Radiation from ultrarelativistic particles passing through perfect or mosaic crystals

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Analytic expressions are obtained for the integral number of x-ray quanta emitted by a charged particle as it passes through a perfect or a mosaic crystal. The question of the distinguishing features of the radiation in mosaic crystals is considered for the first time. It is shown that the integral characteristics of the radiation in perfect and in mosaic crystals differ little.

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1. INTRODUCTION

As shown by Garibyan *et al.*,[1,2] when a charged ultrarelativistic particle passes through a perfect crystal a new channel for radiation from the particle is opened, besides the transition radiation (see, e.g.,[3]). This channel is due to the periodicity of the crystal structure. A similar conclusion was reached by Baryshevskiĭ and Feranchuk.[4–6] 1)

A fast charged particle carries with it an electromagnetic field which is very close to the free radiation field. The particle field can be diffracted in the crystal at Bragg angles. This produces radiation in the frequency band corresponding to the x-ray frequencies, and this radiation, as shown in[1,2], is concentrated in narrow angle and frequency interval and has an appreciable intensity that can be readily observed in experiment. The extent to which the degree of perfection of the crystal affects this radiation, however, has not been determined to this day. The present study was undertaken precisely to investigate the specific features of this radiation process in mosaic crystals. In the course of the investigation of this question we succeeded also in obtaining simple analytic expressions for the integral intensity of this radiation in the case of a perfect crystal. This question is the subject of all of Sec. 3. Radiation from mosaic crystals is considered in Sec. 4. Our results show that the integral intensity of the radiation is not very sensitive to the degree of perfection of the crystal.

2. FIELD OF CHARGED ULTRARELATIVISTIC PARTICLE IN A CRYSTAL

We consider an ultrarelativistic particle having an energy $E \gg mc^2$, where m is the particle rest mass, and a charge e. The electromagnetic field of such a particle in vacuum is given by

$$\mathbf{E}_v(\mathbf{k},\omega) = 8i\pi^2 e\frac{\omega\mathbf{v}-\mathbf{k}c^2}{k^2c^2-\omega^2}\delta(\omega-\mathbf{kv}).$$

As seen from this expression, the distribution of the Fourier component of the electromagnetic-field vector $\mathbf{E}_v(\mathbf{k},\omega)$ has a sharp peak in the $\mathbf{k}$ direction, which is close to the direction of the particle velocity $\mathbf{v}$, and furthermore the field is practically transverse. In fact, for the transverse component we have

$$\mathbf{E}_v^{(n)}(\mathbf{k},\omega) = -iD\frac{\theta}{1-\beta^2+\theta^2}\delta(\omega-\mathbf{kv})\frac{\varkappa}{\varkappa}. \quad (1)$$

Here $D=8\pi^2 ev/\omega$, and the remaining symbols are standard (see, e.g.,[1]). In the case of an ultrarelativistic particle $1-\beta^2=(mc^2/E)^2 \ll 1$ the entire distribution of $\mathbf{E}_v^{(n)}(\mathbf{k},\omega)$ is concentrated in the narrow angle interval $\theta \sim (1-\beta^2)^{1/2}$.

If the particle lands in an amorphous medium, the polarization of the medium alters the particle field. We shall be interested henceforth in a field $\mathbf{E}_{cp}(\mathbf{k},\omega)$ at x-ray frequencies ω. In this case the polarization $\chi_0(\omega)$ of the medium is weak, i.e., $|\chi_0(\omega)| \ll 1$, and it can be readily shown that

$$\mathbf{E}_{med}^{(n)}(\mathbf{k},\omega) = -iD\frac{\theta}{1-\beta^2+\theta^2-\chi_0}\delta(\omega-\mathbf{kv})\frac{\varkappa}{\varkappa} \quad (2)$$

i.e., a slight redistribution of the Fourier components over the directions takes place.

We consider now the field of a charge moving in a crystal. The presence of translational symmetry in the crystal obviously leads to a stronger realignment of the particle field than in an amorphous medium. In a crystal, the susceptibility is a periodic function of the coordinates, i.e., $\chi^{il}(\mathbf{r}) = \chi^{il}(\mathbf{r}+\mathbf{a})$, where $\mathbf{a}$ is the translation vector, which we represent in the form of an expansion in the reciprocal-lattice vectors $\mathbf{K}_h$:

$$\chi^{il}(\mathbf{r},\omega) = \sum \chi^{il}(\mathbf{K}_h,\omega)\exp(i\mathbf{K}_h\mathbf{r}).$$

At x-ray frequencies the interaction of the electromagnetic field with the crystal atoms is predominantly of dipole type, so that the susceptibility tensor takes the form

$$\chi^{il}(\mathbf{K}_h,\omega) = \delta^{il}\chi(\mathbf{K}_h,\omega).$$

An explicit expression for the Fourier components of the susceptibility $\chi(\mathbf{K}_h,\omega)$ can be found, for example, in[7]. A charged particle produces in a crystal a field with Fourier components of $\mathbf{k}$ not only in the direction $\mathbf{k}_0$ close to the direction of its motion, but also in directions $\mathbf{k}_h$ that differ from $\mathbf{k}_0$ by the reciprocal-lattice vectors $\mathbf{K}_h$. In realistic situations, the only nonzero Fourier components, besides $\mathbf{E}_{cr}(\mathbf{k}_0,\omega)$, are those for

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which

$$k_h^2 \approx k_0^2. \tag{3}$$

We consider for simplicity the case when the condition (3), at specified ω and $\mathbf{k}$, is satisfied only for one vector K_1. Recognizing that in our case $|\chi(\mathbf{K}_h, \omega)| \ll 1$, and also that the particle field remains practically transverse in the crystal, i.e., $\mathbf{k} \cdot \mathbf{E}_{cr}(\mathbf{k}, \omega)$ is equal to zero with high accuracy, we can write for the transverse field components the following system of equations:

$$\begin{gathered}\left(\frac{c^2k_0^2}{\omega^2}-1-\chi_0\right)\mathbf{E}_{cr}(\mathbf{k}_0,\omega)-\chi_{01}\mathbf{E}_{cr}(\mathbf{k}_1,\omega)=iD\frac{\mathbf{v}}{v}\delta(\omega-\mathbf{kv}),\\ -\chi_{10}\mathbf{E}_{cr}(\mathbf{k}_0,\omega)+\left(\frac{c^2k_0^2}{\omega^2}-1-\chi_0+\alpha\right)\mathbf{E}_{cr}(\mathbf{k}_1,\omega)=0,\\ \mathbf{k}_p\mathbf{E}_{cr}(\mathbf{k}_p,\omega)=0;\ p=0,1,\end{gathered} \tag{4}$$

where

$$\alpha=(k_1^2-k_0^2)c^2/\omega^2. \tag{5}$$

This is a system of four equations, since each field $\mathbf{E}_{cr}(\mathbf{k}_0)$ and $\mathbf{E}_{cr}(\mathbf{k}_1)$ is specified by two amplitudes corresponding to different polarization. By choosing the π and δ polarization vectors (see [1,2] and [4-6] we can separate this system into two subsystems of two equations each. After simple transformations we can write the solution in the form

$$\mathbf{E}_{cr}(\mathbf{k}_p,\omega)=\sum_{s=\sigma}^{\pi}\mathbf{e}_p^{(s)}E_{cr}^{(s)}(p), \tag{6}$$

$$E_{cr}(0)=iP^{(s)}D\theta\frac{\alpha-\tilde{\chi}_0}{\Delta^{(s)}},\quad E_{cr}(1)=iP^{(s)}D\theta\frac{c_s\chi_{10}}{\Delta^{(s)}}, \tag{7}$$

where

$$\begin{gathered}\Delta^{(s)}=-\tilde{\chi}_0(\alpha-\tilde{\chi}_0)-c_s^2\chi_{01}\chi_{10},\\ P^{(s)}=\mathbf{e}_0^{(s)}\varkappa/\varkappa,\quad \tilde{\chi}_0=\chi_0-(1-\beta^2+\theta^2),\\ c_s=\begin{cases}1, & \text{for } \sigma \text{ polarization},\\ \cos 2\theta_B, & \text{for } \pi \text{ polarization}.\end{cases}\end{gathered} \tag{8}$$

The function $\delta(\omega-\mathbf{k}\cdot\mathbf{v})$ in the right-hand side of (4) relates the longitudinal projection of the vector $\mathbf{k}$ to the frequency. Taking this relation into account, we define the Fourier components of the field only in terms of the frequency ω and the transverse component of $\varkappa$. We have therefore left out of (7) the factor $\delta(\omega-\mathbf{k}\cdot\mathbf{v})$. Formulas (6)–(8) solve completely the problem of the field of an ultrarelativistic particle in a regular crystal.

3. RADIATION IN A CRYSTAL

It is easily seen that the field, defined by formulas (1) and (7), of a particle in vacuum and in a crystal, is not continuous on the interface. This means that on going through the boundary the particle radiates a free electromagnetic field, which will henceforth be designated E_{rad}. Whereas passage through the boundary between the vacuum and an amorphous medium produces a radiation field in a direction close to the direction of motion of the charge, in the case of a vacuum–crystal interface there is produced additional radiation in the Bragg-scattering direction (see [1,2,4-6]).

We consider for the sake of argument the case when the Bragg-reflected wave propagates into the interior of the crystal. This scattering geometry is known as the Laue case. From the condition that the total electromagnetic field be continuous we find that passage through the first boundary produces a radiation field both in the $\mathbf{k}_0$ and in the $\mathbf{k}_1$ direction, with respective amplitudes

$$E_{rad}^{I(s)}(0)=E_v^{(s)}(\mathbf{k}_0)-E_{cr}(\mathbf{k}_0),\quad E_{rad}^{I(s)}(1)=-E_{cr}^{(s)}(\mathbf{k}_1). \tag{9}$$

Relations (9) hold for each of the polarizations π and δ separately. We shall consider henceforth only one of the polarizations and omit the superscript s. Summation of the end result over the polarizations is trivial. The radiation fields $E_{rad}{}^{I}(\mathbf{k}_0)$ and $E_{rad}{}^{I}(\mathbf{k}_1)$ will propagate into the interior of the crystal and will experience multiple diffraction scattering and absorption. By virtue of these processes, the radiation field immediately ahead of the second boundary will be determined by the following formulas:

$$\begin{gathered}E_{rad}^{II}(0)=E_{rad}^{I}(0)A_L^{00}(l)+E_{rad}^{I}(1)A_L^{01}(l),\\ E_{rad}^{II}(1)=E_{rad}^{I}(0)A_L^{10}(l)+E_{rad}^{I}(1)A_L^{11}(l).\end{gathered} \tag{10}$$

The coefficients $A_L^{im}(l)$ have here the following meaning: $A_L^{im}(l)$ determines the amplitude of the field in the i direction on the exit surface if a field of unit amplitude is incident on the crystal in the m direction. The determination of these coefficients is the task of the theory of dynamic scattering of x rays, and in the case of the two-wave approximation their calculation entails no special difficulty. When the particle emerges from the crystal, an additional radiation is produced:

$$E_{rad}^{II}(0)=E_{cr}(0)-E_v(0),\quad E_{rad}^{II}(1)=E_{cr}(1). \tag{11}$$

The resultant radiation field past the crystal is obviously the sum of (10) and (11), i.e.,

$$\begin{gathered}E_{rad}(0)=(E_{cr}(0)-E_v(0))(1-A_L^{00}(l))-E_{cr}(1)A_L^{01}(l),\\ E_{rad}(1)=(E_v(0)-E_{cr}(0))A_L^{10}(l)+E_{cr}(1)(1-A_L^{11}(l)).\end{gathered} \tag{12}$$

Formulas (12) solve the problem of radiation by a particle passing through a crystal, provided that the coefficients $A_L^{im}(l)$ are known. In a number of cases, however, which are of greatest interest from the experimental viewpoint, the radiation field can be obtained also without knowing the actual form of these coefficients. In fact, let us consider a thick crystal, one that absorbs all the incident radiation. In the case of x-ray quanta, absorption takes place in practice in a thickness on the order of several dozen microns. The coefficients $A_L^{im}(l)$, in accord with their definition, should then vanish and the radiation fields are correspondingly determined by formulas (11).

Let us analyze now the structure of the radiation field for this case in greater detail. We are interested only in the field in the $\mathbf{k}_1$ direction, since it is easiest to investigate experimentally.

In accordance with (7) and (8) we have for the intensity of the radiation field past the crystal, with allowance for (11),

$$I_1(\theta, \omega) = \frac{D^2\theta^2}{(1-\beta^2+\theta^2+|\chi_0|)^2} \sum_{s=\sigma} \frac{|P^{(s)} c_s \chi_{10}|^2}{|\alpha - \alpha_s^{(0)} - i\rho_s|^2}, \tag{13}$$

where

$$\alpha_s^{(0)} = (\tilde{\chi}_0'^2 - c_s^2 \chi_{01}' \chi_{10}')/\tilde{\chi}_0', \tag{14}$$

$$\rho_s = [\tilde{\chi}_0'^2 \chi_0'' + c_s^2 \chi_{01}' \chi_{10}' \chi_0'' - -c_s^2 \tilde{\chi}_0' (\chi_{01}' \chi_{10}'' + \chi_{01}'' \chi_{10}')]/\tilde{\chi}_0'^2. \tag{15}$$

Here $\tilde{\chi}_0' = \chi_0' - (1 - \beta^2 + \theta^2)$, and $\chi_{\alpha\beta}'$ and $\chi_{\alpha\beta}''$ are the real and imaginary parts of the polarizability of the crystal. In the case of a monatomic crystal (see [7]),

$$\chi_0' = -4\pi r_0 c^2 F(0)/\omega^2 \Omega_0, \tag{16}$$

$$\chi_{01}' = \chi_{10}' = \chi_0' F(\mathbf{K}_h) \exp[-W(\mathbf{K}_h)]/F(0), \tag{17}$$

$\chi_0'' = \mu c/\omega$, and χ_{01}'' can be represented with high accuracy in the form

$$\chi_{01}'' = \chi_0'' \exp[-W(\mathbf{K}_h)]. \tag{18}$$

Here $\exp[-W(\mathbf{K}_h)]$ is the Debye-Waller factor, and the remaining symbols are standard.

We consider now in greater detail the quantity α in (13). In accord with its definition

$$\alpha = \frac{c^2}{\omega^2}(k_1^2 - k_0^2) = \frac{c^2}{\omega^2}(K_1^2 + 2\mathbf{K}_1\mathbf{k}_0). \tag{19}$$

We now define the frequency ω_0 in such a way that $\alpha - \alpha_s^{(0)} = 0$ at $\varkappa = 0$. It is easily seen that

$$\omega_0 = K_1 c/2 \sin\theta_B, \tag{20}$$

where $\pi/2 + \theta_B$ is the angle between the particle-motion direction and the vector $\mathbf{K}_1$. In the vicinity of $\omega = \omega_0$ and $\varkappa = 0$ we have for α the expansion

$$\alpha - \alpha_s^{(0)} = 4\sin\theta_B\left(-\frac{\Delta\omega}{\omega_0}\sin\theta_B + \theta\cos\Phi\cos\theta_B\right). \tag{21}$$

($\Phi = \sphericalangle(\varkappa, [\mathbf{k}_0 \times \mathbf{k}_1])$). We note now that expression (13), as a function of α, has an unusually sharp peak whose width is of the order of χ_0'', and we can replace the expression $|\alpha - \alpha_s^{(0)} - i\rho_s|^{-2}$ by

$$\frac{\pi}{\rho_s}\delta\left(4\sin\theta_B\left(-\frac{\Delta\omega}{\omega_0}\sin\theta_B + \theta\cos\Phi\cos\theta_B\right)\right)$$

We can thus represent (13) in the form

$$I_1(\theta, \omega) = \frac{D^2\theta^2}{(1-\beta^2+\theta^2-\chi_0)^2} \sum_{s=\sigma}^{\pi} \frac{\pi |P^{(s)} c_s \chi_{10}|^2}{\rho_s(\theta)} \times\delta\left(4\sin\theta_B\left(-\frac{\Delta\omega}{\omega_0}\sin\theta_B + \theta\cos\Phi\cos\theta_B\right)\right). \tag{22}$$

This formula describes completely the distribution of the radiation over the angles and frequencies near the direction corresponding to diffraction scattering along the vector $\mathbf{K}_1$.

For the integral number of quanta emitted by one particle on passing through the crystal, we readily obtain

$$N(\mathbf{k}_1) = \frac{e^2}{\hbar c} \frac{\chi_{01}'}{8\sin^2\theta_B} \frac{\chi_{10}'}{\chi_0''} \sum_s c_s^2 \left\{\frac{1}{2}\ln\frac{1}{x_s^2 + y_s^2} - \frac{x_s}{y_s}\left(\frac{\pi}{2} - \operatorname{arctg}\frac{x_s}{y_s}\right)\right\}. \tag{23}$$

where

$$x_s = 1 - \beta^2 + |\chi_0| - c_s^2 |\chi_{01}| \exp[-W(\mathbf{K}_1)],$$
$$y_s = c_s^2 |\chi_{01}| (1 - c_s^2 \exp[-2W(\mathbf{K}_1)])^{1/2}. \tag{24}$$

The quantity $N(\mathbf{k}_1)$ was determined earlier[2] by numerical integration. The results of calculations by formula (23) and the results of numerical integration agree with good accuracy.

It must be noted first that $N(\mathbf{k}_1)$ tends to infinity if the absorption of the medium is decreased (the coefficient χ_0''). This means that radiation is generated in the entire volume of the crystal, and not at the boundary, as might appear in the course of the derivation of this formula. We have thus in this case an analog of Cerenkov radiation. In an amorphous medium, in the frequency region where the polarizability is $\chi_0(\omega) < 0$, there is no Cerenkov radiation, since the conditions of simultaneous satisfaction of the energy and momentum conservation law, which are needed for the radiation process, are not satisfied. In a crystal, as is well known, the momentum should be conserved accurate to the reciprocal-lattice vector, and this uncovers a possibility of volume radiation in the frequency region with $\chi_0(\omega) < 0$.

It follows also from (23) that the dependence of $N(\mathbf{k}_1)$ on the energy of the incident particle is weak. An analogous procedure is used to solve the problem of radiation by a charged particle in a crystal in the case when the diffracted wave travels not into but out of the crystal (the so-called case of Bragg diffraction). We then have for the field of the radiation emerging from the crystal

$$E_{\mathrm{rad}}(1) = E_{\mathrm{cr}}(1 - A_B^{11}(l)) + (E_v(0) - E_{\mathrm{cr}}(0)) A_B^{10}(l). \tag{25}$$

For the radiation field in the direction of motion of the charged particle, on the other hand, we have

$$E_{\mathrm{rad}}(0) = (E_{\mathrm{cr}}(0) - E_v(0))(1 - A_B^{00}(l)) - E_{\mathrm{cr}}(1) A_B^{01}(l). \tag{26}$$

The coefficients $A_B^{im}(l)$ have a physical meaning similar to that in Laue-geometry scattering, but different values. $A_B^{im}(l)$ determines the amplitude of the field in the "i" direction on the exit surface under the condition when a field of unit amplitude is incident in the direction "m" on the entrance surface. Here the exit surface for the field in the direction 1 is the surface I, and the entrance surface is II, with the reverse for the direction 0. For a thick crystal, the coefficients $A_B^{ii}(\infty)$ vanish but the coefficients $A_B^{01}(\infty)$ and $A_B^{10}(\infty)$ are no longer equal to zero. The radiation fields are therefore determined by the formulas

$$E_{\mathrm{rad}}(1) = E_{\mathrm{cr}}(1) + (E_v(0) - E_{\mathrm{cr}}(0)) A_B^{10}(\infty),$$
$$E_{\mathrm{rad}}(0) = E_{\mathrm{cr}}(0) - E_v(0) - E_{\mathrm{cr}}(1) A_B^{01}(\infty). \tag{27}$$

Just as in the Laue case, let us analyze the radiation field in the $\mathbf{k}_1$ direction. For $A_B^{10}(\infty)$, neglecting the small imaginary parts $\chi''_{\alpha\beta}$, we can obtain

$$A_B^{10}(\infty)=bc_s\chi_{10}/(2\varepsilon-\tilde{\chi}_0). \tag{28}$$

Here

$$2\varepsilon-\tilde{\chi}_0={}^1/_2[x-i(4bc_s^2\chi_{01}'\chi_{10}'-x^2)^{1/2}] \quad \text{if } |x|<2(bc_s^2\chi_{01}'\chi_{10}')^{1/2}, \tag{29a}$$

$$2\varepsilon-\tilde{\chi}_0={}^1/_2[x+x|x|^{-1}(x^2-4bc_s^2\chi_{01}'\chi_{10}')^{1/2}], \quad \text{if } |x|>2(bc_s^2\chi_{01}'\chi_{10}')^{1/2}; \tag{29b}$$

$$x=b\alpha-\tilde{\chi}_0'(1+b),\quad b=|\gamma_0/\gamma_1|,\quad \gamma_p=\cos(\mathbf{n},\mathbf{k}_p),$$

$\mathbf{n}$ is a unit vector normal to the crystal surface. Using formulas (6)–(8), we obtain for the field of the charge in the crystal in the $\mathbf{k}_0$ direction

$$E_{\text{cr}}^{(s)}(0)=E_{\text{med}}^{(s)}(0)+\Delta E_{\text{med}}^{(s)}(0), \tag{30}$$

where

$$E_{\text{med}}^{(s)}=-iP^{(s)}D\theta/\tilde{\chi}_0,$$
$$\Delta E_{\text{med}}^{(s)}(0)=-iP^{(s)}D\theta c_s^2\chi_{01}\chi_{10}/\tilde{\chi}_0\Delta^{(s)}. \tag{31}$$

Substituting (30) and (28) in Eq. (27) for the radiation field intensity in the direction 1, with allowance for the fact that here, too, we have a function $|\alpha-\alpha_s^{(0)}-i\rho_s|^{-2}$ with a sharp peak as a function of α, we get

$$I_1(\theta,\omega)=D^2\theta^2\frac{|P^{(s)}c_s\chi_{10}|^2}{|2\varepsilon-\tilde{\chi}_0|^2}\left\{\frac{1}{1-\beta^2+\theta^2}-\frac{1}{1-\beta^2+\theta^2-\chi_0}\right\}^2 + \frac{D^2\theta^2}{(1-\beta^2+\theta^2-\chi_0)^2}\left(1-\frac{bc_s^2\chi_{01}\chi_{10}}{\tilde{\chi}_0^2}\right)^2 \times\eta(\tilde{\chi}_0^2-6c_s^2\chi_{01}\chi_{10})\frac{\pi|P^{(s)}c_s\chi_{10}|^2}{\rho_s(\theta)}\delta(\alpha-\alpha_s^{(0)}), \tag{32}$$

where $\eta(x)=1$ at $x\geqslant 0$ and $\eta(x)=0$ at $x\leqslant 0$.

The first term in (32) is that part of the transition radiation which is reflected in the direction 1. It is easily seen that it is smaller than the second term by a factor χ_0'' and can be neglected. The interference term is proportional to $\alpha-\alpha_s^{(0)}$, and owing to the presence of $\delta(\alpha-\alpha_s^{(0)})$ it makes a zero contribution after integration; it was therefore left out. We have thus for the intensity of the radiation field, with allowance for the two polarizations,

$$I_1(\theta,\omega)=\frac{D^2\theta^2}{\tilde{\chi}_0^2}\sum_{s=\pi}\frac{\pi|P^{(s)}c_s\chi_{10}|^2}{\rho_s} \times\left(1-\frac{bc_s^2\chi_{01}\chi_{10}}{\tilde{\chi}_0^2}\right)^2\eta(\tilde{\chi}_0^2-bc_s^2\chi_{01}\chi_{10})\delta(\alpha-\alpha_s^{(0)}). \tag{33}$$

For the integral number of quanta emitted by one particle as it passes through the crystal we have

$$N(\mathbf{k}_1)=N_0(\mathbf{k}_1)\frac{1}{2}\sum_s c_s^2\int_0\left(1-\frac{bc_s^2\chi_{01}\chi_{10}}{(x+z_0)^2}\right)^2 \times\eta(\tilde{\chi}_0^2-bc_s^2\chi_{01}\chi_{10})\frac{x\,dx}{(x+x_s)^2+y_s^2}, \tag{34}$$

where

$$N_0(\mathbf{k}_1)=\frac{e^2}{\hbar c}\frac{\chi_{01}}{4\sin^2\theta_B}\frac{\chi_{10}}{\chi_0''},$$
$$z_0=1-\beta^2+|\chi_0|. \tag{35}$$

The exact expression for this integral is quite complicated. We present therefore approximate expressions for a number of cases. We assume first that $b\lesssim 1$. Then, if $z_0\gg x_s$ and $z_0\gg y_s$ we have with logarithmic accuracy

$$N_B(\mathbf{k}_1)=N_0(\mathbf{k}_1)\frac{1}{2}\sum_s c_s^2\left\{\frac{1}{2}\ln\frac{1}{x_s^2+y_s^2} -\frac{x_s}{y_s}\left(\frac{\pi}{2}-\operatorname{arctg}\frac{x_s}{y_s}\right)-\frac{2bc_s^2\chi_{01}\chi_{10}}{z_0^2} \times\left[\frac{1}{2}\ln\frac{z_0^2\xi_1}{x_s^2+y_s^2}-\frac{x_s}{y_s}\left(\frac{\pi}{2}-\operatorname{arctg}\frac{x_s}{y_s}\right)\right]+\frac{(bc_s^2\chi_{01}\chi_{10})^2}{z_0^4} \times\left[\frac{1}{2}\ln\frac{z_0^2\xi_2}{x_s^2+y_s^2}-\frac{x_s}{y_s}\left(\frac{\pi}{2}-\operatorname{arctg}\frac{x_s}{y_s}\right)\right]\right\}, \tag{36}$$

where ξ_1 and ξ_2 are certain numerical constants of the order of unity.

We consider now the case $b\gg 1$. The lower limit in the integral (34) is no longer zero, but $x_{\min}=b^{1/2}c_s|\chi_{01}|-z_0$. Assuming that $x_{\min}\gg z_0$, x_s, y_s we can put $x_s=0$, $y_s=0$, $z_0=0$ in the integral of (34). As a result of these simplifications we get

$$N_B(\mathbf{k}_1)=N_0(\mathbf{k}_1)\frac{1}{2}\sum_s c_s^2\ln\frac{1}{x_{min}^2}. \tag{37}$$

Thus, the formulas (34)–(37) obtained by us solve completely the problem of radiation from a thick crystal in the case of Bragg geometry. As seen from these formulas, the radiation does not differ greatly here from the case of Laue geometry.

4. RADIATION FROM MOSAIC CRYSTALS

Before we proceed to the case of a mosaic crystal consisting of small blocks of characteristic thickness $d_0\ll\gamma_1 c/\omega|\chi_{10}|$, we consider the radiation from a thin crystal. To be specific, we shall deal with Laue geometry. The radiation field past the crystal is determined by formulas (12). We write out the coefficients $A_L^{im}(l)$ in explicit form for a crystal of arbitrary thickness:

$$A_L^{10}(l)=\frac{bc_s\chi_{10}}{2\varepsilon_2^{(s)}-2\varepsilon_1^{(s)}}\left[\exp(iz_2^{(s)})-\exp(iz_1^{(s)})\right],$$
$$A_L^{11}(l)=\frac{2\varepsilon_2^{(s)}-\tilde{\chi}_0}{2\varepsilon_2^{(s)}-2\varepsilon_1^{(s)}}\exp(iz_2^{(s)})-\frac{2\varepsilon_1^{(s)}-\tilde{\chi}_0}{2\varepsilon_2^{(s)}-2\varepsilon_1^{(s)}}\exp(iz_1^{(s)}), \tag{38}$$
$$A_L^{01}(l)=A_L^{10}(l)\chi_{01}/b\chi_{10},\quad A_L^{00}(l)=A_L^{11}(-l)\exp(iz_1^{(s)}+iz_2^{(s)}).$$

Here $z_i^{(s)}=\epsilon_i^{(s)}\omega l/c\gamma_0$ and

$$2\varepsilon_{1,2}^{(s)}={}^1/_2\{(1+b)\tilde{\chi}_0-b\alpha\pm[((1+b)\tilde{\chi}_0-b\alpha)^2-4b\Delta^{(s)}]^{1/2}\}, \tag{39}$$

where $\Delta^{(s)}$ is given by (14).

Substituting (38) and (7) in formulas (12) we obtain after simple transformations

$$E_{\text{rad}}(1)=E_v(0)A_L^{10}(l)+\Delta E, \tag{40}$$
$$E_{\text{rad}}(0)=(E_{\text{med}}(0)-E_v(0))(1-A_L^{00}(l))-\frac{c_s\chi_{01}}{\tilde{\chi}_0}\Delta E, \tag{41}$$

where

$$\Delta E = iP^{(s)} D\theta \frac{c_s\chi_{10}}{\Delta^{(s)}} \cdot \left(1 - \frac{\varepsilon_2^{(s)}\exp(iz_1^{(s)}) - \varepsilon_1^{(s)}\exp(iz_2^{(s)})}{\varepsilon_2^{(s)} - \varepsilon_1^{(s)}}\right). \quad (42)$$

It is easily seen that for a thin crystal of thickness $l \ll \gamma_1 c/\omega|\chi_{10}|$ we can neglect the term ΔE in (39). In fact, assuming $z_1 \ll 1$ and $z_2 \ll 1$ we get (accurate to terms cubic in l)

$$\Delta E = -{}^1/_2 iP^{(s)} D\theta c_s \chi_{10} (\omega l/2\gamma_0 c)^2. \quad (43)$$

It is also important here that the strong dependence of $E_{rad}(1)$ on α, due to the denominator $\Delta^{(s)}$, disappears in the case of a thin crystal.

Thus, the radiation from a thin crystal in the direction 1 is determined entirely by the first term of (40), which is linear in l. We can then use for the coefficient $A_L^{10}(l)$ the approximate expression

$$A_L^{10}(l) \approx \frac{1-\exp(-i\omega l\alpha/2\gamma_1 c)}{\alpha} c_s \chi_{10}. \quad (44)$$

We can similarly neglect also the second term of (41). The first term then describes radiation that does not depend on the crystal structure and corresponds to the ordinary transition radiation.

We proceed now to the mosaic crystal. The blocks located in the immediate vicinity of the surface produce radiation described by formulas (40)–(42). In the interior of the crystal, however, the situation changes somewhat. In fact, the average field of the particle is now described not by $E_v(0)$ but by $E_{med}(0)$. Consequently, to determine the radiation from a block located far from the surface we must replace $E_v(0)$ in (40) and (41) by $E_{med}(0)$. Then $E_{rad}(0)$ vanishes (accurate to the second term), a physically obvious fact. Indeed, the transition radiation vanishes in the interior of the medium. The radiation field in direction 1 is determined in accordance with (40), (44), and (30) by the expression

$$E_{rad}(1) = -iP^{(s)} D\theta \frac{c_s\chi_{10}}{\tilde{\chi}_0} \frac{1-\exp(-i\omega l\alpha_i/2\gamma_1 c)}{\alpha_i}. \quad (45)$$

Here α_i determine the deviation from the exact satisfaction of the Bragg condition on the i-th block, and this quantity changes from block to block. For the radiation-field intensity integrated over the frequencies $\Delta\omega$ we readily get

$$J(\theta) = J_0(\theta)|c_s\chi_{10}|^2 \int \frac{4\sin^2(\omega l\alpha_i/4\gamma_1 c)}{\alpha_i^2} d\frac{\Delta\omega}{\omega_0} = J_0(\theta) r l, \quad (46)$$

where

$$J_0(\theta) = 2\pi \frac{e^2\omega_0}{c} \frac{\theta^2}{(1-\beta^2+\theta^2-\chi_0)^2}$$
$$r = \frac{\pi\omega_0|c_s\chi_{10}|^2}{4\gamma_1 c \sin^2\theta_B}.$$

As seen from (46), the intensity of the radiation field in the direction 1 does not depend on the concrete orientation of the block (under the natural condition that the degree of mosaic disorientation in the crystal is not large) and is proportional to the block thickness. We can therefore introduce the radiation intensity per unit length, given by

$$dJ(\theta)/dl = J_0(\theta) r. \quad (47)$$

Thus, a charged particle traveling through a mosaic crystal generates continuously radiation in the direction 1. This radiation will, on the one hand, be absorbed in the crystal, and on the other hand it will be diffraction-scattered in the direction 0. All these processes can be described by a system of equations that describe diffraction in thick crystals (see, e.g., [8]):

$$\frac{dI_0(\theta,z)}{dz} = -\left(\frac{\mu}{\gamma_0} + \frac{r}{\delta}\right) I_0(\theta,z) + \frac{r}{\delta b^2} I_1(\theta,z),$$
$$\frac{dI_1(\theta,z)}{dz} = -\left(\frac{\mu}{\gamma_1} + \frac{r}{\delta b^2}\right) I_1(\theta,z) + \frac{r}{\delta} I_0(\theta,z) + rJ_0(\theta).$$

Here μ is the linear absorption coefficient [see (17)], δ is a parameter that characterizes the "mosaicity" of the crystal, i.e., the average disorientation of the blocks, expressed in units of the width of the Bragg "flat." In our case we assume that $\delta \gg 1$.

We write down the solution of this system directly for the case of a thick crystal:

$$I_1(\theta) = \frac{\mu + r\gamma_0/\delta}{\mu + r(\gamma_0+\gamma_1/b^2)/\delta} \frac{r\gamma_1 J_0(0)}{\mu}. \quad (48)$$

Since usually $\mu \gg r$, we get from (48)

$$I_1(\theta) = \frac{r\gamma_1}{\mu} J_0(\theta) = \frac{\pi^2 e^2\omega_0}{2c\sin^2\theta_B} \chi_{10} \frac{\chi_{01}}{\chi_0''} \frac{c_s^2\theta^2}{(1-\beta^2+\theta^2-\chi_0)^2}.$$

For the number of quanta per incident particle, integrated over all angles θ, we readily obtain

$$N_m(\mathbf{k}_1) = \frac{1}{2} N_0(\mathbf{k}_1) \sum_{s=\sigma,\pi} c_s^2 \left\{ \ln \frac{1}{1-\beta^2+|\chi_0|} - 1 \right\}. \quad (49)$$

Precisely the same results are obtained in the case of radiation from a mosaic crystal and Bragg scattering geometry (although in the case of perfect crystals, as shown in Sec. 3, there is some quantitative difference).

We compare now expression (49) with the expression (23) that describes radiation from a perfect crystal. It is easily seen that the radiations from perfect and mosaic crystals do not differ greatly. The only difference is in the factor under the logarithm sign, and is small by virtue of this fact. For particles with not too high an energy, $1-\beta^2 > |\chi_0|$, these expressions practically coincide. For ultrarelativistic particles, $1-\beta^2 \ll |\chi_0|$, a slight difference is observed in the character of the radiation, but even in the case of electrons with energy on the order of 1 GeV this difference can be neglected. Thus, our analysis shows that from the point of view of radiation intensity a perfect crystal offers, in fact, no advantages over a mosaic crystal.

1) Unfortunately, errors have crept into [4,6]. A correction is contained in [6].

1. G. M. Garibyan and Yan Shi, Zh. Eksp. Teor. Fiz. **61**, 930 (1971); **63**, 1198 (1972) [Sov. Phys. JETP **34**, 495 (1972); **36**,