Contribution to the theory of the oscillations of a plasmon gas in a weakly turbulent plasma

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A theory is developed of plasmon-gas oscillations in the case of weakly turbulent magnetoactive plasma. The oscillations take place in the drift velocity and in the plasmon temperature. Special investigations are made of waves in a gas of MHD plasmons—secondary MHD waves having a linear dispersion law. The phase velocity of these depends on the relation between the speed of sound and the Alfven velocity. If the speed of sound is less than the Alfven velocity, then the velocity of the secondary MHD waves is close to that of the sound. If the inverse relation holds, then the secondary MHD wave velocity is close to the Alfven velocity.

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1. INTRODUCTION

As shown by Landau,[1] unique waves, which he called second-sound waves, can propagate in a quasiparticle gas. Subsequently waves of this kind were investigated in phonon and magnon cases[2-5] as well as in plasmon gas in the case of an isotropic plasma.[6-8]

We consider in this paper oscillations in a gas of plasmons in an anisotropic magnetoactive plasma. This problem is not only of theoretical but also of practical interest, particularly in connection with the possibility of heating a turbulent plasma by alternating spatially inhomogeneous fields. In the case of a spatially homogeneous field, the theory of this heating method was developed in [9,10]. When spatially inhomogeneous fields are used, it is possible to excite in a plasmon gas oscillations that should increase, under resonance conditions, the efficiency of the heating of the turbulent plasma. From the character of the resonant absorption it is possible to ascertain the presence of oscillations in the plasmon gas and, in addition, it is possible to turbulent-plasma parameters on which the energy absorption depends.

We consider specifically oscillations in a gas of magnetohydrodynamic (MHD) plasmons in the case of a weakly turbulent plasma. We assume equilibrium, i. e., Planck, plasmon distributions. The waves connected with these oscillations will be called secondary MHD waves. The oscillations take place in the drift velocity and in the temperature of the plasmons.

We investigate first the natural oscillations and then consider the excitation of secondary MHD waves with the aid of space-time modulation of the external magnetic field. We shall show that in the frequency region $\Omega_q \ll 1/\tau$, where τ is the average plasmon collision frequency,[11] the secondary MHD waves attenuate weakly and can be revealed by the resonant absorption of energy from an external source, a modulating field. The drift velocity of the MHD plasmons oscillates in the plane made up by the vector of the external magnetic field and the wave vector of the secondary MHD waves. At low values of the level of the turbulent pulsations, the drift velocity of the secondary MHD waves is directed predominantly along the vector of the external magnetic field. The velocity of the secondary MHD waves in the limiting cases $V_S^2 \ll V_A^2$ and $V_S^2 \gg V_A^2$ (V_S is the sound velocity and V_A is the Alfven velocity) is respectively equal to

$$S = {}^7/_9\sqrt{3}V_S, \quad S = {}^7/_9\sqrt{{}^3/_2}V_A.$$

2. KINETIC EQUATION FOR PLASMON GAS

To determine the dispersion and the velocity of the secondary waves we use the kinetic equation for the plasmon distribution function $N_\alpha = N_\alpha(\mathbf{r}, t)$ ($\alpha \equiv (\mu_\alpha, k_\alpha)$, μ_α is the plasmon species and $\mathbf{k}_\alpha$ is its wave vector) at the point $\mathbf{r}$ and at the instant of time t:

$$\frac{\partial N_1}{\partial t} + \mathbf{v}_1\left(\nabla N_1 - \frac{\partial N_1}{\partial \omega_1}\nabla\omega_1\right) = (\dot{N}_1)_c, \tag{1}$$

where $\omega_1 \equiv \omega_1(\mathbf{r}, t)$ is the perturbed plasmon frequency, $\mathbf{v}_1 = \partial\omega_1/\partial\mathbf{k}_1$ is its group velocity, and $(\dot{N}_1)_c$ is the collision integral.

The use of the kinetic equation is legitimate, strictly speaking, if the average plasmon lifetime τ and the plasmon mean free path are much shorter than the oscillation period and the wavelength of the secondary waves. In addition, the dissipative processes should not cause too strong a damping of the secondary waves.

In the study of secondary waves it is customary to consider natural oscillations in a gas of quasiparticles with unperturbed frequency. In the presence of external sources that lead to modulation of the frequencies of the quasiparticles and to excitation of the secondary waves, it is necessary to take into account the dependence of the frequency on $\mathbf{r}$ and t. We represent this dependence in the form

$$\omega_1(\mathbf{r}, t) = \omega_{10}(1 + a_1), \tag{2}$$

where ω_{10} is the plasmon frequency in the absence of modulation, $a_1 \equiv a_1(\mathbf{r}, t)$ is a parameter that characterizes the depth of modulation, which we assume to be small, $|a_1| \ll 1$.

The secondary waves are connected with the deviation of the plasmon distribution function N_1 from the plasmon equilibrium distribution function for which the collision integral vanishes. It is therefore important to determine the equilibrium distribution function. In the presence of sources and damping regions of plasmons separated by an inertial interval, the problem is very complicated and must be solved in each concrete case separately (see e.g., [12,13]).

We shall assume that the number of plasmons is large enough so that their lifetime relative to plasmon interaction with one another is much shorter than the lifetime due to the interaction between the plasmons and the plasma particles. (In the case of MHD plasmons the corresponding inequalities were derived in [10].)

In addition, we shall assume that the intensity of the plasmon sources is small enough. In this case the only terms of importance in the collision integral $(\dot{N}_1)_c$ will be those due to the interaction of the plasmons with one another. The presence of damping region can manifest itself in the calculation of the damping coefficient of the secondary waves.

If these assumptions are satisfied, the collision integral vanishes for a local equilibrium Planck distribution function

$$N_{10}=N_{10}(\mathbf{r},t)=\left[\exp\left(\frac{\omega_{10}-\mathbf{k}_1\mathbf{u}}{T^*}\right)-1\right]^{-1}, \qquad (3)$$

where $T^*=T^*(\mathbf{r},t)$ and $\mathbf{u}\equiv\mathbf{u}(\mathbf{r},t)$ are the temperature field and the plasmon drift-velocity field. In the case of total statistical equilibrium we have $\mathbf{u}=0$, and T^* is independent of $\mathbf{r}$ or t. We assume the drift velocity to be small and the temperature oscillations to be small: $T^*=T^*_0(1+\vartheta)$, where $\vartheta\equiv\vartheta(\mathbf{r},t)$ is a small relative increment of the temperature T^*_0 ($|\vartheta|\ll 1$). The full-equilibrium distribution function is of the form $\bar{N}_1=[\exp(\omega_{10}/T^*_0)-1]^{-1}$. At small perturbations, the functions N_{10} and $\bar{N}_1$ differ little from each other. Expanding N_{10} in a series in the small quantities a_1, ϑ, and $\mathbf{k}_1\cdot\mathbf{u}/\omega_{10}$, we obtain

$$N_{10}=\bar{N}_1+\frac{\partial\bar{N}_1}{\partial\omega_{10}}\omega_{10}\left(a_1-\vartheta-\frac{\mathbf{k}_1\mathbf{u}}{\omega_{10}}\right). \qquad (4)$$

We seek the solution of the kinetic equation (1) in the form $N_1=N_{10}+\delta N_1$, where δN_1 is a small deviation from N_{10} and is proportional, as will be shown later, to the product of two small quantities $(a_1-\vartheta-\mathbf{k}_1\mathbf{u}/\omega_{10})\Omega_q\tau$, where Ω_q is the frequency of the oscillations of the secondary waves. Therefore in the first-order approximation we can neglect in the left-hand side of the kinetic equation (1) the derivatives of δN_1 with respect to the coordinates and the time. These terms must, however, be taken into account in the investigation of the problem of absorption of secondary waves.

Neglecting the damping, we represent the kinetic equation in first-order approximation in the form

$$\frac{\partial\bar{N}_1}{\partial\omega_{10}}\omega_{10}\left[\dot{a}_1+\left(\frac{\partial}{\partial t}+(v_1\nabla)\right)\left(\vartheta+\frac{\mathbf{k}_1\mathbf{u}}{\omega_{10}}\right)\right]=(\dot{N}_1)_c. \qquad (5)$$

3. EQUATIONS FOR THE OSCILLATIONS OF THE PLASMON TEMPERATURE AND THE PLASMON DRIFT VELOCITY

Using the kinetic equation (5) and the laws of energy and momentum conservation in plasmon interaction [the number of plasmons is not conserved, since the decay and coalescence of the plasmons are taken into account in $(\dot{N}_1)_c$], we obtain equations that describe the changes of ϑ and $\mathbf{u}$:

$$\begin{gathered}\dot{\vartheta}+\left\langle\frac{k_iv_j}{\omega_0}\right\rangle\frac{\partial u_i}{\partial r_j}=\langle\dot{a}\rangle,\\ \left\langle\frac{k_ik_j}{\omega_0^2}\right\rangle\dot{u}_j+\left\langle\frac{k_iv_j}{\omega_0}\right\rangle\frac{\partial\vartheta}{\partial r_j}=\left\langle k_i\frac{\dot{a}}{\omega_0}\right\rangle,\end{gathered} \qquad (6)$$

where $\langle\ldots\rangle$ denotes averaging over the phase space of the plasmons:

$$\langle f\rangle=\left(\sum_1\omega_{10}^2\bar{N}_1(\bar{N}_1+1)f_1\right)\Big/\sum_1\omega_{10}^2\bar{N}_1(\bar{N}_1+1). \qquad (7)$$

The summation is carried out here over the wave vectors and the species of the plasmons.

Equations (6) describe the natural oscillations of the secondary waves at $\langle a\rangle=0$ and the forced oscillations at $\langle a\rangle\neq 0$ without allowance for dissipation. Allowance for dissipation leads to a small damping proportional to the ratio of the plasmon lifetime to the period of the oscillations of the secondary waves. This estimate of the damping will be made later on with secondary MHD waves as an example.

Let us examine in greater detail an anisotropic axially symmetrical medium with a symmetry axis along the unit vector $\mathbf{b}$. In this case it is convenient to represent the wave vector $\mathbf{k}_1$ in the form $\mathbf{k}_1=\mathbf{k}_{1\perp}+k_{1\parallel}\mathbf{b}$, where $\mathbf{k}_{1\perp}\perp\mathbf{b}$. The frequency ω_{10} is then a function of $k_{1\perp}$ and $|k_{1\parallel}|$, so that the group velocity can be represented in the form

$$\mathbf{v}_\perp=v_{1\perp}\frac{\mathbf{k}_{1\perp}}{k_{1\perp}}+v_{1\parallel}\frac{k_{1\parallel}}{|k_{1\parallel}|}\mathbf{b};\quad v_{1\perp}=\frac{\partial\omega_{10}}{\partial k_{1\perp}},\quad v_{1\parallel}=\frac{\partial\omega_{10}}{\partial|k_{1\parallel}|}.$$

Using these relations, we get

$$\begin{gathered}\left\langle\frac{k_ik_j}{\omega_0^2}\right\rangle=\frac{1}{2}(\delta_{ij}-b_ib_j)\left\langle\frac{k_\perp^2}{\omega_0^2}\right\rangle+b_ib_j\left\langle\frac{k_\parallel^2}{\omega_0^2}\right\rangle,\\ \left\langle\frac{k_iv_j}{\omega_0}\right\rangle=\frac{1}{2}(\delta_{ij}-b_ib_j)\left\langle\frac{k_\perp v_\perp}{\omega_0}\right\rangle+b_ib_j\left\langle\frac{|k_\parallel|v_\parallel}{\omega_0}\right\rangle.\end{gathered}$$

Substituting these expressions in the system (6) we have for the axially symmetric case with allowance for the fact that terms of the type $\langle k_ia/\omega_0\rangle$ vanish on account of the averaging over the phase space,

$$\begin{gathered}\dot{\vartheta}+\left\langle\frac{|k_\parallel|v_\parallel}{\omega_0}\right\rangle\frac{\partial u_\parallel}{\partial r_\parallel}+\frac{1}{2}\left\langle\frac{k_\perp v_\perp}{\omega_0}\right\rangle(\nabla_\perp\mathbf{u}_\perp)=\langle\dot{a}\rangle,\\ \left\langle\frac{|k_\parallel|v_\parallel}{\omega_0}\right\rangle\frac{\partial\vartheta}{\partial r_\parallel}+\left\langle\frac{k_\parallel^2}{\omega_0^2}\right\rangle\dot{u}_\parallel=0,\\ \left\langle\frac{k_\perp v_\perp}{\omega_0}\right\rangle\nabla_\perp\vartheta+\left\langle\frac{k_\perp^2}{\omega_0^2}\right\rangle\dot{\mathbf{u}}_\perp=0,\end{gathered} \qquad (8)$$

where $\mathbf{u}=\mathbf{u}_\perp+u_\parallel\mathbf{b}$, $\mathbf{r}=\mathbf{r}_\perp+r_\parallel\mathbf{b}$, $\nabla=\nabla_\perp+\mathbf{b}\partial/\partial r_\parallel$.

We consider the natural ocsillations of the plasmons. We seek the solution of Eqs. (8) in the form of plane waves

$$(\mathbf{u}, \vartheta) \propto (\mathbf{u}_q, \vartheta_q) \exp[i(\mathbf{qr}-\Omega t)]$$

with a wave vector **q** and a frequency Ω. From the condition that these equations have a solution, we obtain the dispersion equation for the secondary waves

$$\Omega^2=\Omega_q^2=S_\parallel^2 q_\parallel^2+S_\perp^2 q_\perp^2, \qquad (9)$$

where $\mathbf{q}=\mathbf{q}_\perp+q_\parallel\mathbf{b}$ and the quantities $S_\parallel$ and $S_\perp$, which have the dimension of velocity, are equal to

$$S_\parallel=\left\langle\frac{|k_\parallel|v_\parallel}{\omega_0}\right\rangle\left\langle\frac{k_\parallel^2}{\omega_0^2}\right\rangle^{-1/2}, \quad S_\perp=\left\langle\frac{k_\perp v_\perp}{\omega_0}\right\rangle\left\langle\frac{k_\perp^2}{\omega_0^2}\right\rangle^{-1/2}\frac{1}{\sqrt{2}}. \qquad (10)$$

The plasmons oscillate in the direction of the unit vector e lying in the plane of the vectors q and b:

$$\mathbf{u}_q=u_q\mathbf{e}, \quad \mathbf{e}=e_\perp\mathbf{q}_\perp/q_\perp+e_\parallel\mathbf{b},$$
$$e_\parallel=(1+|u_{q\perp}|^2/|u_{q\parallel}|^2)^{-1/2}, \quad e_\perp=(1+|u_{q\parallel}|^2/|u_{q\perp}|^2)^{-1/2}. \qquad (11)$$

The drift velocities $u_{q\parallel}$ and $u_{q\perp}$ can be easily expressed in terms of the relative change of the temperature:

$$u_{q\parallel}=\frac{S_\parallel q_\parallel}{\Omega_q}\left\langle\frac{k_\parallel^2}{\omega_0^2}\right\rangle^{-1/2}\vartheta_q, \quad u_{q\perp}=\sqrt{2}\frac{S_\perp q_\perp}{\Omega_q}\left\langle\frac{k_\perp^2}{\omega_0^2}\right\rangle^{-1/2}\vartheta_q. \qquad (12)$$

We calculate the momentum density of the plasmon gas

$$P=\sum_1 \mathbf{k}_1 N_1. \qquad (13)$$

Using the expansion of N_1 accurate to terms linear in **u**, we get

$$P_i=m_{ij}^* u_j, \qquad (14)$$

where m^*_{ij} is the tensor of the effective mass per unit volume of the plasmon gas

$$m_{ij}^*=W\langle k_i k_j/\omega_0^2\rangle, \quad W=\frac{1}{T_0}\sum_1 \omega_{10}^2 \bar{N}_1(\bar{N}_1+1). \qquad (15)$$

In the case of the Rayleigh-Jeans distribution, W is equal to the energy density of the turbulent pulsations:

$$W=\sum_1 \omega_{10}\bar{N}_1.$$

In the axially-symmetrical case

$$m_{ij}^*=m_\perp^*(\delta_{ij}-b_i b_j)+m_\parallel^* b_i b_j,$$
$$m_\perp^*={}^1/_2 W\langle k_\perp^2/\omega_0\rangle, \quad m_\parallel^*=W\langle k_\parallel^2/\omega_0^2\rangle. \qquad (16)$$

The secondary-wave energy density averaged over time or space can be represented in the form

$$\varepsilon=\overline{\sum_1 \omega_{10}(N_{10}-\bar{N}_1)}, \qquad (17)$$

where the bar denotes averaging. From this expression we obtain, accurate to terms quadratic in u,

$$\varepsilon={}^1/_2 m_{ij}^*|u_{qi}||u_{qj}|. \qquad (18)$$

In the axially symmetrical case

$$\varepsilon={}^1/_2 m_\perp^*|u_{q\perp}|^2+{}^1/_2 m_\parallel^*|u_{q\parallel}|^2. \qquad (19)$$

Using the expression (12) for the drift velocity and the definition (16) of the mass tensor, we get from (19)

$$\varepsilon_q={}^1/_2 W|\vartheta|^2. \qquad (20)$$

We see that the energy density of the secondary waves is smaller by a factor $2/|\vartheta|^2$ than the plasmon energy density.

We consider now the excitation of secondary waves by an external wave modulation of frequency Ω and wave vector **q**. Solving the system of inhomogeneous equations (8) with respect to ϑ and **u**, we get

$$\vartheta=\frac{\Omega^2}{\Omega^2-\Omega_q^2}\langle a\rangle,$$
$$u_\parallel=\frac{\Omega}{\Omega^2-\Omega_q^2}\left\langle\frac{k_\parallel^2}{\omega_0^2}\right\rangle^{-1/2}S_\parallel q_\parallel\langle a\rangle,$$
$$\mathbf{u}_\perp=\frac{\Omega}{\Omega^2-\Omega_q^2}\left\langle\frac{k_\perp^2}{\omega_0^2}\right\rangle^{-1/2}S_\perp\mathbf{q}_\perp\langle a\rangle, \qquad (21)$$
$$\varepsilon=\frac{\Omega^2\Omega_q^2}{(\Omega^2-\Omega_q^2)^2}W\overline{\langle a\rangle^2}.$$

It is easily seen that when the resonance conditions $\Omega=\Omega_q$ are satisfied small density perturbations lead to large values of ϑ and **u**. This in turn increases the energy density of the secondary waves to values limited by their damping.

4. SECONDARY MHD WAVES

We consider now concretely secondary waves in a magnetoactive weakly turbulent plasma in which low-frequency MHD waves of three types are excited: Alfven (a), and fast (f) and slow (s) magnetosonic. The dispersion of these waves without allowance for the nonlinear interactions between them is determined by the known expressions

$$\omega_a=|k_\parallel|V_A,$$
$$\omega_{f,s}={}^1/_2 k|(V_A^2+V_S^2+2\varkappa_\parallel V_A V_S)^{1/2}\pm(V_A^2+V_S^2-2\varkappa_\parallel V_A V_S)^{1/2}|, \qquad (22)$$

where $V_A=B_0/\sqrt{\rho}$ is the Alfven velocity, $V_S=(\partial p/\partial\rho)_S$ is the speed of sound, $\varkappa_\parallel=k_\parallel$, $\mathbf{b}=\mathbf{B}_0/B_0$, $\mathbf{B}_0$ is the external magnetic field, and p is the gas-kinetic pressure of the plasma. The group velocity of these waves can be represented in the form

$$\mathbf{v}_a=V_A\frac{k_\parallel}{|k_\parallel|}\mathbf{b},$$
$$v_{f\parallel}=\left(\omega_f-\omega_s A\frac{k_\perp^2}{k|k_\parallel|}\right)\frac{|k_\parallel|}{k^2}, \quad v_{f\perp}=\left(\omega_f+\omega_s A\frac{|k_\parallel|}{k}\right)\frac{k_\perp}{k^2}, \qquad (23)$$
$$v_{s\parallel}=\left(\omega_s+\omega_f A\frac{k_\perp^2}{k|k_\parallel|}\right)\frac{|k_\parallel|}{k^2}, \quad v_{s\perp}=\left(\omega_s-\omega_f A\frac{|k_\parallel|}{k}\right)\frac{k_\perp}{k^2},$$
$$A=V_S V_A[(V_A^2+V_S^2)^2-4\varkappa_\parallel^2 V_S^2 V_A^2]^{-1/2}.$$

We note that, independently of the ratio of V_S to V_A, the frequencies ω_a and ω_s vanish at $k_\parallel=0$. This leads to a divergence in the expression for $\langle k_\perp^2/\omega_0^2\rangle$, so that $S_\perp$ and $u_{q\perp}$ vanish. When account is taken of the nonlinear dispersion laws of the MHD waves,[14] these quantities differ from zero and depend on the level of the turbulent pulsations. At low turbulent pulsation levels, $W/\rho V_S^2 \ll 1$, they are small, but they must be taken into account in the case of secondary MHD waves that propagate transversely to the magnetic field, $q_\perp \gg |q_\parallel|$.

Let us examine in detail the limiting case of an MHD plasma with $V_S^2 \ll V_A^2$ the main quantities that characterize the secondary MHD waves, calculated with allowance for the modified dispersion laws,[14] are

$$m_\parallel^* = \frac{W}{3V_s^2} = \frac{1}{3}\rho\left(\frac{W}{\rho V_s^2}\right), \quad m_\perp^* \approx \rho\left(\frac{W}{\rho V_s^2}\right)^{1/4};$$
$$S_\parallel = \frac{7}{9}\left(\frac{W}{m_\parallel^*}\right)^{1/2} = \frac{7}{9}3^{1/2}V_s, \quad S_\perp = \frac{1}{9}\left(\frac{W}{m_\perp^*}\right)^{1/2} \approx 0.1\left(\frac{W}{\rho V_s^2}\right)^{1/4} V_s;$$
$$u_{q\parallel} = \frac{S_\parallel q_\parallel}{\Omega_q}\left(\frac{W}{m_\parallel^*}\right)^{1/2}\vartheta_q = \frac{7}{3}\frac{q_\parallel V_s^2}{\Omega_q}\vartheta_q,$$
$$u_{q\perp} = \frac{S_\perp q_\perp}{\Omega_q}\left(\frac{W}{m_\perp^*}\right)^{1/2}\vartheta_q \approx 0.1\frac{q_\perp V_s^2}{\Omega_q}\left(\frac{W}{\rho V_s^2}\right)^{3/4}\vartheta_q. \quad (24)$$

We see from these expressions that the secondary MHD waves in a plasma with low gaskinetic pressure ($V_S^2 \ll V_A^2$) propagate with a velocity on the order of that of sound in the direction of the magnetic field, the propagation velocity transverse to the magnetic field is much less than the sound velocity, and the transverse density $m_\perp^*$ of the plasmon mass is much larger than their longitudinal density $m_\parallel^*$. The plasmons oscillate predominantly in the direction of the magnetic field.

All these conclusions are valid also in the other limiting case $V_S^2 \gg V_A^2$, except that the numerical coefficients in (24) are different, and the sound velocity V_S is replaced by the Alfven velocity V_A. We present the calculation results in this limiting case for secondary MHD waves propagating at an angle to the magnetic field not close to $\pi/2$, when the nonlinear MHD wave dispersion laws are insignificant:

$$m_\parallel^* = \frac{2}{3}\frac{W}{V_A^2} = \frac{2}{3}\rho\left(\frac{W}{\rho V_A^2}\right), \quad S_\parallel = \frac{7}{9}\sqrt{\frac{3}{2}}V_A,$$
$$u_{q\parallel} = \sqrt{^{3}/_{2}}V_A\vartheta_q, \quad e_\parallel \approx 1. \quad (25)$$

The energy density $\bar{\epsilon}_q$ in the two limiting cases is determined by (20).

We investigate now the secondary MHD-wave damping due to plasmon interaction with one another. To estimate the damping we retain in the left-hand side of the kinetic equation the term $(\partial/\partial t + (\mathbf{v}_1\nabla))\delta N_1$, and linearize the collision integral $(\dot{N}_1)_c$ relative to δN_1. As a result of the linearization the collision integral takes the form

$$(\dot{N}_1)_c = -\tau_1^{-1}\delta N_1 + L_1\{\delta N\},$$

where $1/\tau_1$ is the frequency of the plasmon collisions with one another (τ_1 is the plasmon relaxation time), and $L_1\{\delta N\}$ is a linear functional of δN. As an estimate we can put $(\dot{N}_1)_c = -\tau_1^{-1}\delta N_1$. If the conditions $\Omega\tau_1 \ll 1$ and $|\mathbf{q}\cdot\mathbf{v}|\tau_1 \ll 1$ are satisfied, we have

$$\delta N_1 = -i\frac{\partial \bar{N}_1}{\partial\omega_{10}}\omega_{10}\left(\vartheta + \frac{\mathbf{k}_1\mathbf{u}}{\omega_{10}}\right)(\Omega - \mathbf{q}\mathbf{v}_1)\tau_1. \quad (26)$$

The term discarded in the kinetic equation (5) then turns out to equal

$$-\frac{\partial \bar{N}_1}{\partial\omega_{10}}\omega_{10}(\Omega - \mathbf{q}\mathbf{v}_1)^2\left(\vartheta + \frac{\mathbf{k}_1\mathbf{u}}{\omega_{10}}\right)\tau_1.$$

Taking into account this term in the system (8) of the equations that define ϑ_q and $\mathbf{u}_q$, we obtain from the solvability condition a dispersion equation for the complex frequency $\Omega = \Omega_q + i\gamma_q$, of the secondary MHD waves, where γ_q is the damping coefficient. In order of magnitude it is equal at $V_S^2 \ll V_A^2$ to the product of the square of the frequency of the secondary MHD waves by the time τ_s of relaxation of the plasmons of the slow magnetosonic waves,[11] a time longer than the relaxation times of the plasmons of the other types:

$$\gamma_q/\Omega_q \sim \Omega_q\tau_s. \quad (27)$$

The same estimate can be obtained in a different manner, namely, by using the formula

$$\gamma_q/\Omega_q \sim \overline{T^*\dot{S}^*}/\Omega_q\bar{\epsilon}_q, \quad (28)$$

where S^* is the entropy density of the plasmon gas and was obtained in [9,10]. According to [10] we have $T^*\dot{S}^* \sim \Omega_q\tau_s W|\vartheta|^2$; taking into account also the expression for $\bar{\epsilon}_q$,[20] we obtain $\gamma_q \sim \Omega_q\tau_s$. Using the results of [11], we obtain the damping coefficient, whose order of magnitude is

$$\frac{\gamma_q}{\Omega_q} \sim \left(\frac{W}{\rho V_A^2}\right)^{-1}\frac{\Omega_q}{\omega_i} \quad (29)$$

(ω_i is the ion cyclotron frequency). For weakly damped secondary MHD waves it is necessary to satisfy the inequality

$$W/\rho V_A^2 \gg \Omega_q/\omega_i \sim q_\parallel V_S/\omega_i. \quad (30)$$

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