Two-component Fermi liquid with electron-hole pairing

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A microscopic theory of an electron-hole Fermi liquid in conditions of coexistence of singlet and triplet pairing is constructed in the framework of the Larkin-Migdal scheme (Zh. Eksp. Teor. Fiz. **44**, 1703 (1963) [Sov. Phys. JETP **17**, 1146 (1963)]). The spectrum of the possible excitations in such a system is considered.

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1. Electron-hole systems, both equilibrium, in semimetals and semiconductors have recently attracted much attention from experimentalists and theorists. It is sufficient to point to the flow of papers on electron-hole droplets in semiconductors or to the work on excitonic insulators (see, e.g., the reviews by Keldysh[1] and Kopaev[2]). One of the interesting features of electron-hole systems in solids is the possibility of a phase transition to the dielectric state.[3,4] Unlike in the case of superconductors, here it is possible for electron-hole pairs to be formed not only in the singlet state but also in the triplet state, and this, in particular, can lead to the appearance of ferromagnetic properties.

The situation corresponding to coexistence of singlet and triplet pairings, which is achieved, e.g., by doping or by means of a magnetic field, is giving rise to particular interest.[5,6] In the latter papers the authors investigated in detail the magnetic properties of an excitonic insulator with two types of gap in the high-density approximation.

It should be remarked that the aforementioned rearrangement of the spectrum of the charge carriers cannot fail to affect also their collective properties in the corresponding region of frequencies. Moreover, the interaction in the system is by no means weak, generally speaking, and this requires that many-particle Fermi-liquid correlations be taken consistently into account.

The present paper is devoted to an investigation of the properties of the electron-hole Fermi liquid in systems of the excitonic-insulator type, and to an analysis of the spectrum of the sound and zero-sound excitations. In constructing a theory of this kind of liquid we shall make use of the scheme of Larkin and Migdal,[7,8] having generalized it to the case of a two-component system with two types of pairing. Lying at the basis of this scheme is the assumption that the ratio of the gap to the Fermi energy is small ($\Delta/\epsilon_F \ll 1$); this, as in the theory of superconductivity, corresponds in practice to weakness of the coherent part of the effective interaction of the quasi-particles, which is the part responsible for the phase transition. Consistent use of this parameter in the general case is possible in the so-called "classical regime," in which the role of fluctuations is relatively small (compare the discussion of the analogous range of topics for ^{3}He in the review[9]).

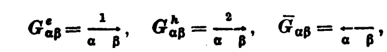
Below, in Sec. 2, equations are obtained in Fermi-liquid theory for the Green functions and the gaps, and in Sec. 3 the equation for the vertex of the interaction with an external field is obtained. In Secs. 4–7 the properties of the dispersion equation are analyzed and the results obtained are discussed.

2. The Green functions of the electrons and holes in the absence of pairing have the usual form

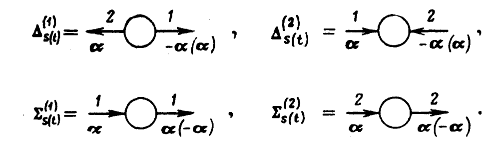
$$G^{(i)}=\frac{a_i}{\varepsilon-\xi_p^{(i)}+i\delta}+G_{reg}^{(i)},\quad \xi_p^{(i)}=\frac{p^2}{2m}+\mu_i, \qquad (1)$$

where $\xi_p^{(i)}$ is the energy of the carriers, measured from the Fermi level ϵ_F, $i=(e,h)\equiv(1,2)$, G_{reg} is a function that is slowly varying near the Fermi surface, $(a_i)^{-1}=(\partial G^{(i)}/\partial\epsilon)_0$, and μ is the shift of the Fermi level as a result of the doping (in equilibrium, $\mu_e=-\mu_h=\mu$). We shall suppose that the presence of the doping impurity weakly alters the chemical potential of the carriers, $\mu\ll\epsilon_F$, which permits us to neglect the corresponding corrections of order μ/ϵ_F in the calculations of the Green functions. However, for coexistence of two types of gap it is necessary, generally speaking, that μ be nonzero.

Diagrammatically, we can write



where α,β are spin indices, taking the value $\pm1/2$. The indices 1, 2 refer to the kind of particle. We also introduce the irreducible pairing and spin-flip amplitudes and the mass operator:



(2)

The mass operator Σ_s, whose action reduces basically to renormalizing the mass and chemical potential, is already taken into account above in formulas (1); therefore, we shall omit it in the following.

When quantities of order μ/ϵ_F are neglected, in the case of coinciding dispersion laws, as adopted below, it follows from considerations of symmetry that

$$\Delta_s^{(1)}=\Delta_s^{(2)}=\Delta_s,\quad \Delta_t^{(1)}=\Delta_t^{(2)}=\Delta_t,\quad \Sigma_t^{(1)}=\Sigma_t^{(2)}=\Sigma_t. \qquad (3)$$

Finally, we denote the set of all graphs that transform one kind of carrier into the other by $F_{\alpha\beta}$ and $\bar F_{\alpha\beta}$:

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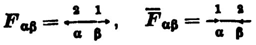
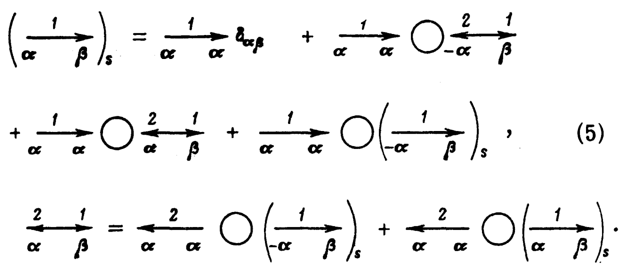
 (4)

and write the equation for the Green function of the system with pairing, which we denote by $(G_{\alpha\beta})_s$:



(5)

In analytic form, we have

$$(G_{\alpha\beta})_s = G^{(1)}\{\delta_{\alpha\beta} + \Delta_t F_{\alpha\beta} + \Delta_s F_{-\alpha\beta} + \Sigma_t (G_{-\alpha\beta})_s\},$$
$$F_{\alpha\beta} = \bar{G}^{(2)}\{\Delta_t (G_{\alpha\beta})_s + \Delta_s (G_{-\alpha\beta})_s\}, \quad (6)$$

whence it is easy to find

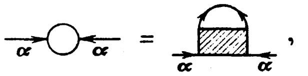
$$(G_{\alpha\beta})_s = {}^1\!/_2 a(\varepsilon+\xi_p)\{\Omega(\Delta_+, \mu_+) + (-1)^{\alpha-\beta}\Omega(\Delta_-, \mu_-)\},$$
$$F_{\alpha\beta} = {}^1\!/_2 a\{\Delta_+\Omega(\Delta_+, \mu_+) + (-1)^{\alpha+\beta}\Delta_-\Omega(\Delta_-, \mu_-)\}, \quad (7)$$

where

$$\Delta_\pm = (\Delta_s \pm \Delta_t)a^{-1}, \quad \mu_\pm = \mu \pm \Sigma_t, \quad \xi_p = p^2/2m,$$
$$\Omega(\Delta, \mu) = [\varepsilon^2 - \xi_p^2 - \Delta^2 - (\varepsilon+\xi_p)2\mu]^{-1}.$$

The presence of two types of excitation in a system with coexisting gaps already follows directly from the form of the solutions (7).

The equation for the gap has the following form:



or

$$\Delta_t = 2\hat{v}\hat{\hat{F}} \quad (8)$$

and analogously for Δ_s. The block $\hat{v}$ does not contain parts linked by two vertical lines, and under the condition $\Delta/\epsilon_F \ll 1$ coincides with the irreducible four-point vertex of the system without pairing (cf. Ref. 7).

In the absence of doping or under the condition $\mu \ll \epsilon_F$, we have $\bar{F}_{\alpha\beta} = F_{\alpha\beta}$ and $(G^{(1)}_{\alpha\beta})_s = (\bar{G}^{(2)}_{\alpha\beta})_s$. However, before making use of the relations (7), it is necessary to change in Eq. (8) to integration over the region of energy close to the Fermi surface, which is achieved by introducing the vertex $\hat{\Gamma}^\xi_1$ (and $\hat{\Gamma}^\xi_2$):

$$\hat{\Gamma}^\xi = \hat{\Gamma}_1{}^\xi + \hat{\sigma}\hat{\sigma}'\hat{\Gamma}_2{}^\xi = \hat{v}[1 + \theta_\xi \hat{G}^{(0)}\hat{\bar{G}}^{(0)}\hat{\Gamma}^\xi], \quad \mu \ll \xi \ll \varepsilon_F. \quad (9)$$

Here $\theta(\xi)$ is the usual step function. The corresponding procedure is described in detail in Ref. 7. In complete analogy we can quote the final result:

$$\Delta_t = a\hat{\Gamma}_2{}^\xi \hat{F}_{\alpha\alpha}, \quad \Delta_s = a\hat{\Gamma}_1{}^\xi \hat{F}_{-\alpha\alpha}. \quad (10)$$

The vertex Γ^ξ_1 corresponds to the interaction of electrons and holes with opposite spins.

In the case of coinciding (or close) dispersion laws, the functions $\Gamma^\xi_j(p, p')$ depend only on the angle between p and p′. In accordance with this, we expand Γ^ξ_j as usual in Legendre polynomials (cf. Ref. 7):

$$(a^2\rho\Gamma_j{}^\xi)_l = \ln^{-1}(\xi/C_j{}^l), \quad j=1, 2, \quad \rho = mp_0/2\pi^2 \quad (11)$$

and confine ourselves to the harmonic with $l = 0$, assuming the interaction to be isotropic. As a result, the system of equations for the gaps Δ_s and Δ_t takes the form

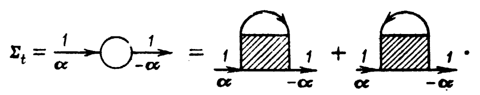
$$2\Delta_s \ln\frac{1}{C_1} = \Delta_+ \ln\frac{1}{\bar{\Delta}_+} + \Delta_- \ln\frac{1}{\bar{\Delta}_-},$$
$$2\Delta_t \ln\frac{1}{C_2} = \Delta_+ \ln\frac{1}{\bar{\Delta}_+} - \Delta_- \ln\frac{1}{\bar{\Delta}_-}, \quad (12)$$

where

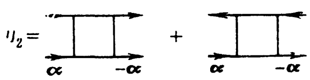
$$\bar{\Delta}_\pm = \mu_\pm + (\mu_\pm^2 - \Delta_\pm^2)^{1/2}.$$

In the large-density limit, when Σ_t is found to be small, $\mu_\pm \to \mu$ and the system (12) goes over into the corresponding system of equations of Refs. 5, 6. A stable solution $\Delta_+ \neq \Delta_-$ of the system (12) exists, as in the limit indicated above, only under the condition $\mu \neq 0$ (see Sec. 5 below). In accordance with the requirements of consistency, the gaps must be chosen to be real.

The system of equations (12) must be supplemented by an equation for the amplitude Σ_t. In diagrammatic form the corresponding equation is

 (13)

As a result of the renormalization of the irreducible four-point vertex



by introducing the new amplitude

$$\hat{\Gamma}_2{}^\omega = \hat{U}_2\{1 + (GG)^\omega \hat{\Gamma}_2{}^\omega\} \quad (14)$$

we obtain the equation

$$\Sigma_t = 2\hat{\Gamma}_2{}^\omega \hat{G}_{\alpha,-\alpha}. \quad (15)$$

Here the index ω, as usual, denotes the limit $k/\omega \to 0$ as $k, \omega \to 0$, and Γ^ω_2 is the spin part of the Landau Fermi-liquid function corresponding to the sum of the electron-electron and electron-hole interactions.

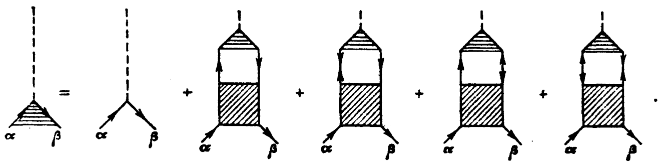
Retaining, as before, only the zeroth harmonic g^ω_0 of the expansion of $a^2\rho\Gamma^\omega_2$ in Legendre polynomials, we have (cf. Ref. 10)

$$\Sigma_t = {}^1\!/_2 g_0{}^\omega\{(\mu_+^2 - \Delta_+^2)^{1/2} - (\mu_-^2 - \Delta_-^2)^{1/2}\}. \quad (16)$$

It is not difficult to see that for $\mu = 0$ (and $g^\omega_0 > -1$) the quantity $\Sigma_t = 0$, and, in analogy with Ref. 5, we arrive at the nonphysical region of the solution for the gaps.

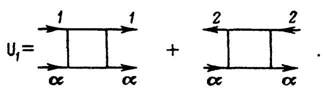
It should be remarked that above we did not take into account that part of the interband interaction which corresponds to virtual single-particle transitions. The role of this interaction in the high-density limit and the corresponding system of equations of the type (12), (13) for the gaps and Σ_t have been analyzed in detail in a recently published paper.[10],1)

3. We shall find the interaction amplitude of an electron with an external field, the poles of which determine the frequencies of the normal vibrations of the system. In graphical form, the corresponding equation is written as follows:



(17)

In all closed contours, summation over the spins and kinds of particles is implied. It is important also that $(G^{(1)})_s = (\overline{G}^{(2)})_s$ under the condition $\mu \ll \epsilon_F$, and the amplitude of the interaction with the field does not depend on the kind of particle. In fact, we arrive at an equation of the form of (22) in Ref. 7, with $\hat{U}$ replaced by



By the above-described renormalization,

$$\hat{\Gamma}_1^\omega = \hat{U}_1\{1+(GG)^\omega \hat{\Gamma}_1^\omega\},$$

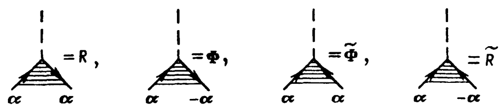
the integrals are reduced to integrals over a region close to ϵ_F. The amplitude Γ_1^ω corresponds to the spinless part of the Landau Fermi-liquid function corresponding to the sum of the electron-electron and electron-hole interactions.

It is natural that (17) should be supplemented by equations for the vertices that take account of pair creation, but in view of the close analogy with the results of Ref. 7 we write these equations immediately in analytic form. Moreover, in the general case it is necessary to add an equation for the polarization operator. We shall not write out this equation, since taking it into account leads to the appearance of the high-frequency plasma branch

$$\omega^2 = 2\omega_{pl}^2 + \alpha(kv_F)^2, \quad \omega_{pl}^2 = 4\pi e^2 N/m, \tag{18}$$

obtained earlier in the paper[11] by Kozlov and Maksimov, in which the excitation spectrum of an excitonic insulator with one gap was treated in the gas approximation, and also to zero-sound modes of the type studied in the papers[12,13] by one of the authors.

Introducing now the notation



and using the renormalized amplitudes $\Gamma_{1,2}^\omega$ introduced above, from Eqs. (17) we finally obtain the following system of four equations:

$$\begin{aligned} R &= R^\omega + \tfrac{1}{2}\Gamma_1^\omega\{(L_+ + L_-)R + (M_+ + M_-)\tilde{R} + (L_+ - L_-)\Phi + (M_+ - M_-)\tilde{\Phi}\}, \\ \tilde{R} &= \tfrac{1}{2}\Gamma_1^\xi\{(N_+ + N_-)\tilde{R} + (O_+ + O_-)R + (N_+ - N_-)\tilde{\Phi} + (O_+ - O_-)\Phi\}. \end{aligned} \tag{19}$$

The equation for $\tilde{\Phi}$ is obtained by the replacements

$$N_- \to -N_-, \quad O_- \to -O_-, \quad \Gamma_1^\xi \to \Gamma_2^\xi,$$

in the equation for $\tilde{R}$, and the equation for Φ by the replacements

$$L_- \to -L_-, \quad M_- \to -M_-, \quad \Gamma_1^\omega \to \Gamma_2^\omega, \quad R^\omega = 0$$

in the equation for R.

The following notation was introduced above:

$$\begin{aligned} L &= \rho a^2\left\{\frac{\mathbf{kv}}{\omega - \mathbf{kv}}[1-\varphi(x,\omega)] - \varphi(x,\omega)\left(\frac{1+\hat{P}}{2}\right)\right\}, \\ M &= \rho a^2 \frac{\omega+\mathbf{kv}}{2\Delta}\varphi(x,\omega); \quad N = \rho a^2\left\{\ln\frac{2\xi}{\Delta} - x^2\varphi(x,\omega)\right\}, \\ O &= -\rho a^2\left\{\frac{\omega+\mathbf{kv}}{4\Delta} + \frac{\omega-\mathbf{kv}}{4\Delta}\hat{P}\right\}\varphi(x,\omega), \\ \varphi(x,\omega) &= \frac{1}{2x(1+x^2)^{1/2}}\left\{2\,\mathrm{arcsh}\,x + \mathrm{arcth}\frac{(\mu^2-\Delta^2)^{1/2}x(1+x^2)^{1/2}}{\omega/2+\mu x^2}\right. \\ &\quad \left. - \mathrm{arcth}\frac{(\mu^2-\Delta^2)^{1/2}x(1+x^2)^{1/2}}{\omega/2-\mu x^2}\right\}, \quad x^2 = \frac{(\mathbf{kv})^2-\omega^2}{4\Delta^2}, \end{aligned} \tag{20}$$

where $\hat{P}$ is the operator that permutes the external points of the vertices, and the subscript plus or minus on the quantities L, M, N, O and φ corresponds to the choice $\Delta = \Delta_\pm$ in the formulas (20). Here R^ω is the renormalized interaction amplitude of an electron with the external field:

$$R^\omega = R^0(1+[G^{(0)}G^{(0)}]^\omega \Gamma_1^\omega),$$

where R^0 is the bare vertex [see (17)].

Thus, unlike in the case of a one-component Fermi liquid, a system of four equations has arisen (new amplitudes Φ and $\tilde{\Phi}$ have appeared), and this is connected with the fact of the coexistence of singlet and triplet pairings.

4. We turn to the solution of the system of equations for R, $\tilde{R}$, Φ and $\tilde{\Phi}$, and thereby to the study of the dispersion equation of the excitation in the Fermi liquid, confining ourselves to the analysis of the longitudinal vibrations. Taking into account the complexity of the problem, we shall consider first the low-frequency long-wavelength limit $\omega - \mathbf{k}\cdot\mathbf{v} \ll \min\{\Delta_+, \Delta_-\}$, ω, and shall seek the scalar vertices R and Φ in the form $A + B\mathbf{k}\cdot\mathbf{v}$. After straightforward but cumbersome calculations we find

$$R = \frac{\partial G^{-1}}{\partial \varepsilon}\,\frac{\omega^2 - c_1^2k^2 + \tfrac{1}{3}f_1^\omega \mathbf{kv}\omega}{\omega^2 - c^2k^2}, \tag{21}$$

where f_1^ω is a coefficient of the expansion of the function $f^\omega \equiv \rho a^2 \Gamma_1^\omega$ and

$$c_1^2 = \tfrac{1}{3}v^2(1+\tfrac{1}{3}f_1^\omega), \quad c^2 = c_1^2(1+f_0^\omega).$$

Thus, in the presence of coexisting gaps the acoustic excitation spectrum preserves the same form as in a one-component superfluid Fermi liquid[7,8] (compare also with the gas limit in Ref. 11 and with the case of superconductors[14,15]).

A nontrivial solution for the amplitude Φ is obtained under the condition

$$\omega^2-c^2k^2-q(1+g_0^\omega)=0, \tag{22}$$

which follows directly from Eqs. (19)–(21). The quantity q is defined by the following expression:

$$q=\frac{4\Delta_+^2\Delta_-^2}{\Delta_+^2-\Delta_-^2}\ln\frac{\bar{\Delta}_-}{\bar{\Delta}_+}. \tag{23}$$

In accordance with the assumption that ω and ck are small, Eq. (22) is valid only in the case of small values of the parameter $\eta=q(1+g_0^\omega)$.

For positive values of η Eq. (22) always has a solution with $\omega^2(0)>0$. The presence of a threshold appears completely natural, since the amplitude Φ corresponds to excitations with a spin flip. The absence of spin degeneracy leads to the appearance of a minimum frequency $\omega^2=q(1+g_0^\omega)$ for a transition between levels with opposite spins. The situation is completely analogous to that considered by Kozlov and Maksimov,[11] where the threshold was determined by interband transitions. In the limit $\Delta_s=\Delta_t$, $\eta=0$ and we arrive at the usual acoustic spin-wave spectrum (cf. the case of a one-component liquid in Ref. 16).

For negative values of the parameter η the picture changes radically—stable vibrations exist only at sufficiently large values of ck, or, more precisely, in the region

$$-\eta<(kc)^2<\mu.$$

Moreover, when $k^2c^2=-\eta$ the frequency vanishes, after which the system becomes unstable with respect to long-wavelength excitations of the type (22). The stabilization effect at finite values of the wave vector indicates the possibility of a transition of the system to a stable nonuniform state with two gaps; however, this question needs further investigation.

The situation described arises, in particular, in the case $\mu=0$, $1+g_0^\omega>0$, which corresponds to the nonphysical solution (in the absence of doping) obtained in Ref. 5 in the limit $k=0, g_0^\omega=0$. In the presence of doping a system with two gaps turns out to be stable. It is interesting that these conclusions also remain valid in the case of the formal limit of a Stoner ferromagnet, when $g_0^\omega\leq-1$.

5. We shall investigate the solution of the system (19) in conditions when the Fermi-liquid parameters that are not responsible for the pairing can be neglected. We shall assume also that the system is uniform ($R^\omega=0$), and that Γ_2^ξ and Γ_1^ξ are isotropic. In this case Eqs. (19) have the following form:

$$\begin{gathered}\bar{R}\{1-{}^1/_2\Gamma_1^\xi(N_++N_-)\}-{}^1/_2\Gamma_1^\xi(N_+-N_-)\Phi=0,\\ {}^1/_2\Gamma_2^\xi(N_+-N_-)\bar{R}-\{1-{}^1/_2\Gamma_2^\xi(N_++N_-)\}\Phi=0.\end{gathered} \tag{24}$$

Next, integrating (24) over the angle and equating the determinant to zero, we obtain the following equation:

$$\begin{gathered}\int_{-1}^{1}(\omega^2-k^2v^2x^2)\varphi_+dx\left\{q+\int_{-1}^{1}(\omega^2-k^2v^2x_1^2)\varphi_-dx_1\right\}\\ +\int_{-1}^{1}(\omega^2-k^2v^2x^2)\varphi_-dx\left\{q+\int_{-1}^{1}(\omega^2-k^2v^2x_1^2)\varphi_+dx_1\right\}=0.\end{gathered} \tag{25}$$

When $\bar{\Delta}_+=\bar{\Delta}_-$, (25) goes over into a dispersion equation of the same type as that investigated in Ref. 15, and has, in particular, the solution

$$\omega=2\Delta\left(1-\exp\left\{-\frac{kv}{\pi\Delta}\ln\frac{kv}{\Delta}\right\}\right) \tag{26}$$

in the case $kv\gg\Delta$. We investigated the limit ω, $kv\ll\mu$ above [(see (21) and (22)].

We consider next the case of large $k, kv\gg\mu$. It is easy to see that the dependence on the frequency is important only near the poles of the function φ, i.e., for $\omega\simeq2\Delta_\pm$. In particular, if $\omega\simeq2\Delta_+$, the system (25) has a solution similar to (26):

$$\omega=2\Delta_+\left(1-\exp\left\{-\frac{kv}{\pi\Delta_+}\ln\frac{kv}{\Delta_+}+\frac{q}{\Delta_+^2}\right\}\right). \tag{27}$$

The presence of the term containing q in the exponent is unimportant in practice, since $(kv)^2\gg\Delta_+^2\simeq q$. This is a natural result, because, for large momenta of the excitations, the lifting of the spin degeneracy has a weak effect on the form of the spectrum. An analogous result is also obtained for $\omega\simeq2\Delta_-$.

6. We shall analyze the limit $k=0$ in more detail. In this case the system of integral equations (19) becomes algebraic and the dispersion equation for the threshold frequencies $\omega(0)$ takes the following form:

$$\begin{gathered}\omega^2\varphi_+\varphi_-[1-f_l^\omega g_l^\omega(\varphi_+-\varphi_-)^2]\\ ={}^1/_2q[\varphi_++\varphi_-+2g_l^\omega\varphi_+\varphi_-+f_l^\omega g_l^\omega(\varphi_++\varphi_-)(\varphi_+-\varphi_-)^2].\end{gathered} \tag{28}$$

In the limit of $\Delta_+\sim\Delta_-$ and frequencies $\omega\ll\mu$, Eq. (28) goes over into (22) for values $k\simeq0$. In the general case the analytic solution of Eq. (28) is difficult, but the very fact that solutions exist can be perceived from a graphical analysis.

Thus, putting $f_l^\omega=g_l^\omega=0$ initially, in place of (28) we obtain the equation

$$\omega^2\varphi_+\varphi_-={}^1/_2q(\varphi_++\varphi_-). \tag{29}$$

Taking into account the asymptotic forms of the functions $\varphi_\pm$, it is easy to see that a solution of Eq. (29) necessarily exists provided only that the gaps $\Delta_\pm$ satisfy the following inequality:

$$\frac{1}{\Delta_+^2-\Delta_-^2}\ln\frac{\mu_++(\mu_+^2-\Delta_+^2)^{1/2}}{\mu_-+(\mu_-^2-\Delta_-^2)^{1/2}}<0. \tag{30}$$

In the general case, Eq. (28) has a solution of the form (22) for small ω when a condition of the type (30) is fulfilled, as is easily discerned from the intersection of the curves

$$\begin{gathered}y_1(\omega)=2\omega^2q^{-1}[1-f_l^\omega g_l^\omega(\varphi_+-\varphi_-)^2],\\ y_2(\omega)=\varphi_-^{-1}+\varphi_+^{-1}+2g_l^\omega+f_l^\omega g_l^\omega(\varphi_+-\varphi_-)^2.\end{gathered}$$

In addition, there are two more zero-sound modes, entirely due to liquid effects. With decrease of $f_l^\omega g_l^\omega$ these solutions are shifted into the region of high frequencies, approaching $2\Delta_+$ or $2\Delta_-$, respectively, so that for $f_l^\omega g_l^\omega \ll 1$ we have

$$\omega(0)=2\Delta_\pm\left(1+\frac{\pi}{4}f_l^\omega g_l^\omega\frac{q+8\Delta_\pm^2}{q-8\Delta_\pm^2}\right). \quad (31)$$

For $f_l^\xi=g_l^\xi$ we have $\Delta_-=q=0$, and we arrive at a solution of the same type as zero-sound in a one-component system:

$$\omega(0)=2\Delta(1-{}^1/_4\pi f_l^\omega g_l^\omega). \quad (32)$$

Finally, in the case $\eta<0$, i.e., $\mu=0$, in the low-frequency region there is a solution only for $\omega^2(0)<0$ and the system becomes unstable, as already noted above in the analysis of Eq. (22).

7. Thus, the scheme developed above permits us to describe correlation effects in electron-hole systems with a phase transition, which can possess a number of interesting properties, e.g., ferromagnetic ordering, superfluidity in the case of electron-hole droplets, etc. In applications to real semiconductors and semimetals it should be borne in mind that the lower (as compared with metals) carrier density enhances the role of Fermi-liquid effects, on the one hand, and, on the other, leads to an increase of the correlation length and correspondingly narrows the region of applicability of the local theory, which assumes a point Fermi-liquid interaction. Moreover, in the static case at zero temperature the screening length in the ground state of the excitonic insulator increases without limit,[17] and for $\omega\sim 0$, even for $T\lesssim T_c$, it is found to be greater than the size of a pair, thereby limiting the region of attainable values of the wave vectors and frequencies.

The spectrum of the longitudinal collective excitations considered above in Secs. 4–6 is made up of the "normal" modes, such as sound, plasma oscillations and zero-sound in the two-component Fermi liquid, and oscillations associated in an essential way with the presence of two order parameters. In this case correlation effects play an essential role and lead to the appearance of new, zero-sound modes, due both to the presence of the two gaps and to liquid effects. And if a branch of the acoustic type can lie in the low-frequency region $\omega\ll\Delta_\pm$, two other modes lie in the region $\omega\sim 2\Delta_\pm$. In typical semimetals and semiconductors this is the region of frequencies $\omega\sim 10^{13}$–10^{14} sec^{-1}, while in droplets the frequencies are one or two orders of magnitude lower. Collisional dissipation, which is small at low temperature, is still further suppressed by the presence of pairing, and this favors observation of the excitations, e.g., by sound absorption or neutron scattering. At nonzero temperatures hybridization of both the spinless and the spin excitations of the normal and ordered phases should arise.

The question of the excitation spectrum is also directly connected with the question of the stability of a system with two gaps. The stability diagram determined from the minimum of the free energy in the high-density limit[5] essentially corresponds to the curve $\eta=0$ for $k=g^\omega=f^\omega=0$; in the region $\eta<0$ a buildup of long-wavelength excitations occurs, and this leads to a rearrangement of the system and to a transition of the system to a state with one gap or, possibly, to a nonuniform state. Correlation effects, as was elucidated above, change the value of the parameter η and deform the phase-stability diagram; in any case, however, electron-hole pairing leads to coexistence of gaps only in the presence of doping.

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