Anomalous magnetic moment of an electron in a polarized electromagnetic wave

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A calculation of the mass operator of an electron moving in a polarized electromagnetic wave in a constant external crossed field is used to determine separately the static anomalous magnetic moment of the particle. The expressions obtained for the anomalous moment of the electron determine it as a function of the characteristics of an essentially nonstationary electromagnetic field.

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1. INTRODUCTION

Calculations of the radiative shifts of the energy levels of particles play a central role in the development of modern quantum electrodynamics. The magnetic moments determined with high accuracy for the electron, muon, and various other elementary particles provide one of the effective means for testing the predictions of the theory, and also make it possible to obtain evermore detailed information about the nature of various interactions. Especial interest attaches to theoretical calculations of the radiative corrections to the motion of particles in external macroscopic fields. The rapid development of laser technology, and also of the technology for producing strong static fields means that it is more important than ever to take into account nonlinear effects in the calculation of interactions with external fields. A quantity that in this sense can in principle be measured in experiments, is, for example, the anomalous magnetic moment of the electron in a strong electromagnetic field.

A very important characteristic of the anomalous magnetic moment is the tendency for its Schwinger value[1] (equal to $\alpha e/4\pi m$, where e, m, and α are the electron charge and mass and the fine structure constant)[1)] to decrease in any external electromagnetic field.[2-5] It would be very desirable to confirm this conclusion experimentally, since it could significantly augment traditional investigations of small spacetime regions in vacuum. And although the characteristic limiting cases where the anomalous moment in a constant external field differs most appreciably from the Schwinger value $\alpha e/4\pi m$ will not be verified that soon, experimentally, the case is altered in an electromagnetic field which is a superposition of a constant field and the field of a coherent electromagnetic wave.[5]

In this connection, it is worth considering once more the anomalous magnetic moment in an external electromagnetic field in order to draw attention to an essentially new aspect which underlines the dynamical nature of the anomalous moment—its dependence on the polarization of a coherent electromagnetic wave.

2. ANOMALOUS MAGNETIC MOMENT OF AN ELECTRON IN AN EXTERNAL ELECTROMAGNETIC FIELD

To obtain information about the anomalous magnetic moment in an external electromagnetic field, one uses the modified Dirac–Schwinger equation and calculates the mass operator of the particle.[4,5] Allowance for interaction of the electron with the vacuum leads to the appearance in the mass operator of a characteristic term, and in the rest frame of the particle this has the form

$$\Delta m \sim \mu \sigma_{\rho\nu} F_1^{\rho\nu}, \qquad (1)$$

where μ is the anomalous magnetic moment, and F_1 is the tensor of the constant field. In a nonstatic electromagnetic field of special form, the mass operator of the electron calculated on the mass shell in general contains other spin-dependent terms. For example, in a superposition of a constant crossed field and the field of a plane monochromatic electromagnetic wave of circular polarization:

$$A_\mu = a e_\mu^1 (nx) + b[e_\mu^1 \cos(kx) + g e_\mu^2 \sin(kx)] \qquad (2)$$

(where $g = \pm 1$; $k_\mu = \omega n_\mu$ is the wave four-vector; $n^2 = (e^1 e^2) = (e^1 n) = (e^2 n) = 0$) the expression for the on-shell mass operator averaged over a period of the wave can be written in the form

$$\langle M(p,F)\rangle = -\frac{\alpha}{2\pi}(2\pi)^4\delta(p-p') \times \frac{m^2}{q_0}\int_0^\infty \frac{dt}{(1+t)^2}\int_0^\infty \frac{dx}{x}\left\{G\exp\left[-i\frac{x^3}{3} - ixz(1+\xi^2) + i\frac{2t\xi^2}{yx}\sin^2\frac{y}{2}\right] - e^{-izx}\right\}; \qquad (3)$$

where

$$G = \frac{2+2t+t^2}{1+t}\left[\left(\frac{x^2}{z} + \frac{1}{2}\xi^2\sin^2\frac{y}{2}\right)J_0(u) + i\xi g\frac{x\sqrt{2}}{\sqrt{z}}\sin\frac{y}{2}J_0'(u)\right] + J_0(u) - i\frac{2x\gamma z}{1+t}\left(J_0 + i\xi\frac{g}{x}\left(\frac{z}{2}\right)^{1/2}\sin\frac{y}{2}J_0'\right) - \frac{\gamma_1\xi t(2+t)}{2\sqrt{2}(1+t)y}\left(y\cos\frac{y}{2} - 2\sin\frac{y}{2}\right)\left(\frac{x}{\sqrt{z}}J_0'(u) + i\frac{g}{\sqrt{2}}\xi\sin\frac{y}{2}J_0(u)\right),$$

$$q_\mu = p_\mu - \frac{e^2\bar{A}_\mu^2}{2(np)}n_\mu;$$

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$J_0(u)$ and $J_0'(u)$ are the Bessel functions of index zero and its derivative with respect to the argument

$$u=\frac{2\sqrt{2}\,\xi t x}{\varkappa\sqrt{z}\,y}\left(y\cos\frac{y}{2}-2\sin\frac{y}{2}\right);$$

the remaining quantities are defined by

$$z=\left(\frac{t}{\chi}\right)^{2/3},\quad \chi^2=-\frac{e^2}{m^6}(F_1^{\mu\nu}p_\nu)^2,\quad y=\frac{\varkappa z x}{t},\quad \varkappa=\frac{2(kp)}{m^2};$$

$$\xi=\frac{eb}{m},\quad \gamma=\frac{e s_\nu p_\lambda F_1^{*\nu\lambda}}{2m^2(np)},$$

$$\gamma_1=\frac{\varepsilon^{\alpha\beta\gamma\delta}s_\alpha e_\beta^1 e_\gamma^2 n_\delta}{(np)m^{-1}},\quad F_1^{*\nu\lambda}=\frac{\varepsilon^{\nu\lambda\mu\sigma}F_{1\mu\sigma}}{2};$$

q_μ is the four-quasimomentum of the electron in the field of the wave, p_μ is the four-momentum of the particle, and s_μ is the polarization four-vector of the electron. The spin structure of the mass operator in the given field is determined by two terms, which depend on the polarization of the electron and are proportional to the parameters γ and γ_1; however, the requirement that the interaction have the form (1) in the rest frame of the particle (see also[6]) makes a unique choice possible.

Thus, only the term containing γ in the mass operator determines the anomalous moment of the electron since the part of the energy corresponding to it in the rest frame of the particle can be interpreted as the energy of the interaction between the magnetic moment of the electron and the external field. Note that off the mass shell the expression for $M(p,p';F)$ contains some other terms proportional to $\sigma_{\mu\nu}F_2^{\mu\nu}$ (contraction of the tensor $\sigma_{\mu\lambda}$ with the tensor of the wave field) which do not contribute to the energy of the electron averaged over the period of the wave and therefore disappear from the expression (3) calculated for real electron ends. Note also that corrections $\sim\gamma_1$ to the electron mass arise only if the field configuration contains an electromagnetic wave with nonzero degree of circular polarization (in particular, in the case of a wave with elliptic polarization). For the configuration of a constant crossed field and an electromagnetic wave of linear polarization, which is described by the vector potential

$$A_\mu=ae_\mu^1(nx)+be_\mu^2\sin(kx), \tag{4}$$

the expression for the mass operator does not contain such a term:

$$\langle M(p,F)\rangle=-\frac{\alpha}{2\pi}(2\pi)^4\delta(p-p')\frac{m^2}{q_0}\int_0^\infty\frac{dt}{(1+t)^2}\int_0^\infty\frac{dx}{x}$$
$$\times\exp\left[-i\frac{x^3}{3}-ixz\left(1+\frac{1}{2}\xi^2\right)+i\frac{2t\xi^2}{y\varkappa}\sin^2\frac{y}{2}\right]$$
$$\times\left\{J_0(u)-2\gamma\frac{xz}{1+t}J_0(u)+\frac{2+2t+t^2}{1+t}\left[\frac{x^2}{z}J_0(u)\right.\right.$$
$$\left.\left.+\frac{1}{2}\xi^2\sin\frac{y}{2}\,(J_0(u)+iJ_1(u))\right]\right\}-e^{-izx}. \tag{5}$$

Here, $J_0(u)$ and $J_1(u)$ are Bessel functions of the argument

$$u=\xi^2\frac{t}{\varkappa y}\left(2\sin^2\frac{y}{2}-\frac{y}{2}\sin y\right).$$

The remaining parameters are the same as in the expression (3).

3. POLARIZATION EFFECTS

In accordance with the definition given above, the anomalous magnetic moment of the electron for the superposition (2) can be calculated, for example, in accordance with the expression

$$\Delta m=\gamma\frac{\alpha}{\pi}\frac{m^2}{q_0}\mathrm{Re}\,i\int_0^\infty\frac{z\,dt}{(1+t)^3}\int_0^\infty dx\left[J_0(u)\right.$$
$$\left.-i\frac{g}{x}\xi\left(\frac{z}{2}\right)^{1/2}\sin\frac{y}{2}J_0'(u)\right]\exp\left\{-i\frac{x^3}{3}-ixz(1+\xi^2)\right.$$
$$\left.+i\frac{2t\xi^2}{y\varkappa}\sin^2\frac{y}{2}\right\}. \tag{6}$$

For the value of the anomalous moment in the case (4) (crossed field + wave of linear polarization) one can separate out from the expression (5) an analogous term containing two integrations, which, however, cannot be performed analytically for arbitrary values of the basic parameters. To make further analysis in the special case of a relatively weak wave, it is more convenient to use a different representation for the vacuum correction to the electron energy due to the interaction of the anomalous moment with the external field; this is expressed as an expansion with respect to the number of photons of the wave that participate in the process. Thus, in the superposition (4) the value of the anomalous moment is determined by the expression

$$\Delta m=-\frac{i\alpha}{2\pi^3}\sum_{(\nu_k)}\exp\left[i\frac{\pi}{2}(\nu_1-\nu_2)\right]\int\frac{t\,dt\,d\tau}{(1+t)^3}$$
$$\times\frac{dl_1dl_2\delta(\nu_1-\nu_2-2\nu_3+2\nu_4)}{(l_1-m^2+i\varepsilon)(l_2+i\varepsilon)}\Phi(\rho)\Phi'(\rho)\prod_{k=1}^4 J_{\nu_k}(u_k), \tag{7}$$

where

$$\rho=\left(\frac{t}{2\chi}\right)^{2/3}\left[1+\tau^2+\frac{1}{2}\xi^2+\frac{l_1}{m^2}\frac{1+t}{t^2}+\frac{l_2}{m^2}\frac{1+t}{t}+\frac{\varkappa}{t}(\nu_2-2\nu_4)\right],$$
$$u_1=u_2=\xi\frac{2\tau t}{\varkappa},\quad u_3=u_4=\xi^2\frac{t}{4\varkappa};$$

$\Phi(\rho)$ and $\Phi'(\rho)$ are the Airy function and its derivative with respect to the argument ρ; $J_{\nu_k}(u_k)$ are Bessel functions of the arguments u_k, $k=1,2,3,4$.

Bearing in mind that the characteristic laser intensities are $\xi<1$, expanding (7) in a series in ξ (to order ξ^2), and integrating with respect to τ, l_1, and l_2, we find

$$\mu^{lin}=\frac{\alpha}{\sqrt{\pi}}\mu_0\,\mathrm{Re}\int_0^\infty\frac{dt}{(1+t)^3}\left\{zf+\frac{1}{2}\xi^2\left[z^2f'\right.\right.$$
$$\left.\left.-\frac{t^2}{\varkappa^2}(2f_1-f_1^--f_1^+)\right]\right\}, \tag{8}$$

where

$$\mu_0=\frac{e}{2m},\quad f(z)=\frac{i}{\sqrt{\pi}}\int_0^\infty\exp\left(-i\frac{x^3}{3}-izx\right)dx,$$

$$f_1(z^\mp)=f_1^\mp,\quad f_1(z)=\int_z^\infty dx\left(f(x)-\frac{1}{x}\right),\quad z^\mp=z\left(1\mp\frac{\varkappa}{t}\right).$$

For modern experiments, the region $\chi\ll1$ is probably of greatest interest. For it, we finally obtain (to terms linear in χ)

$$\mu^{lin}=\frac{\alpha}{2\pi}\mu_0\Big\{1+\frac{1}{2}\xi^2\Big[-1-\frac{2}{\varkappa}\Big(\frac{\pi^2}{3}+F(-1-\varkappa)+F(-1+\varkappa)\Big)+\frac{2}{1-\varkappa^2}+\frac{6\varkappa^2-2}{(1-\varkappa^2)^2}\ln\varkappa\Big]\Big\}, \tag{9}$$

where

$$F(x)=\int_0^x\frac{\ln(1+t)}{t}dt\ .$$

Making lengthier but similar calculations for a plane-wave superposition of a constant crossed field and an electromagnetic wave of elliptic polarization:

$$A_\mu=ae_\mu^1(nx)+2^{1/2}b[e_\mu^1\sin\psi\cos(kx)+ge_\mu^2\cos\psi\sin(kx)], \tag{10}$$

we obtain an expression for the anomalous magnetic moment which depends explicitly on the wave polarization:

$$\mu=\frac{\alpha}{\sqrt{\pi}}\mu_0\,\mathrm{Re}\int_0^\infty\frac{z\,dt}{(1+t)^3}\Big\{f+\xi^2\Big[zf'+2\sin^2\psi\Big(r^-f+\frac{r^+}{2}(f_-+f_+)-\frac{t}{2\varkappa}(f_--f_+)+4z\frac{t\chi^2}{\varkappa^3}(f_-'-f_+')\Big)-\frac{1}{z}\frac{t^2}{\varkappa^2}\cos2\psi(2f_1-f_1^--f_1^+)\Big]\Big\},$$

$$r^\pm=\frac{2t^2}{\varkappa^2}\Big(1\pm\frac{4\chi^2}{\varkappa^2}\Big). \tag{11}$$

This result for an electromagnetic wave of fairly general form can also be represented in the slightly different form

$$\mu=\mu^{cir}+(\mu^{lin}-\mu^{cir})\cos2\psi. \tag{12}$$

Here, μ^{cir} corresponds to the value of the anomalous moment in a superposition of a crossed field and a wave of circular polarization, and it is equal to

$$\mu^{cir}=\frac{\alpha}{\sqrt{\pi}}\mu_0\,\mathrm{Re}\int_0^\infty\frac{zdt}{(1+t)^3}\Big\{f+\xi^2\Big[zf'+r^-f+\frac{1}{2}r^+(f_-+f_+)-\frac{t}{2\varkappa}(f_--f_+)+4z\frac{t\chi^2}{\varkappa^3}(f_-'-f_+')\Big]\Big\}. \tag{13}$$

The expression (13) agrees with the result obtained earlier in[5] by the method of analytic continuation for a configuration representing a combination of a constant magnetic field and the field of a plane electromagnetic wave propagating along the vector of the magnetic field strength. This agreement is a known general consequence (see, for example,[4]) of the equivalence of the conditions under which different quantum processes take place in electromagnetic fields in the case of relativistic energies of the interacting particles extended to the region of radiative effects.

In the region $\chi\ll1$ in the limiting cases $(\varkappa)^{\pm1}\gg1$ we have the following expressions for the correction to the anomalous electron moment in the field of an electromagnetic wave of elliptic polarization:

$$\mu=-\frac{\alpha}{2\pi}\mu_0\xi^2\begin{cases}6\varkappa^2\Big(\frac{1}{2}+\sin^2\psi\Big)\ln\Big(\frac{1}{\varkappa}\Big), & \varkappa\ll1\\ 1+\frac{2}{\varkappa^2}\ln\varkappa(3+\ln\varkappa\cos2\psi), & \varkappa\gg1\ .\end{cases} \tag{14}$$

Note the symmetry of the complete expression for μ in the limit $\chi\ll1, \varkappa\ll1$:

$$\mu=\frac{\alpha}{2\pi}\mu_0\Big\{1-12\Big[\chi^2\ln\Big(\frac{1}{\chi}\Big)+\chi_1^2(1+2\sin^2\psi)\ln\Big(\frac{1}{\varkappa}\Big)\Big]\Big\}, \tag{15}$$

where $\chi_1=m^{-3}eb(kp)$ and it is assumed that not only the arguments of the logarithms but also the logarithms themselves are large:

$$\ln(1/\chi)\gg1,\quad \ln(1/\varkappa)\gg1.$$

We also give the expression for the anomalous moment in the limit $\chi\gg1, \xi<1$ (we do not write out the term which is proportional to $\chi^{-4/3}$ but does not depend on ξ):

$$\mu=\frac{\alpha\Gamma(1/3)}{9\sqrt{3}\,(3\chi)^{2/3}}\mu_0\Big[1-\frac{2}{9}\frac{b^2}{a^2}\Big(\frac{1}{2}+\cos^2\psi\Big)\Big]. \tag{16}$$

Note that in this case the anomalous magnetic moment contains a correlation term $\sim b^2/a^2$.

It is characteristic that the expressions obtained for the anomalous moment do not depend on the handedness of the wave polarization, $g=\pm1$. Such a dependence can arise in the next order in the expansions with respect to the energy of relativistic particles moving in a magnetic field, but it is not important unless the resonance situation $((kp)=ea)$ is realized; allowance for this requires an additional investigation.

In the general case of a wave of elliptic polarization and stationary motion of electrons in a magnetic field, the value of the anomalous magnetic moment for the two values of the angle ψ ($\psi=0,\pi/2$) is the same, as follows from the symmetry of the problem. It is to be expected that in the quasiclassical region ($\chi\ll1$) the anomalous magnetic moment in such a field will be again described by the expression (12) if one bears in mind that in this case ψ varies in the interval from 0 to $\pi/4$ (where $\psi=\pi/4$ corresponds to an electromagnetic wave of circular polarization). It would seem that this is the situation that could be most readily realized in an experiment.

1) Here and below we use units for which $c=\hbar=1$ and $\alpha=e^2$ and a four-metric with signature (+ – – –). The scalar product is written as $(ab)=a_0b_0-\mathbf{ab}$.

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