal and EHD material, respectively, and r is the drop radius) is realized in the observation of PR in EHD. In this case the cross section for wave absorption by a drop is quite accurately given by the expression[4-6]

$$\sigma(\omega)=\frac{2\pi}{k_0^2}\left[2(k_0r)^3\operatorname{Im}\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}+2}+\frac{1}{15}(k_0r)^5\operatorname{Im}\bar{\varepsilon}+\ldots\right], \quad (1)$$

which takes into account, in the order in which the terms appear, the two principal mechanisms of wave-energy absorption: the electric- and magnetic-dipole mechanisms. The first mechanism is due to the damping of the displacement currents induced in the drop by the electric field of the wave, while the second is due to the damping of the eddy currents excited by the magnetic field of the wave. In the formula (1) $\bar{\varepsilon}=\varepsilon(\omega)/\varepsilon_0$, where ε_0 is the permittivity of the Ge lattice.

The presence of an external magnetic field makes the description of the interaction of electromagnetic waves with EHD significantly difficult. Mie's theory is inapplicable in this case, since the permittivity of the EHD material in a magnetic field is a tensor quantity and the shape of the drops ceases to be spherical. Nevertheless, in the Rayleigh approximation, which is realized in experiments on MPR observation in EHD in Ge,[7-12] we can obtain analytic expressions for the absorption cross section of EHD with allowance for both the electric- and magnetic-dipole mechanisms. The inequalities k_0r, $|kr|\ll 1$ imply that a drop can be regarded as a relatively transparent particle located in a spatially homogeneous electric (or, correspondingly, magnetic) field varying in time with frequency ω. Let us consider the interaction of the EHD with the electric field, $\mathbf{E}^\omega$, of the light wave. We shall assume that the drops have the shape of a triaxial ellipsoid, characterized by the depolarization-coefficient tensor L_{kj} ($\operatorname{Sp}L_{kj}=4\pi$).[18] Using the results of the well-known problem of the dielectric ellipsoid in a homogeneous electric field,[18,19] we can write down the following expressions for the components of the polarization vector, $\mathbf{P}$, of the electron-hole gas in a drop:

$$P_i=\frac{1}{4\pi}(\varepsilon_{ik}-\varepsilon_0\delta_{ik})(E_k^\omega-\varepsilon_0^{-1}L_{kj}P_j),$$
$$P_i=\frac{1}{4\pi}\left[\delta_{ij}+\frac{1}{4\pi}(\varepsilon_{ik}-\varepsilon_0\delta_{ik})\varepsilon_0^{-1}L_{kj}\right]^{-1}(\varepsilon_{jn}-\varepsilon_0\delta_{jn})E_n^\omega. \quad (2)$$

Here $\varepsilon_{ik}(\omega,\mathbf{H})$ is the permittivity tensor of the EHD material in a magnetic field $\mathbf{H}$. With allowance for (2), we obtain the following expression for the cross section for electric-dipole absorption by EHD in a magnetic field:

$$\sigma(\omega,H)=\frac{1}{2}V_{\text{EHD}}\operatorname{Re}\left[\frac{dP_i}{dt}(E_i^\omega)^*\right]\Big/\frac{c\varepsilon_0^{1/2}}{8\pi}|E^\omega|^2. \quad (3)$$

Let us note that we did not, in deriving (2) and (3), specify the form of ε_{ik}; therefore, the obtained relations are applicable not only to the case of a magnetic field, but are also of a more general character. Below we shall be interested in the $\mathbf{H}\|[100]$ and $\mathbf{H}\|[111]$ orientations, which are the most symmetric, and in which the ε_{ik} tensor can be reduced to the diagonal form by going over to circular polarizations of the electromagnetic wave in the plane perpendicular to $\mathbf{H}$ while retaining the linear polarization along $\mathbf{H}$. In this case the tensor L_{kj} is also diagonal, since we can on the basis of symmetry considerations regard the EHD as having the shape of an ellipsoid of revolution about $\mathbf{H}$. With allowance for this, we obtain

$$\sigma_{\pm,\|}(\omega,H)=\frac{4\pi\varepsilon_0^{1/2}}{c}V_{\text{EHD}}\omega\operatorname{Im}\left[\frac{\bar{\varepsilon}_{\pm,\|}-1}{4\pi+L_{\perp,\|}(\bar{\varepsilon}_{\pm,\|}-1)}\right]. \quad (4)$$

Here $\bar{\varepsilon}_+$, $\bar{\varepsilon}_-$, $\bar{\varepsilon}_\|$, $L_\perp$, $L_\|$ are the components of the diagonal tensors $\bar{\varepsilon}_{ik}$ and L_{kj} in the system of coordinates with the axis $z\|\mathbf{H}$. The expressions for $\bar{\varepsilon}_{\pm,\|}(\omega,H)$ have the form[14]

$$\bar{\varepsilon}_\pm(\omega,H)=1-\frac{1}{\omega}\left[\frac{(1-\Delta_\perp)\omega_{pe}^2}{\omega+i\gamma\mp\omega_{ce}}+\frac{\Delta_\perp\omega_{pe}^2}{\omega+i\gamma\mp(-\omega_{ce})}+\sum_j\frac{\omega_{pj}^2}{\omega+i\gamma\mp\omega_{cj}}\right];$$
$$\bar{\varepsilon}_\|(\omega,H)=1-\frac{1}{\omega^2+i\gamma\omega}\left[\omega_p^2+\omega_{pe}^2\frac{\Delta_\|\omega_{ce}^2}{(\omega+i\gamma)^2-\omega_{ce}^2}\right]. \quad (5)$$

Here ω_{cj} and ω_{pj} are the cyclotron (with allowance for the sign of the charge) and plasma frequencies of the carriers whose effective masses are isotropic in the plane perpendicular to $\mathbf{H}$; ω_{ce} and ω_{pe} are the analogous quantities for the electrons whose effective-mass ellipsoids are inclined to $\mathbf{H}$;

$$\Delta_\perp={}^1/_2(1-m_e/m_{ce})$$

is the anisotropy parameter for the effective mass of these electrons in the plane perpendicular to $\mathbf{H}$; $\Delta_\|$ is a parameter which, depending on the crystal orientation, is equal to

$$\Delta_\|=(1-m_{ce}^2m_e/m_\|m_\perp^2)$$

for $\mathbf{H}\|[100]$ and

$$\Delta_\|=[1-81m_\|m_\perp/(8m_\|+m_\perp)(m_\|+8m_\perp)]$$

for $\mathbf{H}\|[111]$. The meanings of the remaining symbols are given in Ref. 14.

As can be seen from (4) and (5), the electric-dipole absorption has a resonance character, the resonance frequencies in the case of weak damping being determinable from the condition that the denominator of (4) should vanish:

$$\bar{\varepsilon}_\pm(\omega,H,\gamma=0)=1-4\pi/L_\perp,\quad \mathbf{E}^\omega\perp\mathbf{H},$$
$$\bar{\varepsilon}_\|(\omega,H,\gamma=0)=1-4\pi/L_\|,\quad \mathbf{E}^\omega\|\mathbf{H}. \quad (6)$$

Thus, the determination of the resonance frequencies of the electric-dipole type amounts essentially to the analysis of the frequency dependence of the permittivity of a multicomponent plasma in a magnetic field (Fig. 1). Such an analysis (it is most conveniently carried out graphically) allows the establishment of the following facts:

1. In the case of the $\mathbf{H}\|[100]$ and $\mathbf{H}\|[111]$ (i.e., most symmetric) orientations, we can separate the interactions of the EHD with the electromagnetic waves of

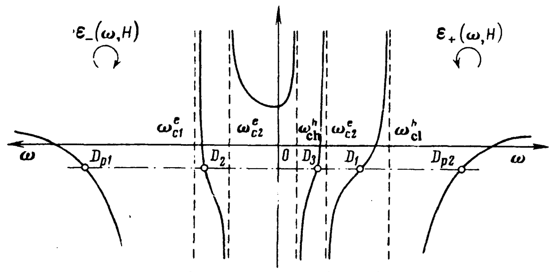

FIG. 1. Scheme of the graphical analysis of the position of the resonance frequencies of MPR in EHD in Ge for H || [111] and electromagnetic waves of circular polarization in the "electron" (ε_-) and "hole" (ε_+) directions. D_{p1}, D_{p2} and D_1, D_2, D_3 are respectively the frequencies of the plasma and cyclotron branches of the MPR in EHD; ω^e_{c1}, ω^e_{c2} and ω^h_{cl}, ω^h_{ch} are the cyclotron frequencies of the free (light and heavy) electrons and the free (light and heavy) holes in Ge, respectively.

left-hand (e_+) and right-hand (e_-) circular polarizations ($\mathbf{E}^\omega \perp \mathbf{H}$), as well as of linear ($e_\parallel$) polarization ($\mathbf{E}^\omega \parallel \mathbf{H}$).

2. The possibility of resonance interaction of the carriers with an electromagnetic wave of definite polarization (e_+, e_-, or $e_\parallel$) depends on the orientation of the principal axes of the corresponding effective-mass ellipsoid relative to **H**. If one of the principal axes (the axis of revolution of the ellipsoid) is directed along **H**, then resonance interaction takes place only with $\mathbf{E}^\omega \perp \mathbf{H}$ waves circularly polarized in a definite direction (e_+ or e_-, depending on the sign of the charge of the carriers). If along the magnetic field is directed a less symmetric principal axis of the ellipsoid, then the carriers are resonantly active toward waves of both circular polarizations, e_+ and e_-, but inactive toward $e_\parallel$. Finally, if all the principal axes of the ellipsoid are inclined to **H**, then the carriers are resonantly active toward all the three polarizations. To this type of carrier correspond the discontinuities at $\omega = |\omega_c|$ for all the three components $\tilde{\varepsilon}_{\pm,\parallel}(\omega, H, \gamma = 0)$.

3. There is always one solution representing the electric-dipole-type MPR in EHD on the short-wave side of each cyclotron-resonance (CR) frequency of the free carriers (in particular, between every two neighboring cyclotron frequencies corresponding to the same polarization). As $(\omega_{pj}/\omega_{cj}) \to 0$, the MPR frequencies tend asymptotically to the corresponding CR frequencies of the free carriers in the drop. This leads, generally speaking, to a nonmonotonic short-wave shift of the resonance frequencies of the MPR with increasing H even when the particle concentration in the EHD remains constant (Fig. 3).

4. The frequencies of the MPR in EHD can be split up into two types: the "cyclotron" MPR branches, which shift toward $\omega = 0$ as $H \to 0$, and the "plasma" branches, which tend to $\omega_0 = \omega_p/3^{1/2}$ as $H \to 0$ (see Fig. 3 below). The number of "cyclotron" branches depends on the orientation of H and the number of valleys in the energy spectrum of the carriers, whereas the number of "plasma" branches is always three—one for each of the three types of wave polarization.

5. The intensity of the electric-dipole-type resonances is, as can be seen from (4), proportional to $\omega_{res}[\mathrm{Im}\tilde{\varepsilon}(\omega_{res})]^{-1}$, i.e., decreases with decreasing frequency faster than ω^2. As a result, when $|\omega_{cj}| < \omega_{pj}$ the absorption in the region of the "cyclotron" branches is significantly weaker than the absorption at the plasma frequencies.

The electric-dipole approximation turns out to be sufficient for the description of the experimental data on MPR in EHD in Ge[7-12] discussed in the present paper. At the same time, as can be seen from example (1), as the wavelength of the probing radiation, or the size of the drops, increases, it becomes more and more necessary to take into account the subsequent terms of the expansion in powers of $k_0 r \ll 1$, first and foremost the term corresponding to the magnetic-dipole absorption. As a simple analysis of (1) and (4) shows, the relative intensity of this absorption increases with wavelength, and at $\lambda_{vac} = 1-2$ mm, $r = 1\ \mu$, it becomes comparable to the electric-dipole absorption. But in the far IR region, where $\lambda = 0.04-0.8$ mm, the fraction of the magnetic-dipole absorption in the total absorption by small EHD ($r = 1\ \mu$) remains, both in the presence of

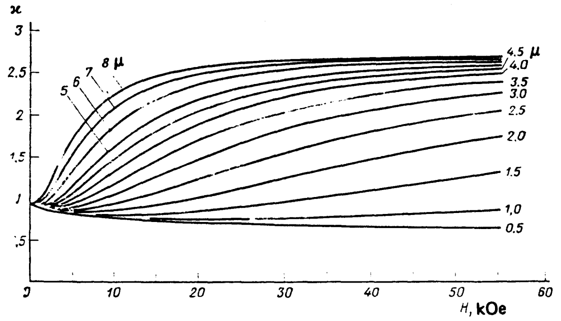

FIG. 2. Variation of the shape of EHD of different sizes in a magnetic field, computed for Ge from the formula (7): $n_k = 3\times10^{17}$ cm^{-3}, $\sigma = 2\times10^{-4}$ erg/cm^2, $\tau = 2.7\times10^{-10}$ sec, $\tau_k = 2\times10^{-5}$ sec, $\rho_0 = 250$ g$^{1/2}$-cm$^{-3/2}$-sec^{-1}, $r_{eff} = 0.5-8\ \mu$ (these r_{eff} values are indicated on the respective curves).

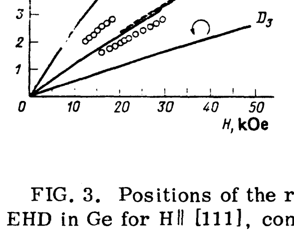

FIG. 3. Positions of the resonance frequencies of the MPR in EHD in Ge for H ∥ [111], computed in the electric-dipole approximation: a) without allowance for the quantum deformations of the valence band—the continuous lines ($n_k = 2\times10^{17}$ cm^{-3}); b) with allowance for the quantum deformations of the valence band in a magnetic field—the dash-dot lines ($n_k = 2\times10^{17}$ cm^{-3}) and the dashed lines ($n_k = 3.7\times10^{17}$ cm^{-3}). The masses of the carriers in the EHD were assumed to be equal to the masses of the free carriers in Ge. Experiment: ○) taken from Refs. 7, 8, 11, and 12; △) taken from Refs. 9 and 10.

an external magnetic field and in zero H field, quite insignificant.

As the relations (6) show, the locations of the resonance frequencies of the MPR in EHD are essentially connected with the shape of the drops. In its turn, as will be seen from what follows, the shape of the drops depends on their size and the intensity of the magnetic field. Such a dependence is a fundamental characteristic of magnetoplasma phenomena in EHD, as distinct from the analogous phenomena occurring in fairly small samples of semiconductor crystals. Thus, the construction of an experimentally adequate theory of MPR in EHD should also include an analysis of the drop shape.

The shape of EHD in a magnetic field is determined by a relation connecting the following factors[16,20]: the surface tension, the self-phonon wind,[21] recombination magnetization,[22] as well as magnetization due to the paramagnetic and diamagnetic susceptibilities of the carriers. As estimates show,[16,20] the contribution of the last factor is negligibly small in comparison with the influence of the rest. The equilibrium shape of a drop is determined by the minimum energy, W_t, corresponding to the total contribution of the above-enumerated factors to the total EHD energy. Assuming that the drop has roughly the shape of an ellipsoid of revolution about the direction of **H**, with a volume $V = \frac{4}{3}\pi a b^2$ and a semiaxis ratio $\varkappa = b/a$, we can derive for W_t the following expression[16,20]:

$$W_t(\varkappa, H) = 4\pi\sigma_\perp \left(\frac{3V}{4\pi}\right)^{2/3} \left\{ \frac{1}{2} f_1(\varkappa) + \frac{V}{V_0(0)} \left[f_2(\varkappa) - \left(\frac{H}{H_0}\right)^2 f_3(\varkappa) \right] \right\} \tag{7}$$

$$H_0^2 = 16\pi \frac{c^2}{e^2} \frac{\tau_k}{\tau} \frac{m}{n_k} \rho_0^2, \quad V_0(0) = \frac{5\sigma_\perp}{\rho_0^2}.$$

Here τ_k is the lifetime, τ is the momentum-relaxation time, m is the characteristic mass, n_k is the carrier concentration in the EHD, ρ_0 is a constant characterizing the force of the phonon wind, $\sigma_\perp$ is the coefficient of surface tension of the EHD in the case when the normal to the surface of the drop is perpendicular to **H**. The functions $f_i(\varkappa)$, which depend only on the ratio of the semiaxes of the ellipsoid, are given in their explicit form in Refs. 16 and 20.

By minimizing (7), we can compute the shapes of EHD of different sizes as functions of the magnetic-field strength. In Fig. 2 we present the results of such computations, as applied to EHD in germanium, for the most characteristic values of the effective drop radius, $r_{eff} = (3V/4\pi)^{1/3}$. The quantities H_0 and $R_0 = [3V_0(0)/4\pi]^{1/3}$ in (7) are, for typical values of the corresponding parameters,[20] ~1 kOe and ~16 μ. As can be seen from (7), with allowance for the specific H_0 and R_0 values, the indicated dependences can also be used to analyze the shape of the drops in other semiconductors.

The dependences presented in Fig. 2 show that the increase of the magnetic-field intensity leads at first to the elongation of the drops along the field ($\varkappa < 1$). The principal effect in this case is due to the anisotropy of the coefficient of surface tension of EHD in a magnetic field (the first term in (7)). As H is increased further, however, the tendency, due to the recombination magnetization of the EHD (the third term in (7)), of the drops to flatten along the field ($\varkappa > 1$) begins to predominate. The results of the present computations indicate (Fig. 2) that the EHD in Ge should, apparently, undergo severe deformation ($\varkappa \approx 1.5-2.3$) even in the relatively weak fields $H \approx 5-10$ kOe.

3. COMPARISON OF THE THEORY OF MPR IN EHD WITH EXPERIMENT

As can be seen, the totality of the experimental data obtained in the investigation of the magnetoplasma-absorption spectra of EHD in Ge in a broad band of long-wave IR and submillimeter waves ($\lambda = 0.04-0.8$ mm)[7-12] corresponds to the picture, discussed in Sec. 2, of electric-dipole magnetoplasma absorption in EHD. In fact, both the "plasma" and the substantially less intense "cyclotron" MPR branches are experimentally observed, the number of the latter branches being dependent on the orientation of **H**. Thus, the cyclotron branch D_1 (Fig. 1), which is due to the presence of light and heavy holes, has been recorded in **H**‖[111],[9-12] as well as in **H**‖[100].[10] At the same time, the solution D_2, which exists in **H**‖[111] (Fig. 1), and which is due to the existence of "light" and "heavy" electrons, is absent in **H**‖[100], when all the electron ellipsoids occupy equivalent positions. Finally, measurements on submillimeter waves of circular polarization show that the D_2 line (the polarization of the other solutions has thus far not been investigated) is observed in the case of circular polarization in the electron-active direction.[12] This result corroborates one of the main consequences of the above-considered theory of MPR in EHD.

Thus, we can, in discussing the experimental data,[7-12] limit ourselves to the electric-dipole approximation, and use the expressions (4) and (5). In this case remaining in the theory as free parameters are the particle concentration n_k, the attenuation constant γ, the depolarization factor $L_\perp$, the effective electron masses $m_\parallel^{EHD}$, $m_\perp^{EHD}$, and the effective hole mass m_h^{EHD}. According to Ref. 23, the many-particle renormalization of the energy spectrum of the carriers in a drop amounts to changing their effective masses: $m_\parallel^{EHD} = \beta_\parallel m_\parallel$, $m_\perp^{EHD} = \beta_\perp m_\perp$, and $m_h^{EHD} = \beta_h m_h$. As the analysis of the shape of the drops in a magnetic field shows (see Sec. 2), the magnitude of the depolarization factor for $r \sim 1$ μ and $H \approx 20-40$ kOe differs little from $L_\perp = 4\pi/3$. Thus, the adjustable parameters in the numerical computations, expounded below, of the spectral dependence of MPR in EHD in Ge were n_k, γ, $\beta_\parallel$, $\beta_\perp$, and β_h. As shown below, owing to the presence of several resonance branches of the MPR, the values of these parameters can, to a reasonable extent, be determined independently and, consequently, uniquely.

Figure 3 shows the positions, computed from the formulas (4) and (5), of the electric-dipole resonance frequencies of the MPR in EHD in Ge as functions of the magnetic-field intensity for **H**‖[111]. In the computations we took into account the change in the density of states of the carriers in the magnetic field, a change which leads, in particular, to a change in the relative population of the various valleys. The solid curves correspond to $n_k = 2 \times 10^{17}$ cm^{-3}, $\beta_i = 1$, and the simplest assumption about the spectrum of the valence band in a magnetic field: the light- and heavy-hole model. It is not difficult to see that, although this simple model is adequate for a qualitative interpretation of the MPR phenomenon in EHD, it is quite unsuitable for a quantitative description of the results of measurements. Therefore, in the subsequent computations we took into account the real quantum spectrum, obtained in Refs. 24 and 25,[1)] of the holes in the magnetic field. In the course of the computations with each of the filled j-th Landau subband (at fixed n_k and H) was associated its own type of isotropic carriers and, consequently, its own Drude-type term in the formula (5). The plasma frequencies figuring in such terms were determined on the basis of the dispersion curves $\mathfrak{E}_j(\zeta)$[25] and the formulas

$$n_j=\frac{1}{2\pi^2}\left(\frac{eH}{\hbar c}\right)^{3/2}\zeta_j^F,\quad m_j^{-1}=\frac{1}{\zeta_j^F}\frac{\partial\mathfrak{E}_j}{\partial\zeta}\Big|_{\zeta_j^F},\qquad(8)$$

where ζ_j^F is the dimensionless Fermi momentum of the carriers in the j-th subband,

$$\mathfrak{E}=\mathscr{E}\Big/\frac{eH}{mc}\hbar,\quad \zeta=k_H\Big/\left(\frac{eH}{\hbar c}\right)^{1/2}.$$

As the corresponding cyclotron frequencies we chose the energy distances, averaged over $0\leq\zeta\leq\zeta_j^F$, from the Landau subbands under consideration to the higher-lying subbands transitions to which are allowed by the selection rules formulated in Ref. 24. In the computations we took into account only the most intense M_0-type transitions,[24] whose matrix elements were assumed to be equal to each other. The signs with which the corresponding cyclotron frequencies entered into the Drude expression (5) were also determined on the basis of the selection rules given in Ref. 24.

The above-described procedure is equivalent to taking account of the transitions of the carriers within the heavy- and light-hole branches. Besides this, as shown in Ref. 5 for the $H = 0$ case, it is also necessary to take into consideration the transitions between the valence-band branches (the 1–2 transitions). Because of the large magnitude of the attenuation constant of the plasma oscillations in the EHD ($\gamma\approx 2$ meV), the quantization of the valence band of Ge could, in considering the 1–2 transitions in magnetic fields of intensities of up to 30–40 kOe, be neglected. Accordingly, to account for the contribution of the 1–2 transitions to the expression (5) we included the term $\bar{\varepsilon}_{12}(\omega)$ in the form in which it is presented in Ref. 5.

Allowance for the actual structure of the valence band of Ge in a magnetic field permitted us to give a fairly good account of both the locations of the maxima (Fig. 4) and the structure of the measured spectra of the MPR in EHD. Here two characteristic regularities pertaining to the "cyclotron" branches D_1 and D_2 manifested themselves. First, the position of these branches in fields of up to 35 kOe turned out to be not very sensitive to changes in the equilibrium particle concentration in the drops: the dash-dot curves in Fig. 3 were computed for $n_k = 2\times10^{17}$ cm^{-3}; the dashed curves, for $n_k = 3.7\times10^{17}$ cm^{-3}. This is not unexpected, since in the limit

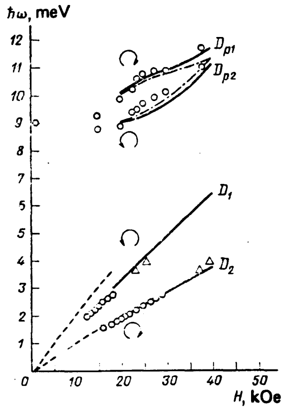

FIG. 4. Comparison of the measured (○—from Refs. 7, 8, 11, and 12; △—from Refs. 9 and 10) and computed values of the resonance MPR frequencies in EHD in Ge for H∥[111]. The continuous curves give the results of the computation with allowance for the quantum deformations of the valence band of Ge and the renormalization of the carrier masses in EHD and with $n_k=3.7\times10^{17}$ cm^{-3}. The dash-dot curves are the results of a similar computation, but one in which the circular polarizations of all the transitions in the valence band of Ge are taken to be the same. The dashed curves are the results of a computation performed for the renormalized carrier masses, but with allowance for the quantum deformations.

$|\omega_{cj}|\ll\omega_{pj}$ the positions of the D_1 and D_2 branches should not depend on n_k at all, being determined only by the effective masses of the carriers.[14] Secondly, the choice of one or another model for the spectrum of the valence band significantly affects only the position of the D_1 branch and virtually does not affect the computed location of the D_2 branch (the solid and dashed curves in Fig. 3 and 4).

The indicated distinctive features allow us to determine, on the basis of measurements in the region of the "cyclotron" MPR branches, the effective electron and hole masses in the drops (i.e., to find the renormalization parameters β_i). Since the position of the D_1 branch turned out to be mainly dependent on β_h, while the position of the D_2 turned out to be mainly dependent on $\beta_\perp$, these coefficients could be determined independently of each other. The best agreement between theory and experiment was obtained for

$$\beta_h=1.15\pm0.03,\quad \beta_\perp=1.10\,{}^{+0.04}_{-0.02}$$

(Fig. 4). The parameter $\beta_\parallel$, which almost does not affect the position of the "cyclotron" branches in the **H** orientation in question, was assumed in the computations to be equal to unity, in accord with the estimates in Ref. 23.

Having thus determined the effective masses of the carriers in the EHD, we can then find their concentration by comparing the theory with experiment in the region of the "plasma" MPR branches, whose position depends on both n_k and β_i. As can be seen from Fig. 4, the concentration $n_k = 3.7\times10^{17}$ cm^{-3} makes it possible in the first approximation to describe satisfactorily the results of the measurements in 20–40 kOe **H**∥[111]

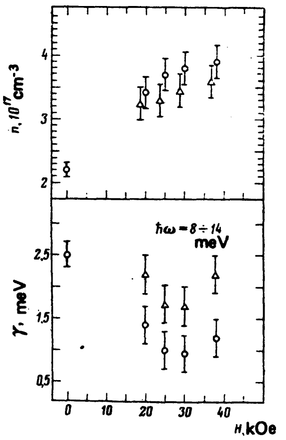

FIG. 5. Dependence of the carrier concentration, as well as of the attenuation constant of the plasma oscillations in EHD in Ge, on the intensity and orientation of the magnetic field ($\hbar\omega = 8$–14 meV; $T = 1.5$ K): ○) **H** || [111], △) **H** || [100].

fields. A more detailed comparison of the location and structure of the experimental MPR spectra in EHD with the theoretical results allows us to determine more accurately the values of n_k and γ, as well as to follow their dependence on the intensity of the magnetic field in different orientations. The results of such a comparison, which are shown in Fig. 5 for the two orientations **H**||[111] and **H**||[100], indicate a substantial increase in the equilibrium particle concentration in EHD under the action of a magnetic field, a greater increase in n_k being observed in the case of the **H**||[111] orientation. There also occurs in these same fields an appreciable decrease in the attenuation constant of the plasma oscillations, this decrease being more substantial again in **H**||[111] (Fig. 5).

The presence of several MPR branches in EHD in principle enables us to follow the frequency dependence of the attenuation of the magnetoplasma oscillations in H = const (Figs. 3 and 4). Such a dependence, constructed on the basis of the experimental data on the MPR-line width in EHD in Ge,[7-12] is shown in Fig. 6. As can be seen from the figure, the quantity γ increases by more than an order of magnitude in the frequency band $\hbar\omega = 2$–10 meV.

4. DISCUSSION OF THE OBTAINED RESULTS

Magnetoplasma resonance in electron-hole drops in germanium is a fairly complex resonance phenomenon. This, on the one hand, makes it quite informative in connection with the study of the properties of the electron-hole liquid, but at the same time it makes the extraction of such information from the experimental data extremely difficult. One of the main products of the present paper consists in the fact that a fairly reliable quantitative description of the experiments on MPR in EHD in Ge has been carried out in it on the basis of the MPR theory developed here, which takes into consideration both the change in the drop shape and the complex quantum spectrum of the charge carriers in Ge in a magnetic field.

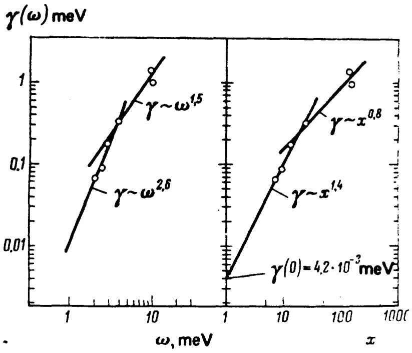

FIG. 6. The frequency dependence of the attenuation constant of the plasma oscillations in EHD in Ge: $T = 1.5$ K, **H** || [111], $H = 20$–25 kOe, $x = 1 + (\hbar\omega/2\pi kT)^2$.

As a result of such a comparison, it has been shown that the effective electron and hole masses in the electron-hole liquid differ from the masses of the free carriers in the crystal. The obtained data characterize the initial (at $H = 0$) many-particle renormalization of the spectrum of the carriers, and correlate with the theoretical estimates.[23]

The substantial increase in the thermodynamic-equilibrium particle concentration in EHD in relatively weak magnetic fields of up to 40 kOe in intensity (Fig. 5) is due, apparently, to a sharp increase in the correlation energy of the EHD in the magnetic field—the so-called "effect of self-compressibility" of the electron-hole liquid.[26] According to Ref. 26, the equilibrium particle density in EHD in a semiconductor located in an ultra-high-intensity ($H \gg H_0$) magnetic field

$$n_k \sim (H/H_0)^{6/7} a_0^{-3}. \tag{9}$$

Thus, in the limit of ultra-high-intensity magnetic fields, the theory predicts an almost linear growth of n_k with the field. In this case the coefficient of proportionality depends on the orientation of H. Estimates from the formula (9) show that in identical fields $n_k([111]) > n_k([100])$. Our measurements of the n_k(**H**) dependence pertain to the region of intermediate fields, 20–40 kOe, where the variation of the particle concentration in EHD in germanium goes over from being oscillatory to being monotonic. As is well known, a quantitative comparison of experiment with theory turns out to be most difficult in this range. However, the qualitative characteristics of the observed variation of n_k(**H**) in different orientations are, as can be seen from Fig. 5, in good agreement with the results of the theory.[26]

Let us discuss, in conclusion, the experimental data concerning the dependence of the attenuation of the plasma oscillations in EHD on ω and H. The information obtainable about the frequency dependence of γ in investigations of MPR in EHD pertains to a broad spectral band. This band extends to frequencies of the order of the plasma frequency, and includes the region of photon energies $\hbar\omega \sim \mathcal{E}^F$ ($\mathcal{E}^F$ is the Fermi energy of the electrons or holes in the EHD), which is extremely interesting for the optics of metals.

In Fig. 6 we show the frequency dependence of the parameter γ, which gives the width of the various peaks of the magnetoplasma absorption in EHD in Ge in H = 20–25 kOe fields (Fig. 4).[7-12] In this dependence we can separate out two regions of power-law growth: in the region $\hbar\omega \lesssim \mathcal{E}^F \approx 3$ meV the attenuation increases in proportion to $\omega^{2.6}$, while in the region $\hbar\omega > \mathcal{E}^F$, as we approach ω_p, this growth slows down substantially and is characterized by a law close to $\omega^{1.5}$.

In principle, the range of attenuation mechanisms for the plasma oscillations in EHD is fairly wide. These are single-particle processes—the collisions of the carriers with each other, with the phonons, with the impurities, and with the EHD boundaries—as well as collective attenuation processes, in the course of which the plasmon, as a quasiparticle, transfers all its energy either to the photon (radiative damping), or to an individual carrier (Landau damping). In the case of Ge, which has a degenerate valence band, plasmon attenuation can also occur as a result of the transfer of carriers from the heavy-hole band to the light-hole band.

Of the single-particle attenuation mechanisms, the most effective, as shown in Refs. 4 and 5, is the electron-hole collision mechanism, whose contribution is given by the following expression:

$$\gamma_{e-h}(\omega)=\gamma_{e-h}(0)\left[1+\left(\frac{\hbar\omega}{2\pi kT}\right)^2\right]. \qquad (10)$$

If we use the dependence (10) and the $\gamma_{e-h}(0)$ value corresponding to the low frequency carrier mobility in EDH, $\mu_0=(0.5-2)\times10^6$ cm^2/V-sec,[27,28] which is the same for the electrons and the holes, then the quantity $\hbar\gamma_{e-h}$ in the region of the plasma frequencies of EHD ($\hbar\omega\approx10$ MeV) at $T=1.5$ K turns out to be of the order of 1 meV. The assumption that the electron-hole collisions make an appreciable contribution to the observable attenuation of the plasma oscillations in EHD is also corroborated by the nature of the $\gamma(H)$ dependence. As is well known, $\gamma_{e-h}(0)\sim\mu_0^{-1}\sim n^{-4/3}$ in the case of electron-hole scattering.[27,28] The experimental data presented in Fig. 6 indicate a clear correlation between the growth of n_k and the decrease of γ in 20–40-kOe fields.

It should, however, be borne in mind, when comparing the experimental $\gamma(\omega)$ dependence with the formula (10), that this formula was derived under the assumption that $\hbar\omega\ll\mathcal{E}^F_{e-h}$. In the other limiting case, $\hbar\omega\gg\mathcal{E}^F_{e,h}$, the scattering of a high-energy electron (hole), after virtually absorbing a quantum $\hbar\omega$, is rather like the Rutherford scattering of charged particles.

It may be inferred, therefore, that in the $\hbar\omega\gtrsim\mathcal{E}^F_{e,h}$ region the frequency dependence $\gamma_{e-h}(\omega)$ should be saturated, and should, generally speaking, no longer be described by the formula (10).[4,5] Thus, the above-given estimate for the attenuation constant γ_{e-h} is evidently too high. Furthermore, as can be seen from the right-hand part of Fig. 6, the experimentally observed dependence turns out in some section of the spectrum to be even stronger than the dependence predicted by the expression (10). All this apparently indicates that, along with the single-particle mechanisms, there also appear collective mechanisms of plasmon attenuation in EHD. One of the most effective among them may turn out to be Landau damping. This is due to the fact that the plasmon wave vector in EHD is, in contrast to the situation in an unbounded plasma, not a "good" quantum number and is, in accordance with the uncertainty principle, conserved only to within terms of the order of $\hbar/d$, where d is the drop diameter.[29] The value of the Landau-damping constant in the case of a one-component confined plasma is, according to Ref. 29, given by

$$\gamma_L\approx\frac{e^2}{d}\frac{(\mathcal{E}^F)^{3/2}(\mathcal{E}^F+\omega_0)^{1/2}}{\omega_0^2}, \qquad (11)$$

where $\omega_0=\omega_p/3^{1/2}$. In the case of Ge the quantity γ_L is in the first approximation the sum of attenuation constants computed from the formula (11) for each of the four electron valleys, as well as for the transitions within and between the branches of the valence band. The numerical estimates $n_k=2\times10^{17}$ cm^{-3}, $\omega_0=8.7$ meV, and $d=2$ μ lead to a total value $\gamma_L\approx1$ meV.

Thus, the experimentally observed plasma- and magnetoplasma-oscillation damping in EHD under the conditions of the experiments published in Refs. 4 and 7–12 ($d\approx1-2$ μ) is due mainly to the electron-hole collisions and Landau damping. Since, as can be seen from (11), the magnitude of the latter depends on the drop size, it is of interest to carry out a similar comparison of experiment with the theory for "large" EHD, which are observed in inhomogeneously deformed Ge crystals. According to investigations of plasma resonance,[30] as well as of oscillating magneto-acoustic absorption,[31] in the case of "large" EHD ($d\approx500$ μ) $\omega_p=6.8$ meV, $\omega_p\tau=100$ ($\hbar\gamma\approx6.8\times10^{-2}$ meV), and $\mathcal{E}^F_e=2.6$ meV. The substitution of these data and of the static relaxation time $\tau_0\approx10^{-10}$ sec ($\hbar\gamma_0\approx6.6\times10^{-3}$ meV)[28,31] into (10) and (11) leads to the values $\gamma_{e-h}(\mathcal{E}^F_e)\approx7.5\times10^{-2}$ meV and $\gamma_L\approx6\times10^{-3}$ meV. These estimates show that in "large" EHD, in which, as was to be expected, Landau damping turns out to be negligibly weak ($\gamma_L\approx\gamma_0$), the experimentally observed plasma-oscillation attenuation is due primarily to the electron-hole collisions, the corresponding frequency dependence of $\gamma_{e-h}(\omega)$ apparently becoming saturated in the region $\hbar\omega\gtrsim\mathcal{E}^F$.

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Forces produced by conduction electrons in metals located in external fields

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The forces produced by conduction electrons in metals located in external fields are considered by a unified approach. The body forces which the electron exert on the lattice as a whole, and the electron "wind" forces acting on the defects, are found. Criteria are obtained which can be used to assess the relative magnitude of the forces created by electrons in metals. The causes of the discrepancies of the results of previous researches are found.

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If a metal is located in external field—electric or magnetic—under the action of mechanical stresses, then various types of forces arise in it, due to the fact that the conduction electrons transfer to the lattice the action of the external fields and strains that they experience. These forces can be divided into two essentially different groups; body forces, which act on the lattice as a whole, and forces that act on the lattice defects — the "electron wind" forces. The forces of the first group are important in the phenomena of interaction of a current with elastic and plastic deformations. The forces of the second group produce motion of the defects in the lattice: electron transfer, attraction of ions by electrons, acceleration of dislocations by electrons, and so on.

A significant number of works have been devoted to the analysis of body forces[1-9] and electron wind forces[10-17]; however, the situation at the present time is unsatisfactory in two respects. First, the expressions for the body forces, obtained in previous researches,[1-7] differ significantly among themselves (a critical analysis of some of these researches is contained in Ref. 8), but even the expressions for the body forces obtained in the latest, most complete researches[8,9] do not reduce to one another. Second, there is an essential difference in the methods of calculation of body forces and electron wind forces. The methods used for the calculation of the forces in the volume[5,7,8] cannot be used for obtaining the forces acting on the defects. Thus there is no single approach to the force problem. The aim of the present work is to consider the forces created by electrons in metals within the framework of common approach. The derivation of the body forces and the electron wind forces based on the use of quantum equations of motion of the electrons, written down in the form of Newton's equations (the quantum theorems of Ehrenfest for motion of electrons in a periodic field of the lattice and for the electron-quasiparticle in field external relative to the periodic

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