Nonlinear effects in the propagation of a strong sound wave and a weak one in metals

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Nonlinear effects of the "wave–particle–wave" type in the simultaneous propagation of strong and weak sound in metals are considered. The gain of a weak wave propagating in the same direction as a strong wave that captures resonant electrons is determined, as well as the damping of a weak wave propagating counter to a strong one. It is shown that when the system of inequalities $\omega_1 \ll \tau^{-1} \ll \omega_0$ is satisfied (ω_0 is the oscillation frequency of the captured particles, τ is the relaxation time, and ω_1 is the frequency of the weak wave in a coordinate frame connected with the strong wave), the gain (damping) of the weak wave is proportional to $a^{-1} = \omega_0\tau \gg 1$ and its absolute value is much larger than the linear damping factor.

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INTRODUCTION

An intense sound wave propagating in a metal distorts strongly the trajectories of the electrons whose velocity component in the wave-propagation direction is close to the sound velocity. This is known to decrease the sound absorption coefficient[1,2] with increasing wave intensity. If the strong nonlinearity condition $a = (\omega_0\tau)^{-1} \ll 1$ is satisfied, where ω_0 the oscillation frequency of the trapped particles and τ is the relaxation time, then the absorption coefficient is proportional to a. If a strong and a weak wave propagate in the crystal simultaneously, nonlinear effects of the "wave–particle–wave" type should be observed. Obviously, the influence of the strong wave on the weak one will be strong in the case when the weak wave interacts with particles whose distribution function is strongly distorted by the strong wave, or in other words if the phase veloicity of the weak wave w_1 falls in the interval

$$w-\tilde{v}<w_1<w+\tilde{v},$$

where w is the velocity of the strong wave and $\tilde{v}$ is the characteristic oscillation velocity of the particles in the potential wells produced by the strong wave.

To investigate the influence of the strong wave on the weak one it is useful to consider the electron distribution function in the field of the strong wave. As will be shown below, the electron distribution function $f(v_x, d', \epsilon_\perp)$ (the wave propagates in the x direction with a velocity component v_x), integrated over the motion energy in a plane perpendicular to the wave vector $\epsilon_\perp$ at $T=0$, takes the form shown in Figs. 1 and 2: $(F(v_x, x') = \int f(v_x, x', \epsilon_\perp)d\epsilon_\perp, x' = x - wt)$.

It follows from the figures that the electron distribution is dome-shaped in the velocity interval $(w-\tilde{v}, \omega+\tilde{v})$, and tends to an equilibrium form outside this interval. The derivative $\partial F/\partial v_x$ on the peak of the dome is equal to zero. In the velocity region $0<v_x<w$ the derivative $\partial F/\partial v_x$ is positive. In the linear theory, when a contribution is made to the absorption by resonant particles moving in phase with the wave, a positive derivative would correspond to enhancement of weak waves propagating with velocities $0<w_1<w$. Even though in the case considered here the particle trajectories are substantially distorted by the strong-wave field and the interaction with the weak wave is not resonant, we shall show that when the inequalities

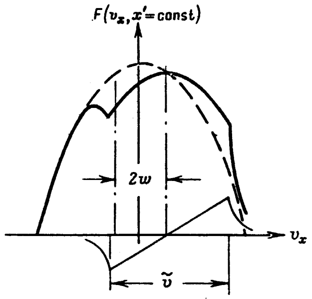

FIG. 1. Electron distribution function $F(v_x, x'=\text{const})$ in the field of a strong longitudinal wave at $\tilde{v} > 2w$. The thin line shows the nonequilibrium increment to the distribution function. The velocity interval $w-\tilde{v}(x) < v_x < w+\tilde{v}(x)$ corresponds to the trapped particles.

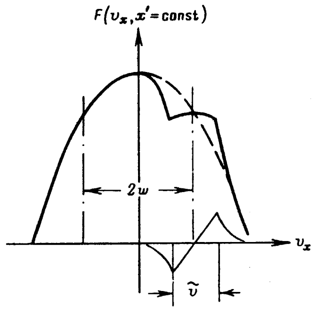

FIG. 2. Electron distribution function $F(v_x, x'=\text{const})$ in the strong-wave field at $\tilde{v} < 2w$. The thin line shows the nonequilibrium increment to the distribution function.

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(16) are satisfied a positive derivative corresponds, just as in the linear theory, to enhancement of the weak wave. If the weak wave travels counter to the strong wave, and if $2w > \tilde{v}$, then it interacts effectively with the particles whose trajectories are weakly distorted by the strong-wave field, and the distribution function differs little from the equilibrium one (Fig. 2). The absorption coefficient of the weak wave differs little from the linear one in this case. If the condition $2w < \tilde{v}$ is satisfied and the waves propagate in opposite direction, then the weak wave is damped and, as will be shown below, the damping coefficient can be much larger than the linear one.

Effects produced when a strong and a weak wave propagate with equal phase velocities were considered in [3]. It was shown, in particular, that at integer k/q, where k and q are the wave vectors of the weak and strong waves, and the nonlinearity is strong ($a \ll 1$), higher harmonics are effectively generated.

The purpose of the present paper is to study the interaction of a strong and weak sound waves whose phase velocities are unequal. We consider cases when the strong and weak waves propagate in the same direction and in opposite directions. We show that weak sound traveling in the direction of the strong wave can be effectively enhanced. At the same time, a weak propagating counter to the strong one is damped. The gain (damping) is proportional to the large parameter $1/a$ and can exceed the linear damping coefficient by many times.

1. SOLUTION OF THE KINETIC EQUATION

We consider the interaction of the electrons with the field of the sound wave. Assume that the interaction can be described by a deformation potential. The complete system of equations describing the propagation of the sound wave consists then of the inelasticity-theory equation

$$\rho\frac{\partial^2 u}{\partial t^2}=C\frac{\partial^2 u}{\partial x^2}+\Lambda\frac{\partial n}{\partial x} \tag{1}$$

and the Boltzmann kinetic equation for the electrons in the strong and weak sound waves:

$$\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}-\frac{\partial}{\partial x}[\varphi_0(x,t)+\varphi_1(x,t)]\frac{\partial f}{\partial p_x}+I\{f\}=0. \tag{2}$$

Here

$$\varphi_0(x,t)=-\varphi_0\cos q(x-wt), \tag{3}$$

$$\varphi_1(x,t)=-\varphi_1\cos k(x-w_1t) \tag{4}$$

are the potentials of the strong and weak waves that move with velocities w and w_1; u is the lattice displacements, C is the elastic modulus, ρ is the crystal density, Λ is the deformation-potential constant, and $I\{f\}$ is the collision integral. The electron density n is connected with the distribution function f by the relation

$$n(x,t)=\frac{2}{(2\pi)^3}\int d\mathbf{p}\, f(\mathbf{p},x,t). \tag{5}$$

In the derivation of the elasticity-theory equation (1) a number of simplified assumptions were made; they are discussed in greater detail, e.g., in [4]. As shown in [3], the amplitudes of the higher harmonics of the strong wave are small if the crystal length is shorter than the nonlinear damping wave $\sim(a\Gamma_L)^{-1}$. We assume this condition to be satisfied.

We seek the solution of (2) in the form

$$f=f_0[\varepsilon(p)+\varphi_0(x,t)+\varphi_1(x,t)]+g_0(\mathbf{p},x,t)+g_1(\mathbf{p},x,t), \tag{6}$$

where $f_0[\varepsilon(p)+\varphi_0(x,t)]+g_0(\mathbf{p},x,t)$ is the solution of the kinetic equation in the strong-wave field, and g_1 is the distribution-function increment due to the weak wave $\varphi_1(x,t)$. We recognize that at sufficiently low temperatures, when the scattering is mainly by impurities, the collision integral is linear in f. We substitute (6) and (2) and linearize with respect to φ_1 and g_1:

$$\frac{\partial g_1}{\partial t}+v_x\frac{\partial g_1}{\partial x}-\frac{\partial\varphi_0}{\partial x}\frac{\partial g_1}{\partial p_x}+I\{g_1\}=\frac{\partial\varphi_1}{\partial x}\frac{\partial g_0}{\partial p_x}-\frac{\partial\varphi_1}{\partial t}\frac{\partial f_0}{\partial\varepsilon}. \tag{7}$$

In the derivation of (7) we have discarded small terms of the order of φ_0/ε_F. The relative "narrowness" of the resonant velocity region

$$|v_x-w|\leqslant\max(1/\tau q,\tilde{v})\ll v_F,$$

in which the functions g_0 and g_1 differ from zero, makes it possible to represent the collision integral in the form $I\{g_1\}=\tau^{-1}g_1$, where τ is the time of departure from the state p. When solving (7) it is convenient to change over to the variables s and ξ and introduce the notation $\omega_0=q\tilde{v}$ and $\tilde{v}=(\varphi_0/\mu)^{1/2}$:

$$\frac{1}{\omega_0}\frac{\partial g_1}{\partial t}+s\frac{\partial g_1}{\partial\xi}-\sin\xi\frac{\partial g_1}{\partial s}+ag_1=\frac{k}{q}\varphi_1\sin\left[\frac{k}{q}\xi+k(w-w_1)t\right]\times\left[\frac{1}{\varphi_0}\frac{\partial g_0}{\partial s}+\frac{w_1}{\tilde{v}}\frac{\partial f_0}{\partial\varepsilon}\right]. \tag{8}$$

Solving (8) by the method of characteristics, we easily see that the particle trajectories are determined by the potential of the strong wave and

$$s^2/2-\cos\xi=2/\varkappa^2-1 \tag{9}$$

is the integral of motion. It follows therefore that all the particles moving in the strong sound field can be divided into trapped ($|\varkappa|>1$) and untrapped ($|\varkappa|<1$). Introducing the dimensionless time of motion along the trajectory

$$y=\int_0^{\xi(y)}\frac{d\xi}{s(\varkappa,\xi)} \tag{10}$$

and changing over to the complex function $\tilde{g}_1$($g_1 = -\mathrm{Re}\,\tilde{g}_1$), we obtain from (8)

$$\frac{d\tilde{g}_1}{dy}+a\tilde{g}_1=i\varphi_1\frac{k}{q}\exp\left(i\frac{k}{q}\xi+i\frac{\omega_1}{\omega_0}y\right)\left[\frac{1}{\varphi_0}\frac{\partial g_0}{\partial s}+\frac{w_1}{\tilde{v}}\frac{\partial f_0}{\partial\varepsilon}\right] \tag{11}$$

($\omega_1=k(w-w_1)$ is the frequency of the weak wave in the coordinate system connected with the strong wave). It follows from (11) that the stationary solution should satisfy the following boundary conditions:

$$\tilde{g}_{1t}[y+T_0(\varkappa)]=\exp\left[i\frac{\omega_1}{\omega_0}T_0(\varkappa)\right]\tilde{g}_{1t}(y), \tag{12}$$

where $T_0(\varkappa)=4K(\varkappa^{-1})$ is the period of motion of the trapped particles,

$$\tilde{g}_{1ut}[y+2|\varkappa|K(\varkappa)]=\exp\left[i\frac{k}{q}2\pi+i\frac{2\omega_1}{\omega_0}|\varkappa|K(\varkappa)\right]\tilde{g}_{1ut}(y), \tag{13}$$

where $2|\varkappa|K(\varkappa)$ is the time during which the untrapped particle negotiates the distance $\xi=2\pi$. In the derivation of (12) and (13) we took into account the periodicity of the function $g_{0t}(\varkappa,y)$ and $g_{0ut}(\varkappa,y)$ in y.

Equation (11) has a solution satisfying the boundary conditions (12) and (13) in the form

$$\tilde{g}_1(y,\varkappa)=\int_{-\infty}^{y}d\tau\exp[a(\tau-y)]U(\tau,\xi(\tau,\varkappa)), \tag{14}$$

where

$$U(\tau,\xi,\varkappa)=i\frac{k}{q}\varphi_1\exp\left[i\frac{k}{q}\xi(\tau,\varkappa)+i\frac{\omega_1}{\omega_0}\tau\right]\times\left[\frac{1}{\varphi_0}\frac{\partial g_0}{\partial s}+\frac{w_1}{\tilde{v}}\frac{\partial f_0}{\partial \varepsilon}\right]. \tag{15}$$

At integer and half-integer k/q the integration in (14) can be carried out by expanding $\exp[\frac{1}{2}im\xi(\varkappa,y)]$, $2k/q=m$ in a Fourier series (see [3]). The corresponding formulas are rather cumbersome. However, the solution takes a simpler form when the frequency of the weak wave, in the coordinate frame connected with the strong wave, is lower than the collision frequency and the frequency ω_0:

$$\omega_1\ll\tau^{-1}\ll\omega_0, \tag{16}$$

i.e., the particle manages to execute several oscillations and to be scattered during the time of variation of the weak wave. The electron "sees" in this case the weak-wave field averaged over the fast oscillations. The second inequality in (16) means that we assume satisfaction of the strong-nonlinearity condition $a=(\omega_0\tau)^{-1}\ll 1$.

Let us calculate the integral (14) in the principal approximation in the parameter $a\ll 1$. We examine first the electron distribution function in the field of a strong longitudinal wave

$$f=f_0[\varepsilon(p)+\varphi_0(x-wt)]+g_0(\mathbf{p},x-wt). \tag{17}$$

Here $f_0[\varepsilon(\mathbf{p})+\varphi_0(x-wt)]$ is the local-equilibrium distribution function, $\varepsilon(p)=p^2/2\mu$ is the electron energy, and $\varphi_0(x-wt)$ is the potential produced by the strong wave. The function $g_0(\mathbf{p},x-wt)$ obtained in [1] is conveniently represented, by introducing the new variables

$$\xi=q(x-wt),\quad s=(v_x-w)/\tilde{v},$$

in the form

$$g_0(t,\varkappa)=\frac{\varphi_0 w}{\tilde{v}}f_0'(\varepsilon_\perp)\int_{-\infty}^{t}d\tau\exp[a(\tau-t)]\sin\xi(\tau,\varkappa), \tag{18}$$

where $\xi(\tau,\varkappa)$ is determined from the equation of motion,

$$E={}^1/_2s^2-\cos\xi$$

is the integral of motion ($\varkappa=[2/(E+1)]$), and

$$t/\varkappa=F(\xi/2,\varkappa)$$

is an incomplete elliptic integral of the first kind.

Formula (18) was obtained under the assumption that the inequalities $\varphi_0/\varepsilon_F\ll 1$ and $w\tilde{v}/v_F^2\ll 1$ are satisfied. Expanding in (18) in powers of the parameter $a\ll 1$ (this is just the case to be considered hereafter), we can write down the distribution functions for the trapped and untrapped particles respectively in the form

$$g_{0t}(s,\xi)=\frac{\varphi_0 w}{\tilde{v}}f_0'(\varepsilon_\perp)(a\xi-s),\qquad |s|\leqslant 2\cos\frac{\xi}{2}, \tag{19}$$

$$g_{0ut}(s,\xi)=\frac{\varphi_0 w}{\tilde{v}}f_0'(\varepsilon_\perp)[a(\xi-\bar{\xi})-(s-\bar{s})],\qquad |s|\geqslant 2\cos\frac{\xi}{2}. \tag{20}$$

In (2), $\bar{s}=\pi/\varkappa K(\varkappa)$ is the average dimensionless velocity,

$$\bar{\xi}=\pi F\left(\frac{\xi}{2},\varkappa\right)\Big/K(\varkappa)$$

is the distance traversed in the time t by a particle moving with average velocity $\bar{s}$; $K(\varkappa)$ is a complete elliptic integral of the first kind; ε_F is the Fermi energy and $\varepsilon_\perp$ is the energy of the transverse motion.

Expanding the right-hand side in (14) and (15) in a Fourier series and calculating the elementary integrals with respect to τ, we obtain, taking (19) and (20) into account, for the trapped particles

$$\tilde{g}_{1t}=i\frac{k}{q}\frac{\varphi_1}{\tilde{v}}f_0'(w_1-w)\exp\left(i\frac{\omega_1}{\omega_0}y\right)\times\sum_{l=-\infty}^{\infty}\left[a+i\left(\frac{\omega_1}{\omega_0}+\frac{\pi l\,\mathrm{sign}\,\varkappa}{2K(\varkappa^{-1})}\right)\right]^{-1}\exp\left[i\frac{\pi l|\varkappa|F(\xi/2,\varkappa)}{2K(\varkappa^{-1})}\right]b_l^m(\varkappa^{-1}) \tag{21a}$$

and for the untrapped particles

$$\tilde{g}_{1ut}=i\frac{k}{q}\frac{\varphi_1}{\tilde{v}}f_0'\exp\left(i\frac{\omega_1}{\omega_0}y\right)\times\sum_{l=-\infty}^{\infty}\left[a+i\left(\frac{\omega_1}{\omega_0}+\frac{\pi l}{2\varkappa K(\varkappa)}\right)\right]^{-1}\exp\left(i\frac{\pi l F(\xi/2,\varkappa)}{2K(\varkappa)}\right)\times a_l^m(\varkappa)\left[(w_1-w)-\frac{\pi l\varkappa^2 w}{4mK(\varkappa)}\frac{dZ}{d\varkappa}\right], \tag{21b}$$

where b_l^m and a_l^m are the Fourier coefficients of the expansion of the function $\exp(ikq^{-1}\xi(\tau,\varkappa))$ for the trapped and untrapped particles ($Z(\varkappa)=\pi/\varkappa K(\varkappa)$):

$$b_l^m(\varkappa^{-1})=\frac{1}{4K(\varkappa^{-1})}\int_{-2K(\varkappa^{-1})}^{2K(\varkappa^{-1})}du\left(\mathrm{dn}(u,\varkappa^{-1})+\frac{i}{|\varkappa|}\mathrm{sn}(u,\varkappa^{-1})\right)^m\exp\left(\frac{i\pi lu}{2K(\varkappa^{-1})}\right), \tag{22}$$

$$a_l^m(\varkappa)=\frac{1}{4K(\varkappa)}\int_{-2K(\varkappa)}^{2K(\varkappa)}du(\mathrm{cn}(u,\varkappa)+i\,\mathrm{sn}(u,\varkappa))^m\exp\left(\frac{i\pi lu}{2K(\varkappa)}\right). \tag{23}$$

It follows from (21a), (21b), and (22) that the terms with $l=0$ are of order a^{-1}:

$$\tilde{g}_{1t}=\frac{i}{a}\frac{k}{q}\frac{\varphi_1}{\tilde{v}}f_0'(\varepsilon_\perp)(w_1-w)\exp\left(i\frac{\omega_1}{\omega_0}y\right)b_0^m(\varkappa^{-1}), \tag{24}$$

$$\tilde{g}_{1ut}=\frac{i}{a}\frac{k}{q}\frac{\varphi_1}{\tilde{v}}f_0'(\varepsilon_\perp)\exp\left(i\frac{\omega_1}{\omega_0}y\right)a_0^m(\varkappa)(w_1-w). \tag{25}$$

It is easy to verify that all the terms with $l\neq 0$ make a contribution of higher order in a. This is obvious in the $\varkappa$ region where $\pi l/2\varkappa K(\varkappa)>a$ and $\pi l/2K(\varkappa^{-1})>a$. The last inequalities fail to hold only in the exponentially narrow region $\varkappa\approx 1\mp e^{-\pi l/a}$ corresponding to trajectories that pass near the separatrix. A contribution is made to the concentration and to the absorption coefficient (after integrating with respect to $\varkappa$) only by the first few terms of the series. Using the asymptotic expressions for $a_l^m(\varkappa)$ and $b_l^m(\varkappa^{-1})$ as $\varkappa\to 1$, we can show that each of these terms is exponentially small. The corresponding calculations show that the contributions of all terms with $l\neq 0$ to the concentration produced upon integration in a wide range of $\varkappa$ is of the order of a.

Changing from $\tilde{g}_1$ to g_1 and returning to the variable t, we write down the distribution function of the trapped and untrapped particles in the form

$$g_{1t}=\frac{1}{a}\frac{k}{q}\frac{\varphi_1}{\tilde{v}}f_0'(\varepsilon_\perp)(w_1-w)b_0^m(\varkappa^{-1})\sin\omega_1 t, \tag{26}$$

$$g_{1ut}=\frac{1}{a}\frac{k}{q}\frac{\varphi_1}{\tilde{v}}f_0'(\varepsilon_\perp)(w_1-w)a_0^m(\varkappa)\sin\omega_1 t. \tag{27}$$

We note that if m is odd (half-integer k/q) the distribution function of the untrapped particles is equal to zero because of the condition $a_0^{2n+1}(\varkappa)=0$. Since the functions g_{1t} and g_{1ut} describe the averaged reaction of the electron system to the slowly varying field of the weak wave they are proportional, as in the static problem, to the relaxation time τ. We discuss now the influence of the strong wave on the weak-sound propagation conditions.

2. SOUND AMPLIFICATION COEFFICIENT

The coefficient of amplification (absorption) of the weak sound, which is defined by the relation $\Gamma=-(2S)^{-1}\times dS/dx$ ($S=\frac{1}{2}\rho u_1^2\omega^2 w$ is the sound-energy flux), is obtained from the law of conservation of the field and particle energy:

$$\frac{dS}{dx}=-\left\langle n_1(x,t)\frac{\partial\varphi_1(x,t)}{\partial t}\right\rangle, \tag{28}$$

where

$$n_1(x,t)=\frac{2}{(2\pi)^3}\int d\mathbf{p}\, g_1(x,t,\mathbf{p})$$

is the nonequilibrium concentration connected with the function g_1, ρ is the density, u_1 is the wave amplitude, and the averaging is over the coordinates and the time. The right-hand side of (28) determines the work of the weak wave on the particles.

To calculate the concentration we proceed from integration over the momenta to the variables $\epsilon_\perp$ and $\varkappa$. In terms of the new variables we have

$$n_1(\xi,t)=\frac{8\mu^2\tilde{v}}{(2\pi)^2}\int_0^\infty d\varepsilon_\perp\int_0^{|\sin(\xi/2)|^{-1}}\frac{d\varkappa}{\varkappa^2}\frac{g_1^{\text{even}}(\varkappa,t)}{[1-\varkappa^2\sin^2(\xi/2)]^{1/2}}, \tag{29}$$

where g_1^{even} is that part of the distribution function which is even in $\varkappa$. The interval $(0,1)$ in the integral with respect to $\varkappa$ corresponds to the region of the untrapped particles, and the interval $(1,|\sin(\xi/2)|^{-1})$ to the region of trapped particles. Substituting (29) in (28) and changing the order of integration with respect to ξ and $\varkappa$, we obtain expressions for the gain (absorption) of the weak wave:

$$\Gamma=\frac{4}{a\pi^2}\frac{(w_1-w)}{w_1}\Gamma_L m\gamma(m), \tag{30}$$

where $\Gamma_L=\Lambda^2\mu^2k/4\pi\rho w_1$ is the linear absorption coefficient of the weak wave, and

$$\gamma(m)=\gamma_t(m)+\gamma_{ut}(m). \tag{31}$$

The coefficients

$$\gamma_t(m)=\int_1^\infty\frac{d\varkappa}{\varkappa^3}K(\varkappa^{-1})(b_0^m(\varkappa^{-1}))^2, \tag{32}$$

$$\gamma_{ut}(m)=\int_0^1\frac{d\varkappa}{\varkappa^2}K(\varkappa)(a_0^m(\varkappa))^2 \tag{33}$$

determine the contribution made to the absorption by the trapped and untrapped particles proper.

At fixed parameters of the strong wave, the ratio of the nonlinear absorption coefficient of the weak wave to the linear one is determined by the function $m\gamma(m)$. To analyze this function, it is convenient to represent the coefficients a_0^m and b_0^m in the form

$$a_0^m(\varkappa)=\frac{1}{4K(\varkappa)}\int_0^{2\pi}\frac{\cos(m\xi/2)\,d\xi}{[1-\varkappa^2\sin^2(\xi/2)]^{1/2}}, \tag{34a}$$

$$b_0^m(x)=\frac{1}{2K(x)}\int_0^{2\arcsin x}\frac{\cos(m\xi/2)\,d\xi}{[x^2-\sin^2(\xi/2)]^{1/2}},\quad x=\varkappa^{-1}. \tag{34b}$$

In the case of odd m (half-integer k/q), when only trapped particles contribute to $\Gamma(a_0^{2n+1}=0)$, we obtain by changing in (34b) from x to $y=1-2x^2$ and using the integral representation of Legendre polynomials,[5]

$$b_0^{2n+1}(y)=\frac{\pi}{2K(\sqrt{(1-y)/2})}P_n(y). \tag{35}$$

Changing next in (32) to the variable y, we obtain at $m=2n+1$

$$m\gamma(m)=\frac{(2n+1)^2\pi^2}{16}\int_{-1}^{1}\frac{P_n^2(y)\,dy}{K(\sqrt{(1-y)/2})}. \tag{36}$$

Since the function $K(\sqrt{(1-y)/2})$ is almost constant in practically the entire integration region and the Legendre polynomials are normalized to $2/(2n+1)$, the function (36) depends quite weakly on m. We can analogously consider the function $m\gamma(m)$ at even m and explain its slow dependence on m. To this end it is necessary to express the coefficients $b_0^{2n}(y)$ in terms of the Legendre function of first order $P_{n-1/2}(y)$. It is more difficult to estimate analytically the contribution of the untrapped particles to the function $m\gamma(m)$. Numerical calculation, whoever, shows that it amounts to a small fraction of the contribution of the trapped particles ($\gamma_{ut}/\gamma_t \sim 1/10$ at $m=2$). The conclusion that the function $m\gamma(m)$ changes slowly is confirmed by computer calculations. $\gamma(1) = 0.64$ at $m=1$ and at $m=2,3,4$ the function $m\gamma(m)$ differs from this value only in the second significant figure.

3. DISCUSSION OF THE RESULTS

We proceed to a discussion of the results. If the strong and weak waves propagate in the same direction, and the strong wave runs ahead of the weak one ($w_1 < w$), then $\Gamma < 0$ according to (30)–(33), corresponding, to amplification of the weak sound. If the velocity of the weak wave exceeds that of the weak wave exceeds that of the strong one, then the weak wave attenuates ($\Gamma > 0$). In the case when the weak wave propagates counter to the strong one, the weak sound is absorbed at both $w > w_1$ and $w < w_1$. To interpret these results, we turn again to the electron distribution function (17) in the strong-wave field. Integration of (17), with account taken of (19) and (20), with respect to the energy of motion in a plane perpendicular to x yields

$$F_t(s,\xi) = \varepsilon_F - \varphi_0(\xi) - a\mu\tilde{v}w\xi - \frac{\mu\tilde{v}^2 s^2}{2} - \frac{\mu w^2}{2},$$
$$|s| \leqslant 2\cos(\xi/2); \qquad (37)$$
$$F_{ut}(s,\xi) = \varepsilon_F - \varphi_0(\xi) - a\mu\tilde{v}w(\xi - \bar{\xi}) - \frac{\mu\tilde{v}^2 s^2}{2} - \frac{\mu w^2}{2} - \mu\tilde{v}w\bar{s},$$
$$|s| \geqslant 2\cos(\xi/2). \qquad (38)$$

We recall that $\bar{s} = \bar{s}(s, \xi)$;

$$F(s,\xi) = \int_0^\infty d\varepsilon_\perp f(\varepsilon_\perp, s, \xi).$$

Plots of the distribution functions (37) and (38) in the dimensional coordinates v_x and x are shown in Figs. 1 and 2.

In the expansion of $f_0(\varepsilon)$ in (37) and (38) we took into account the terms linear and quadratic in s and proportional to $f_0'(\varepsilon_\perp)$. The terms quadratic in s and proportional to $f_0''(\varepsilon_\perp)$ have been left out from the expansions of $f_0(\varepsilon)$ and $g_0(s, \xi, \varepsilon_\perp)$. The point is that in the case of Fermi statistics integration of terms proportional to $f_0''(\varepsilon_\perp)$ with respect to $\varepsilon_\perp$ yields the exponentially small factor $e^{-\mu/T}$, while integration of $f_0'(\varepsilon_\perp)$ yields unity.

The terms quadratic in s which were included by us in the expansion of $f_0(\varepsilon)$ cause the distribution function to become dome-shaped in the velocity region corresponding to the trapped particles. It is readily seen that the sign of Γ is determined in final analysis by the factor in the square brackets in the equation (11) for g_1. In the approximation considered by us, when the sign of $\partial g_0/\partial s$ is constant in the entire interal of the velocities corresponding to the trapped particles (these are precisely the particles that make the main contribution to the damping), the factor in the square brackets in (11) coincides with the derivative

$$\left.\frac{\partial f}{\partial v_x}\right|_{v_x = w_1}$$

and its sign is determined by the difference $w - w_1$. Therefore the sign of the coefficient Γ corresponds in each of the cases considered to the sign of the derivative of the distribution function with respect to the projection of the velocity on the wave propagation direction. This statement is illustrated by Fig. 1.

The sign of the factor in the square brackets in (11) is not constant for the untrapped particles. In our approximation, however, when only the terms with $l=0$ are significant in the sum over l in (21b), the sign of the function g_{1ut}, and accordingly the contribution to the damping of the untrapped particles, is also determined by the difference $w - w_1$.

According to (30), the gain (damping) of the weak wave can exceed greatly the linear damping coefficient $\Gamma_L(k)$, since $\Gamma \sim 1/a \gg 1$. (We recall that the damping coefficient of the strong wave is proportional to $a = (\omega_0\tau)^{-1}$.)

Our result, as already noted, is valid if the inequalities

$$k|w - w_1|/q\bar{v} \ll a \ll 1$$

are satisfied. It must be noted, however, that when the velocities of the strong and weak waves are close, i.e., the inequality $(w - w_1)/w \ll 1$ is satisfied, the coefficient Γ can become smaller than the next terms of the expansion in a. As shown in [3], the coefficients describing the interaction of the strong and weak waves at $w = w_1$ are of the order of a at integer m and of $a\ln(1/a)$ at half-integer m.

Since the dispersion of the sound velocity in crystals is exceedingly small, and the methods for artificially increasing the dispersion in the considered frequency region are of low efficiency, the enhancement effect considered above can be observed only if two acoustic modes with different phase velocities are simultaneously excited. If each of these waves is partially longitudinal, then our theory accounts for the order of magnitude of their interaction. The increase of the damping of the weak wave in the field of the strong one can be observed when both waves are purely longitudinal. To this end it is necessary that the strong and weak waves propagate in opposite directions. According to (32), the damping coefficient of the weak is then

$$\Gamma = \frac{8}{\pi^2 a}\Gamma_\pi m\gamma(m). \qquad (39)$$

Numerical estimates show that the conditions (22), which are necessary for the observation of the considered effects, can be satisfied, for example, in bismuth and indium antimonide, as well as in pure metals. The required strong-wave intensity is of the order of 10 W/cm^2, and the frequencies of the strong and weak waves should be of the order of 10^9 Hz.

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Dielectric properties of a helical smectic liquid crystal

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The temperature and frequency dependences of the permittivity were determined in the vicinity of the phase transition (in either direction) from the smectic *C* phase to the smectic *A* phase in a chiral smectic liquid crystal DOBAMBC. The results obtained were compared with the temperature dependences of the tilt angle θ of the molecules in the smectic layers, spontaneous polarization P_s, and helix pitch p_0. The experimental results were analyzed using a dynamic model based on the phenomenological theory of the chiral smectic *C* phase put forward by Pikin and Indenbom. The relative role played by piezoelectric effects of various kinds in the spontaneous polarization of DOBAMBC was analyzed and conclusions were drawn.

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INTRODUCTION

The existence of ferroelectric properties in chiral smectic *C* liquid crystals, pointed out by Meyer *et al.*,[1] has now been confirmed by numerous experimental measurements of the optical and polarization switching properties of this class of compounds.[2-8] A distinguishing feature of liquid-crystal ferroelectrics is the "orientational" nature of the dipole ordering, associated with the short-range nature of the forces which give rise to a parallel tilt of the molecules in the smectic layers. In this case the polarization is due to piezoelectric effects of various kind allowed by the symmetry of chiral smectic *C* liquid crystals.

A characteristic feature of these crystals is also a helical distribution of the permanent dipole moments of the molecules in the smectic layers (Fig. 1a). In each helix pitch (~4μ) there are ~2×10^3 discrete directions of the dipole moment in the layers (apart from allowance for thermal fluctuations of the directions of the dipole moment). If a sample is sufficiently thick, this helical structure corresponds to the equilibrium state of the chiral smectic *C* phase and its appearance as a result of a phase transition is due to the limiting nature of the symmetry group of the paraelectric high-temperature phase ($\infty/2$). As in the case of splitting of solid ferroelectrics into domains, the appearance of a helical structure helps to retain this macrosymmetry also in the polar phase.

The interaction of the dipole moments of the molecules with an external electric field results in a homogeneous polarization of a liquid crystal, i.e., it causes "untwisting" of the helix. Therefore, we may assume that the dielectric response of a liquid crystal system to an external alternating field includes important information on the features of the dipole ordering, dynamic processes associated with the deformation of the helix, and relative role of the piezoelectric effects of various kinds in the appearance of the spontaneous polarization. We shall report the temperature and frequency dependences of the permittivity in the vicinity of a phase transition (in either direction) from the smectic *C* phase

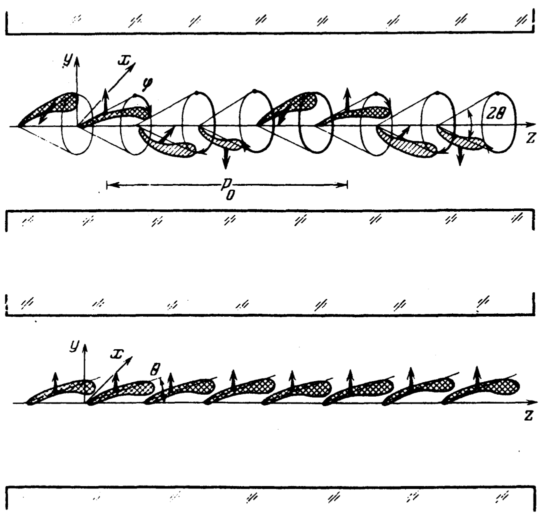
a
b


FIG. 1. Distribution of molecules and their permanent dipole moments in the chiral smectic *C* phase. The smectic layers are parallel to the *xy* plane. Inside the layers the molecules are titlted at an angle θ relative to the normal of the smectic planes: (a) helical structure in the absence of external perturbations; (b) homogeneous orientation of the director **n** and dipole moments **P** in an external electric field $E_= \geq E_p$.

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