Strong fluctuations of laser radiation intensity in a turbulent atmosphere—the distribution function

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A method is proposed for calculating the distribution function of the intensity fluctuations in a medium with a Kolmogorov turbulence in a region of saturated flicker. For the case when the initial type of radiation is a plane wave, the distribution function is obtained in the form of a piecewise approximation that is well joined together in the transition regions.

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1. INTRODUCTION

The physics of the propagation of laser radiation under conditions of multiple scattering in a turbulent medium has been recently intensively investigated.[1] The interest in this subject is due to the extensive possibilities of using laser sources in physical research on plasma, hydrodynamic flow, and the earth's atmosphere.

By now, kinetic equations have been derived for the evolution of the moments of the field of a wave propagating in a medium with specified turbulence statistics.[1-4] The solutions of the equations have been written out for the most important even moments of the field (the odd moments are exponentially small at large distances) for different initial types of radiation.[1,4,5] The results for weak turbulence,[6] based on the Rytov method, are obtained in the new theory[1] in natural fashion as a particular case (first order of perturbation theory). At the same time, the problem of the form of the distribution function of the intensity fluctuations under conditions of multiple scattering is still unsolved, although many articles are devoted to it.[1,7,8]

We propose in the present paper a method of calculating the distribution function of the intensity fluctuations in the region of large propagation distances, where the radiation flicker is already saturated.

The proposed method is based on physical arguments that take into account the wide range of scales (several orders of magnitude) of the inhomogeneities in the spectrum of a real Kolmogorov turbulence. The distribution function is obtained in the form of a piecewise approximation that is well joined together in the transition regions. In the limiting case of low intensity, the results agree with those obtained by using a formal expansion of the distribution function for the field in an Edgeworth series.[1]

2. THEORY

Assume that a plane wave of unit amplitude is incident on a half-space $z>0$ filled with a turbulent medium. The statistics of the turbulence will be described by a Kolmogorov inhomogeneity spectrum

$$\phi(q)\propto q^{-(\alpha+2)},\quad 1<\alpha<2, \tag{1}$$

which is valid in the inertial interval of wave numbers, i.e., at $1/L_0 \ll q < 1/l_0$, where l_0 and L_0 are the internal and external turbulence scales.

The wave perturbation at a certain point of space, at large propagation distances, is the result of a superposition of waves that are multiply reflected by different inhomogeneities of the medium. If these multiply scattered components are statistically independent and their number is large enough, then the resultant field has a normal distribution by virtue of the central limit theorem. For a wide range of propagation distances in a medium with a real Kolmogorov turbulence (1), however, the normalization condition can be satisfied only for a superposition of components that are scattered by minute inhomogeneities. The characteristic scale of the inhomogeneities from which scattering leads to local normalization of the field is of the order of the coherence radius of the field[1]

$$b\approx a_{\rm Fr} m_0^{-2/\alpha}; \tag{2}$$

here $a_{\rm Fr}=(z/k)^{1/2}$ is the radius of the first Fresnel zone and m_0^2 is the flicker index calculated in the Born approximation by solving the equation for the fourth moment of the field.

We assume that the action of the small-scale turbulence on the field is statistically independent of the action of the large-scale inhomogeneities with dimension $L\gg b$. Under these assumptions, in the region $m_0^2\gg 1$ (in the "large distance region") the field distribution function can be locally normal, and the intensity distribution function can be exponential:

$$f_1(I,\langle I\rangle)=\langle I\rangle^{-1}\exp\{-I/\langle I\rangle\}. \tag{3}$$

The angle brackets $\langle\ldots\rangle$ denote here the operation of averaging over the statistics of the small-scale turbulence. The large-scale turbulence modulates the small-scale turbulence, so that the field will be normal only locally, and the average value of the intensity will be a function of the coordinates, $\langle I\rangle=\langle I(\mathbf{r})\rangle$.

To determine the distribution function $f(I)$ that takes into account also the action of the large-scale turbulence, it is necessary to know the distribution function $f_2(\langle I\rangle)$. The total distribution function $f(I)$ is given by

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$$f(I)=\int_0^\infty f_1(I,\langle I\rangle)f_2(\langle I\rangle)d\langle I\rangle. \quad (4)$$

We shall continue the analysis separately for the region of small, intermediate, and large intensity values.

2a) Case of sufficiently small intensity:

$$I<m_0^{(2/\alpha)(2-\alpha)}.$$

In this case there is no need to know the complete form of the function $f_2(\langle I\rangle)$. We represent $\langle I\rangle$ in the form

$$\langle I\rangle=1+\delta, \quad (5)$$

where $\bar\delta=0$

$$\overline{\delta^2}={}^1/_2N_3(\alpha)[k_3(\alpha)]^{(2/\alpha)(2-\alpha)}m_0^{-(4/\alpha)(2-\alpha)} \quad (6)$$

for a definition of the coefficients $N_3(\alpha)$ and $k_3(\alpha)$ see [1], and the superior bar denotes averaging over the statistics of the large-scale turbulence.

Expanding $f_1(I,\langle I\rangle)$ in powers of δ and retaining only the quadratic terms, we obtain after averaging

$$f(I)=\overline{f_1(I,\langle I\rangle)}=\{1+\overline{\delta^2}(1-2I+{}^1/_2I^2)\}\exp(-I). \quad (7)$$

This expansion is valid at $I<m_0^{(2/\alpha)(2-\alpha)}$. For moments of the intensity we get from (7)

$$\overline{\langle I^n\rangle}=n!\{1+{}^1/_2n(n-1)\overline{\delta^2}\}. \quad (8)$$

From (8) and from the form of the solution of the equation for the fourth moment of the field in the region $m_0^2\gg1$ [9] we obtain directly (6).

Expression (7) is easily obtained by using a formal expansion of the distribution function for the field in the region $m_0^2>1$ in an Edgeworth series, as was in fact indicated in [1]. It turns out here that the asymmetry coefficients of the distribution function for the field can be set equal to zero (by virtue of the exponential smallness of the solution of the equation for the third moment of the field), and the excess coefficient is equal to $3\overline{\delta^2}$. This result was recently obtained also in [10], where a complicated procedure was used to solve the equation for the n-th moment of the field.

2b) Case of intermediate values of the intensity:

$$m_0^{(2/\alpha)(2-\alpha)}<I<m_0^{(4/\alpha)(2-\alpha)}.$$

The modulation of the intensity by the large-scale turbulence in the case of a phase screen can be represented in the form[11]

$$\langle I(\mathbf{r}_t,z)\rangle=\int I_0(\mathbf{r}_t-\boldsymbol{\vartheta}z)J(\boldsymbol{\vartheta})d^2\boldsymbol{\vartheta}, \quad (9)$$

where I_0 is the intensity fluctuation due only to the large-scale turbulence and $J(\vartheta)$ is the direction distribution function of the intensity of the radiation scattered by the small-scale inhomogeneities. It follows from this relation that an essential role in the intensity modulation can be played only by scales $L\gtrsim a_{\rm Fr}m_0^{(2/\alpha)}$. The intensity modulation at these scales is weak, its properties have been well investigated, [6] and the distribution function for $f_2(\langle I\rangle)$ can be regarded in this case as normal. This reasoning can be easily generalized also for an extended medium.

Using the fact that δ is normal and is small in comparison with unity, we obtain

$$f(I)=\exp[-I(1-{}^1/_2\overline{\delta^2}I)]. \quad (10)$$

The significant values of δ are of the order of $I\overline{\delta^2}$ and the condition of the smallness of δ in comparison with unity leads to the inequality

$$I<(\overline{\delta^2})^{-1}\sim m_0^{(4/\alpha)(2-\alpha)}. \quad (11)$$

2c) Large values of the intensity: $I>m_0^{(4/\alpha)(2-\alpha)}$.

The tail of the distribution function in the region of large intensity can be described by using the known statistics of random phase shifts of radiation refracted by large-scale ($L\gg b$) inhomogeneities of the medium. Indeed, large intensity spikes in the observation plane can be realized only if the radiation is randomly focused. It must be recognized here that the probability of simultaneous focusing in both transverse coordinates is quadratic in the probability of focusing in only one of them.

Thus, let radiation locally normalized by small-scale turbulence be incident on a cylindrical lens of dimension L. The average intensity in the focal spot will be

$$\langle I\rangle_{\rm foc}=Lb/a_{\rm Fr}^2, \quad (12)$$

since the dimension of the spot is determined by the refraction from a dimension $b\ll L$. We assume that $\langle I\rangle_{\rm foc}\gg1$, i.e.,

$$L\gg a_{\rm Fr}^2/b=a_{\rm Fr}m_0^{2/\alpha}. \quad (13)$$

The focusing condition connects the phase shifts S_L with the scale (i.e., it specifies the radius of curvature of the phase front after passing through the lens):

$$S_L\approx L^2/a_{\rm Fr}^2. \quad (14)$$

The mean squared phase shift over the scale L is[1]

$$\langle\overline{S_L^2}\rangle\approx(L/b)^\alpha. \quad (15)$$

Eliminating L from the system of equations (12), (14), and (15), we obtain

$$S_L=(b/a_{\rm Fr})^{-2}\langle I\rangle_{\rm foc}^2, \quad (16)$$

$$\langle\overline{S_L^2}\rangle\approx(b/a_{\rm Fr})^{-2\alpha}\langle I\rangle_{\rm foc}^\alpha. \quad (17)$$

The average characteristic $\langle\overline{S_L^2}\rangle$, however, depends parametrically on the random dimension L of the lenses. It is in this sense that the last relation (17) should be understood, where $\langle I\rangle_{\rm foc}$ in the right-hand side is a random quantity.

Recognizing that the phase distribution function is Gaussian

$$f(S_L)=(2\pi)^{-1/2}(\langle\overline{S_L^2}\rangle)^{-1/2}\exp\{-{}^1/_2S_L^2(\langle\overline{S_L^2}\rangle)^{-1}\}, \quad (18)$$

and using relation (16), we get

$$f_2(\langle I\rangle)=(2/\pi)^{1/2}(b/a_{\mathrm{Fr}})^{-(2-\alpha)}\langle I\rangle^{1/2(2-\alpha)} \times \exp\{-{}^1/_2(b/a_{\mathrm{Fr}})^{-2(2-\alpha)}\langle I\rangle^{4-\alpha}\}, \quad (19)$$

here and below we shall omit the subscript of $\langle I\rangle_{\mathrm{foc}}$.

We calculate the integral (4), with (3) and (19) included, by using the saddle-point method; as a result we get

$$f(I)\approx\frac{2}{(5-\alpha)^{1/2}}\left(\frac{2}{4-\alpha}\right)^{(3-\alpha)/2(5-\alpha)}\left\{\left(\frac{b}{a_{\Phi\mathrm{p}}}\right)^{2(2-\alpha)}I\right\}^{-1/(5-\alpha)} \times\exp\left\{-\left(\frac{2}{4-\alpha}\right)^{-1/(5-\alpha)}\left(\frac{5-\alpha}{4-\alpha}\right)\left[\left(\frac{b}{a_{\Phi\mathrm{p}}}\right)^{2(2-\alpha)}I\right]^{-1/(5-\alpha)}I\right\} \quad (20)$$

This result can be written in a different form

$$f(I)\approx\frac{2}{(5-\alpha)^{1/2}}\left(\frac{2}{4-\alpha}\right)^{(3-\alpha)/2(5-\alpha)}m_0^{4(2-\alpha)/\alpha(5-\alpha)}I^{-1/(5-\alpha)} \times\exp\left\{-\left(\frac{2}{4-\alpha}\right)^{-1/(5-\alpha)}\left(\frac{5-\alpha}{4-\alpha}\right)m_0^{4(2-\alpha)/\alpha(5-\alpha)}I^{(4-\alpha)/(5-\alpha)}\right\}. \quad (21)$$

Relations (20) and (21) are valid at $I>m_0^{(4/\alpha)(2-\alpha)}$.

3. CONCLUSIONS

We have calculated above the distribution functions of the intensity fluctuations in the region of saturated flicker, for the case when the initial type of radiation is a plane wave (i.e., in the Fresnel zone of a laser emitter). Analogous calculations can also be carried out for other boundary conditions. It thus becomes possible to interpret also in the saturation region the extensive experimental material on flicker observation, which until recently was analyzed for the intensity logarithm.

However, the results are also of independent interest. They are necessary for the calculation of the parameters of the random intensity-field spikes,[1] for the determination of the error probability in two-way communication systems and in laser radars,[12] and for other applications.

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Advance and delay effects in photon echo signals

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An experimental investigation was made of the advance and delay of photon echo signals. The experiments were carried out on a ruby single crystal kept at a temperature of 2.2°K and the wavelength was 6935 Å. A theoretical description of these effects is given. Dephasing of the electric dipoles during the action of the exciting pulses and in the course of the first echo is allowed for the first time in the case of stimulated and multiple echo signals. The results of a theoretical calculation of the shifts of the optical coherent response are in agreement with the experimental data.

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INTRODUCTION

Optical (photon) echo is a coherent optical response of a resonant system to the action of two (or more) laser pulses separated by a time interval. The ratio of the power of an optical echo signal to the power of the exciting pulses is frequently 10^{-4}–10^{-5}, i.e., for pulses of powers of hundreds of kilowatts a coherent response is a relatively strong signal. Therefore, optical echo may find technical applications, particularly in dynamic holography, memory cells of optical computers, and quantum counters of low-intensity signals.[1] A very important task is the control of the process of generation of the optical echo. The present paper is concerned with changes in the shape of an optical echo signal and in the time of its appearance, which occur when the parameters of the exciting pulses are varied under conditions such that we cannot ignore the dephasing of electric dipoles during the action of these pulses. Such investigations are also of interest in connection with possible applications of the echo method in high-resolution optical spectroscopy.

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