

of the radiation intensity. A frequency and angle region can then exist, in which the vacuum radiation and the radiation from the medium are in phase. We wish to call particular attention to this circumstance, since it can take place both in oblique incidence of the particle on the sample and when the particle moves in the channel.

This analysis is not restricted to the  $\gamma$ -resonance region, and can hold also at frequencies close to the natural frequencies of the medium (optical-transition frequencies, absorption edges,<sup>[10]</sup> and others<sup>4)</sup>).

In conclusion, the author thanks V. V. Fedorov for useful remarks, A. F. Tyunis for help with the numerical calculations, and T. B. Mezentseva for help in readying this paper for publication.

<sup>1</sup>We neglect the imaginary part, which is smaller by almost two orders of magnitude than the real part in the frequency band of interest to us.

<sup>2</sup>In this case we omit the third component—the field of the dipole images in the medium as the particle moves in vacuum, since this component is small compared with

$\Pi_{1\omega}$  and  $\Pi_{2\omega}$ .

<sup>3</sup>The trajectory region in which the phases of the waves emitted from it differ by not more than  $\pi$ .

<sup>4</sup>The  $\gamma$ -resonance region was chosen because it was simplest to describe.

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## Interaction of two-level system with a strong monochromatic wave and with a thermostat

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An expression is obtained for the stationary distribution of a two-level system, produced as a result of interaction with the thermostat, in the field of a strong monochromatic wave; the times to establish this distribution are also determined. It is shown that when the external-field frequency is lower than a certain critical value the state of the two-level system is described by a Boltzmann distribution in the quasi-energy, and at high frequencies it contains matrix elements of the interaction with the thermostat. It is also shown that the corresponding relaxation constants are half as large in the resonant case as in the nonresonant one, so that it can be concluded that the presence of the signal suppresses the noise in a quantum amplifier.

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Progress in the theory of irreversible processes raises the question of the behavior of quantum-mechanical objects that are coupled to a thermostat, i.e., to a system that has in the limit an infinite number of degrees of freedom. Since this problem is extremely complicated, solutions can be expected for only the very simplest models, as was first done by Weisskopf and Wigner in their classical work on radiative damping.<sup>[1]</sup> In 1963, Gordon, Walker, and Louisell<sup>[2]</sup> solved the problem for a damped harmonic oscillator coupled to a set of independent oscillators distributed at the initial instant in accord with a canonic ensemble.<sup>[2]</sup> Glauber has shown later<sup>[3]</sup> that "large systems not consisting, of course, of harmonic oscillators have very frequently collective-excitation modes whose amplitudes behave dynamically like oscillator amplitudes," so that such a

thermostat model is quite general. In the present paper, an attempt is made to develop further the Weisskopf-Wigner theory to include a two-level system in the field of a strong monochromatic wave, using the aforementioned model of the thermostat.

The first questions raised when this problem is formulated are:

1. Are there any memory effects whatever in the stationary distribution of a two-level system?
2. Does this distribution depend on the details of the mechanism of the interaction with the thermostat, or is the latter only a temperature, as is postulated in equilibrium statistical physics?

We describe our system by the Hamiltonian

$$\begin{aligned}
H &= H_0 + H_T + V_F + V_T, \\
H_0 &= \frac{1}{2} \omega_0 \sigma_z, \quad H_T = \sum_k \omega_k b_k^* b_k, \\
V_F &= \frac{1}{2} \omega_1 (\sigma_+ e^{-i\omega t} + \sigma_- e^{i\omega t}), \quad V_T = \sum_k (c_k \sigma_+ b_k + c_k^* \sigma_- b_k^*),
\end{aligned}
\tag{1}$$

where  $\omega_0$  is the level difference,  $\omega_1$  is the amplitude of the monochromatic wave in energy units,  $\sigma_+$ ,  $\sigma_-$ , and  $\sigma_z$  are Pauli operators. The interaction terms are written in the rotating-wave approximation. We assume that the interaction is turned on at the instant  $t=0$ .

The solution of the equation for the density operator

$$i \frac{d\rho}{dt} = [H, \rho] \tag{2}$$

can be written in the form

$$\rho = \rho_0 - i \int_0^t U(t, t') [V(t'), \rho_0] U(t', t) dt', \tag{3}$$

where  $U(t, t')$  is the system-evolution operator

$$\left( i \frac{d}{dt} - H \right) U(t, t') = 0, \quad U(t, t) = 1, \tag{4}$$

$$V_i = V_{iF} + V_{iT}, \quad \rho_0 = \rho_{s_0} \rho_T,$$

and  $\rho_{s_0} = 1/2(1 + s_0 \sigma_z)$  is the initial distribution of a two-level system with arbitrary overpopulation  $s_0$ , while  $\rho_T$  is the initial distribution of the thermostat.

Relation (3) can be verified by differentiating both its parts with respect to time and taking (4) into account. We shall be interested hereafter in the reduced density matrix, and for this purpose it is necessary to take in (3) the trace over the variables of the thermostat.

Making the transformation

$$\begin{aligned}
U(t, t') &= L^+(t) U'(t, t') L(t), \\
V &= L^+(t) V(t) L(t), \quad L(t) = \exp [i(\frac{1}{2} \omega_0 \sigma_z + H_T) t],
\end{aligned}$$

we obtain for the reduced density matrix (the term "reduced" will henceforth be omitted for brevity) the following expression:

$$\text{Tr}_T \rho = \frac{1}{2} \exp(-\frac{1}{2} i \omega_0 \sigma_z t) (1 + s \sigma) \exp(\frac{1}{2} i \omega_0 \sigma_z t),$$

where  $s = (s^x, s^y, s^z)$  are the components of the density matrix in the rotating coordinate system

$$s = s_0 - i \int_0^t \text{Tr}_T \rho_0 \langle [G^A(t', t) \sigma G^R(t, t'), V(t')] \rangle dt', \tag{5}$$

$$s_0 = (0, 0, s_0), \quad \langle \dots \rangle = \text{Tr}_T \rho_T \{ \dots \},$$

$\sigma$  and  $b$  under the trace sign designate the variables with respect to which this operation is carried out.

The retarded and advanced Green's function introduced in (5) are determined by the relations

$$\begin{aligned}
G^R(t_1, t_2) &= \begin{cases} -iU'(t_1, t_2), & t_1 > t_2, \\ 0, & t_1 < t_2, \end{cases} \\
G^A(t_1, t_2) &= \begin{cases} 0, & t_1 > t_2, \\ iU'(t_1, t_2), & t_1 < t_2, \end{cases}
\end{aligned}
\tag{6}$$

where

$$\begin{aligned}
\left( i \frac{d}{dt} - \frac{\Delta \omega}{2} \sigma_z - \frac{\omega_1}{2} \sigma_x - V_T(t) \right) G^{R(A)}(t, t') &= \delta(t-t'), \\
V_T(t) &= \sum_k \left( c_k \sigma_+ b_k e^{-i\omega_k t} + c_k^* \sigma_- b_k^* e^{i\omega_k t} \right), \\
\Delta \omega &= \omega_0 - \omega, \quad \bar{\omega}_k = \omega_k - \omega.
\end{aligned}$$

Finally, with the aid of the quantities

$$\begin{aligned}
\Pi_T(t_1 - t, t - t_2) &= \langle G^A(t_1, t) \sigma G^R(t, t_2) V(t_2) - V(t_1) G^A(t_1, t) \sigma G^R(t, t_2) \rangle, \\
\Pi_T(t_1 - t, t - t_2) &= \langle G^A(t_1, t) \sigma G^R(t, t_2) \rangle,
\end{aligned}
\tag{7}$$

we complete the mathematical formulation of the problem:

$$s = s_0 + \int_0^t S(t-t') dt', \tag{8}$$

where

$$S = S_F + S_T, \quad S_F(t-t') = -\frac{1}{2} s_0 \omega_1 \text{Tr}_T \sigma_i \Pi_F(t'-t, t-t'),$$

$$S_T(t-t') = -i \text{Tr}_T \rho_0 \Pi_T(t'-t, t-t'),$$

which has thus been reduced to the calculation of the quantities in (7).

Expanding the Green's functions in (7) in the interaction with the thermostat and using Wick's theorem<sup>[4]</sup> for the mean value of the products of an arbitrary number of Boson operators, we arrive at perturbation theory in diagram form. Some of the diagrams for the Fourier components of  $\Pi_F$ ,

$$\begin{aligned}
\Pi_F(t_1 - t, t - t_2) &= \frac{1}{(2\pi)^2} \int \Pi_F(\epsilon_-, \epsilon_+) \exp[-i\epsilon_-(t_1 - t) - i\epsilon_+(t - t_2)] d\epsilon_- d\epsilon_+, \\
\epsilon_- &= \epsilon - \chi/2, \quad \epsilon_+ = \epsilon + \chi/2,
\end{aligned}
\tag{9}$$

are shown in Figs. 1a and 1b, where the solid lines correspond to the Green's functions averaged over the thermostat

$$g^{R(A)}(\epsilon) = \frac{1}{2\pi} \int \langle G^{R(A)}(t+\tau, t) \rangle e^{-i\epsilon\tau} d\tau,$$

and the dashed lines correspond to the thermostat functions

$$d^+(\epsilon) = 2\pi \sum_k |c_k|^2 (N_k + 1) \delta(\epsilon - \bar{\omega}_k),$$

$$d^-(\epsilon) = 2\pi \sum_k |c_k|^2 N_k \delta(\epsilon + \bar{\omega}_k),$$

$$N_k = \frac{1}{\exp(\omega_k/T) - 1}.$$

Calculating the irreducible energy part  $\Sigma^{R(A)}(\epsilon)$  in first order in the interaction with the thermostat, we get

$$(g^{R(A)}(\epsilon))^{-1} = \epsilon - \frac{1}{2} \Delta \omega \sigma_z - \frac{1}{2} \omega_1 \sigma_x - \Sigma^{R(A)}(\epsilon), \tag{10}$$

where

$$\Sigma^{R(A)}(\epsilon) = (\mp)^{1/2} i (\gamma_1(\epsilon) \sigma_+ \sigma_- + \gamma_2(\epsilon) \sigma_- \sigma_+). \tag{11}$$

Here

$$\begin{aligned}
\gamma_1(\epsilon) &= u^2 d^+(\epsilon + \Omega/2) + v^2 d^+(\epsilon - \Omega/2), \\
\gamma_2(\epsilon) &= v^2 d^-(\epsilon + \Omega/2) + u^2 d^-(\epsilon - \Omega/2), \\
u^2 &= \frac{1}{2} (1 + \Delta \omega / \Omega), \quad v^2 = \frac{1}{2} (1 - \Delta \omega / \Omega), \quad \Omega = [(\Delta \omega)^2 + \omega_1^2]^{1/2}.
\end{aligned}$$

We shall not write out the expressions for the level shifts, since they are not needed in the subsequent cal-

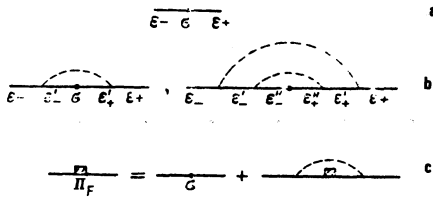


FIG. 1.

culations. We have used in fact the Weisskopf-Wigner theory, assuming the thermostat spectrum to be almost continuous and the interaction with it to be weak.

We proceed now directly to the calculation of  $\Pi_F$  and consider for this purpose first  $\chi \lesssim \gamma_{1,2}$ . In this case the integration of the products of the two Green's functions  $g^0$  and  $g^R$ , which have close arguments, offsets the smallness of the interaction with the thermostat, so that it becomes necessary to sum a "ladder" of diagrams. The latter lead to the integral equation shown graphically in Fig. 1c for  $\Pi_F$ .

It is convenient, however, to solve the equation not for  $\Pi_F$  but for the vertex function connected with  $\Pi_F$  by the relation

$$\Pi_F(\epsilon_-, \epsilon_+) = g^A(\epsilon_-) (\sigma + \Lambda(\epsilon_-, \epsilon_+)) g^R(\epsilon_+). \quad (12)$$

We write the equation for  $\Lambda$  in the form

$$\Lambda = \Lambda_1 \sigma_+ \sigma_- + \Lambda_2 \sigma_- \sigma_+, \quad (13)$$

where

$$\Lambda_1(\epsilon_-, \epsilon_+) = \frac{1}{2\pi} \int d^+ (\epsilon - \epsilon') (g^A(\epsilon_-') (\sigma + \Lambda(\epsilon_-', \epsilon_+')) g^R(\epsilon_+'))_{22} d\epsilon',$$

$$\Lambda_2(\epsilon_-, \epsilon_+) = \frac{1}{2\pi} \int d^- (\epsilon - \epsilon') (g^A(\epsilon_-') (\sigma + \Lambda(\epsilon_-', \epsilon_+')) g^R(\epsilon_+'))_{11} d\epsilon',$$

$$(\dots)_{22} \sigma_+ \sigma_- = \sigma_+ (\dots) \sigma_-, \quad (\dots)_{11} \sigma_- \sigma_+ = \sigma_- (\dots) \sigma_+.$$

Since the kernels of the integral equations differ substantially from zero only in the narrow region ( $\epsilon' \mp \Omega/2 \sim \gamma_{1,2}$ ), while  $\Lambda_1$  and  $\Lambda_2$  are functions that depend little on  $\epsilon'$ , we arrive at the system of linear equations

$$\begin{aligned} \Lambda &= (2uv\Lambda, 0, (u^2 - v^2)\Lambda), \\ \Lambda_1 &= Xu^2 d^+ (\epsilon - \Omega/2) + Yu^2 d^+ (\epsilon + \Omega/2), \\ \Lambda_2 &= Xu^2 d^- (\epsilon - \Omega/2) + Yu^2 d^- (\epsilon + \Omega/2), \end{aligned} \quad (14)$$

$$X = \frac{1}{\Gamma_1 - i\chi} (u^2 \Lambda_1 + v^2 \Lambda_2 + 1)_{\epsilon = -\Omega/2}, \quad Y = \frac{1}{\Gamma_2 - i\chi} (v^2 \Lambda_1 + u^2 \Lambda_2 - 1)_{\epsilon = -\Omega/2}, \quad (15)$$

where

$$a = u^2 d^+ (\Omega) + v^2 d^- (\Omega), \quad b = v^2 d^+ (-\Omega) + u^2 d^- (-\Omega), \quad c = (uv)^2 (d^+(0) + d^-(0)).$$

From (14) and (15) we obtain for  $X$  and  $Y$  the expressions

$$X = \frac{a - b + i\chi}{i\chi(a + b - i\chi)}, \quad Y = \frac{a - b - i\chi}{i\chi(a + b - i\chi)}, \quad (16)$$

the  $y$ -th component of  $\Lambda$  vanishes in the limit of weak interaction with the thermostat, and will not be considered here.

Integrating  $\Pi_F$  with respect to  $\epsilon$ , we get

$$\begin{aligned} S_F^{(n)}(\chi) &= \frac{uv s_0}{\Gamma_1 - i\chi} \{u^2 (\gamma_1 - i\chi) (1 + 2v^2 \Lambda_2) - v^2 (\gamma_2 - i\chi) (1 + 2u^2 \Lambda_1)\}_{\epsilon = -\Omega/2} \\ &+ \frac{uv s_0}{\Gamma_1 - i\chi} \{u^2 (\gamma_2 - i\chi) (1 - 2v^2 \Lambda_1) - v^2 (\gamma_1 - i\chi) (1 - 2u^2 \Lambda_2)\}_{\epsilon = -\Omega/2}, \end{aligned} \quad (17)$$

$$\begin{aligned} S_F(\chi) &= -\frac{uv s_0}{\Gamma_1 - i\chi} \{(\gamma_1 - i\chi) (1 - (u^2 - v^2) \Lambda_2) + (\gamma_2 - i\chi) (1 + (u^2 - v^2) \Lambda_1)\}_{\epsilon = -\Omega/2} \\ &- \frac{uv s_0}{\Gamma_1 - i\chi} \{(\gamma_1 - i\chi) (1 - (u^2 - v^2) \Lambda_2) + (\gamma_2 - i\chi) (1 + (u^2 - v^2) \Lambda_1)\}_{\epsilon = -\Omega/2}, \end{aligned}$$

where  $S_F(\chi)$  is the Fourier component of the function  $S_F(t - t')$ :

$$S_F(t - t') = \frac{1}{2\pi} \int S_F(\chi) e^{-i\chi(t-t')} d\chi.$$

Substituting (14) in (17) we arrive after algebraic transformations at

$$S_F^{(n)} = 2uv S_F, \quad S_F^{(s)} = -s_0 + (u^2 - v^2) S_F,$$

where

$$\begin{aligned} S_F &= s_0 \left\{ u^2 - v^2 + \frac{2(uv)^2}{\Gamma - i\chi} \sum |c_n|^2 (2N_n + 1) \right. \\ &\left. \times (u^2 \delta(\Omega - \bar{\omega}_n) - v^2 \delta(\Omega + \bar{\omega}_n)) \right\}, \quad \Gamma = a + b. \end{aligned} \quad (18)$$

$S_T$  is calculated analogously. Figure 2 shows the diagram form of  $\Pi_T$ , whose analytic form is

$$\begin{aligned} \Pi_T(\epsilon_-, \epsilon_+) &= g^A(\epsilon_-) (\sigma + \Lambda(\epsilon_-, \epsilon_+)) g^R(\epsilon_+) \Sigma^R(\epsilon_+) + g^A(\epsilon_-) \Lambda(\epsilon_-, \epsilon_+) \\ &- \Sigma^A(\epsilon_-) g^A(\epsilon_-) (\sigma + \Lambda(\epsilon_-, \epsilon_+)) g^R(\epsilon_+) - \Lambda(\epsilon_-, \epsilon_+) g^R(\epsilon_+). \end{aligned} \quad (19)$$

After integrating with respect to  $\epsilon$  we arrive at an expression for

$$\begin{aligned} S_T &= (2uv S_T, 0, (u^2 - v^2) S_T), \\ S_T(\chi) &= 1/2 \{ u^2 (1 + s_0) (\Lambda_1 - \gamma_1 X) + v^2 (1 - s_0) (\Lambda_2 - \gamma_2 X) \}_{\epsilon = -\Omega/2} \\ &+ 1/2 \{ v^2 (1 + s_0) (\Lambda_1 - \gamma_1 Y) + u^2 (1 - s_0) (\Lambda_2 - \gamma_2 Y) \}_{\epsilon = -\Omega/2}, \end{aligned} \quad (20)$$

and finally

$$S_T(\chi) = \frac{1}{\Gamma - i\chi} \sum |c_n|^2 (v^4 \delta(\Omega + \bar{\omega}_n) - u^4 \delta(\Omega - \bar{\omega}_n)) ((2N_n + 1) s_0 + 1). \quad (21)$$

At  $|\chi| \gg \Gamma$  the calculation of  $S$  is very simple, since in this case the poles of the Green's functions are far from one another and the only contribution to  $S$  is made by the term (Fig. 1a)

$$\Pi_F(\epsilon_-, \epsilon_+) = g^A(\epsilon_-) \sigma g^R(\epsilon_+). \quad (22)$$

After integrating (22) with respect to  $\epsilon$  we have together with (18) and (21)

$$S^{(n)}(\chi) = \begin{cases} 2uv \left( \frac{\Gamma}{\Gamma - i\chi} s_0 - \frac{i\chi s_0 (u^2 - v^2)}{\Gamma - i\chi} \right), & \chi \lesssim \Gamma, \Delta, \\ s_0 uv (u^2 - v^2) \left( \frac{1}{\chi + \Omega + i\Delta} - \frac{1}{\chi - \Omega + i\Delta} \right), & \chi \gg \Gamma, \Delta, \end{cases} \quad (23)$$

$$S^{(s)}(\chi) = \begin{cases} -s_0 + (u^2 - v^2) \left( \frac{\Gamma}{\Gamma - i\chi} s_0 - \frac{i\chi s_0 (u^2 - v^2)}{\Gamma - i\chi} \right), & \chi \lesssim \Gamma, \Delta, \\ -2s_0 (uv)^2 \left( \frac{1}{\chi + \Omega + i\Delta} - \frac{1}{\chi - \Omega + i\Delta} \right), & \chi \gg \Gamma, \Delta, \end{cases}$$

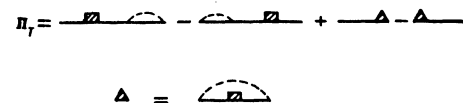


FIG. 2.

where  $\Delta = (\Gamma_1 + \Gamma_2)/2$  and

$$s_\infty = - \frac{\sum_k |c_k|^2 (u^4 \delta(\Omega - \bar{\omega}_k) - v^4 \delta(\Omega + \bar{\omega}_k))}{\sum_k |c_k|^2 (2N_k + 1) (u^4 \delta(\Omega - \bar{\omega}_k) + v^4 \delta(\Omega + \bar{\omega}_k))}. \quad (24)$$

Rewriting (8) in terms of the Fourier components  $S(\chi)$

$$s(t) = s_0 + \frac{i}{2\pi} \int e^{-i\chi t} \frac{S(\chi)}{\chi + i\delta} d\chi, \quad (25)$$

we obtain ultimately

$$s^{(2)}(t) = \frac{s_0 \omega_1 \Delta \omega}{\Omega^2} (e^{-\Gamma t} - e^{-\Delta t} \cos \Omega t) + \frac{\omega_1}{\Omega} s_\infty (1 - e^{-\Gamma t}), \quad (26)$$

$$s^{(1)}(t) = s_0 e^{-\Gamma t} - \frac{\omega_1^2}{\Omega^2} (e^{-\Gamma t} - e^{-\Delta t} \cos \Omega t) + \frac{\Delta \omega}{\Omega} s_\infty (1 - e^{-\Gamma t}),$$

which goes over into the Weisskopf-Wigner result<sup>[1]</sup> in the limit as  $\omega_1 \rightarrow 0$  and  $T \rightarrow 0$ . As seen from (26), the stationary distribution, in the limit as  $t \rightarrow \infty$ , is equal to  $s_\infty$  and is directed along the "effective field."

We can now answer the questions raised at the beginning of the article. First, there are no memory effects in the stationary distribution, i.e., it does not depend on the initial distribution of the two-level system; second, there is a critical frequency

$$\omega_c = \frac{1}{2} \omega_0 [1 + (\omega_1/\omega_0)^2], \quad (27)$$

such that at  $\omega \leq \omega_c$  the second delta-function vanishes and we have

$$s_\infty = - \text{cth} \frac{\omega + \Omega}{2T}, \quad (28)$$

which corresponds to a Boltzmann distribution with

quasienergy  $\omega + \Omega$ . At  $\omega > \omega_c$  the terms with  $\delta(\Omega + \bar{\omega}_k)$  begin to play an essential role and we find that at these frequencies the stationary distribution is determined by the matrix elements of the operator of the interaction with the thermostat.

Turning to a practical application of (26), we note that it can have a bearing on the theory of quantum amplifiers. It is known that the relaxation processes due to interaction with thermal radiation are the cause of the noise in quantum amplifiers. The corresponding relaxation constants

$$\Gamma = 2\pi \sum_k |c_k|^2 (2N_k + 1) (u^4 \delta(\Omega - \bar{\omega}_k) + v^4 \delta(\Omega + \bar{\omega}_k)), \quad (29)$$

$\Delta = 2\pi \sum_k |c_k|^2 (2N_k + 1) (\frac{1}{2} u^4 \delta(\Omega - \bar{\omega}_k) + \frac{1}{2} v^4 \delta(\Omega + \bar{\omega}_k) + (uv)^2 \delta(\bar{\omega}_k))$ , assume in the resonance case  $\Delta \omega \ll \omega_1 \ll \omega_0$  the values

$$\Gamma_p = \Delta_p = \Gamma_0/2, \quad (30)$$

where  $\Gamma_0$  is the reciprocal decay time in the absence of a signal (the Weisskopf-Wigner constant in the optical region), so that without a signal the noise in the amplifier is double the noise in the resonant case.

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## The fluctuation-dissipation relation in nonlinear electrodynamics

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The linear and nonlinear responses to an external perturbing field in a plasma are considered. It is shown that, apart from the usual fluctuation-dissipation relation connecting the binary correlation function for the charge-density-fluctuations with the linear electric susceptibility, there also exist a number of additional relations connecting correlation functions of higher order with the nonlinear susceptibilities. A number of integral relations between the linear and nonlinear susceptibilities in a plasma are established.

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### 1. INTRODUCTION

As is well known, in linear electrodynamics, for systems in thermodynamic equilibrium, a fluctuation-dissipation relation establishes a general connection between the dissipative properties of the system and the fluctuations of various quantities. Since the dissipative properties of an electro-dynamic system are deter-

mined by the macroscopic coefficients in the linear relationship between the induced charges or currents and the fields, specifying these coefficients determines completely the spectral distributions of the fluctuations of the electrodynamic quantities.<sup>[1-4]</sup> For an equilibrium plasma the spectral distribution of the electromagnetic fluctuations is determined by specifying the permittivity tensor. Conversely, knowing the spectrum