Stimulated Mandel'shtam-Brillouin scattering in an expanding plasma

I. M. Gorbunov and A. N. Polyanichev

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted 20 July 1977)
Zh. Eksp. Teor. Fiz. 74, 552–562 (February 1978)

The decrease in the intensity of an electromagnetic wave as a result of SMBS in an expanding inhomogeneous laser plasma is investigated. It is shown that the SMBS depends strongly on the gasdynamics of the plasma corona and decreases with decreasing rate of plasma expansion and with decreasing inhomogeneity scale.

PACS numbers: 52.25.Ps, 52.35.Hr

INTRODUCTION

The penetration of light into dense plasma layers, where it is absorbed, can be hindered by scattering in the rarefied plasma corona. One of the important nonlinear processes of scattering of high-power laser radiation is stimulated Mandel'shtam-Brillouin scattering (SMBS) (see e.g.,[1-4])

We investigate here the decrease of the intensity (attenuation) of a pump wave by SMBS in an expanding inhomogeneous laser plasma. It is shown that the SMBS depends strongly on the gasdynamics of the plasma corona and decreases with increasing plasma expansion rate. The incident-flux energy rate below which the attenuation due to SMBS can be neglected is determined.

It is shown in Sec. 1 that in the case of SMBS in a moving plasma the frequencies of the scattered waves can be not only lower (Stokes scattering) but also higher (anti-Stokes scattering) than the frequency of the incident light. In Sec. 2 we use the dispersion equation to determine the gains of the scattered waves for supersonic motion of the inhomogeneous plasma. In Sec. 3 we obtain an equation that determines the change of the pump-wave intensity with changing coordinate. This equation is solved numerically and treated analytically in Sec. 4. The main conclusions and consequences of the calculation are formulated in the conclusion.

1. INITIAL PROBLEM

It will be shown in this section that in the case of SMBS in a moving plasma the frequency of the scattered radiation can be both lower (Stokes scattering) and higher (anti-Stokes scattering) of the frequency of the incident radiation.

The dispersion equation for the spectrum of the coupled density perturbations and electromagnetic field in a moving plasma through which a linearly polarized monochromatic pump wave passes can be obtained from the equations of plasma hydrodynamics in a high-frequency field.[5] It takes the form

$$(\omega-\mathbf{ku})^2-k^2s^2=\frac{v_E^2k^2\omega_{Li}^2}{4}\left\{\frac{\sin^2\theta_-}{(\omega-\omega_0)^2-\omega_p^2-c^2(\mathbf{k}-\mathbf{k}_0)^2}+\frac{\sin^2\theta_+}{(\omega+\omega_0)^2-\omega_p^2-c^2(\mathbf{k}+\mathbf{k}_0)^2}\right\},\qquad(1.1)$$

where $\mathbf{u}$ is the plasma velocity, $s=(zT_e/m_i)^{1/2}$ is the speed of sound, T_e is the electron temperature, m_i and z are the mass and charge of the plasma ions, $\mathbf{E}_0$, ω_0, and $\mathbf{k}_0$ are respectively the amplitude, frequency, and wave vector of the pump wave, $v_E=eE_0/m\omega_0$, ω_{Li} is the Langmuir frequency of the ions, $\sin^2\theta_\pm=[\mathbf{n}_0(\mathbf{k}\pm\mathbf{k}_0)]^2/(\mathbf{k}\pm k_0)^2$, $\mathbf{n}_0=\mathbf{E}_0/E_0$ is the polarization vector, $\omega_p=(4\pi e^2N/m)^{1/2}$ is the plasma frequency, and N is the electron density. In Eq. (1.1) we have neglected dissipative effects and omitted terms that determine the perturbations of the longitudinal field.

Decay instabilities correspond to the approximation with weak parametric coupling of the waves, when Eq. (1) can be solved by perturbation theory in the pump-wave amplitude.[5] In the zeroth approximation ($\omega=\omega^{(0)}$), putting $v_E=0$ in (1.1), we obtain the dispersion law for the acoustic waves in the moving plasma:

$$\omega^{(0)}-\mathbf{ku}=\pm ks.\qquad(1.2)$$

The pump wave is most effectively coupled with acoustic waves for which the denominator of one of the terms in the right-hand side of (1.1) is small. We shall therefore assume in the zeroth approximation that the following relations are satisfied:

$$(\omega^{(0)}\pm\omega_0)^2=\omega_p^2+c^2(\mathbf{k}\pm\mathbf{k}_0)^2,\qquad(1.3)$$

these being the dispersion laws for the scattered waves with frequencies $\omega'=\omega_0\pm\omega^{(0)}$ and wave vectors $\mathbf{k}'=\mathbf{k}_0\pm\mathbf{k}$. When the dispersion law $\omega_0^2=\omega_p^2+k_0^2c^2$ of the pump wave is taken into account and quantities of order $\omega^{(0)}/\omega_0$ are neglected, relations (1.3) take the form $\mathbf{k}^2\pm2\mathbf{k}\cdot\mathbf{k}_0=0$.

In first-order approximation ($\omega=\omega^{(0)}+\omega^{(1)}$), if relation (1.2) and one of the equalities in (1.3) are satisfied, we get from (1.1)

$$(\omega^{(1)})^2=\pm\frac{v_E^2k^2\omega_{Li}^2\sin\theta_\pm}{16\omega_0(\omega^{(0)}-\mathbf{ku})},\qquad(1.4)$$

where the plus and minus signs correspond to the signs in (1.3). The initial perturbations increase with time if $(\omega^{(1)})^2$ is negative. Let us see when this is possible.

Let the dispersion law for the Stokes scatteredwave be satisfied ($\omega'=\omega_0-\omega^{(0)}, k^2=2\mathbf{k}\cdot\mathbf{k}_0$). Then $(\omega^{(1)})^2<0$ if the plus sign is used in (1.2) and $\omega^{(0)}=ks+\mathbf{k}\cdot\mathbf{u}$.

 ©1978 American Institute of Physics

If the dispersion law for the anti-Stokes scattered wave ($\omega' = \omega_0 + \omega^{(0)}, k^2 = 2\mathbf{k}\cdot\mathbf{k}_0$ is satisfied, then increasing initial perturbations correspond to a minus sign in formula (1.2) and $\omega^{(0)} = -ks + \mathbf{k}\cdot\mathbf{u}$. From the condition $\omega^{(0)} > 0$ it follows then that the anti-Stokes scattering is possible only in the case of supersonic motion of the plasma ($u > s$).

The conclusion that the plasma motion gives rise to anti-Stokes scattering can be explained with simple physical arguments. Let the plasma move counter to the pump wave. For an observer in the coordinate frame of the plasma, the only growing waves are Stokes scattered waves of frequency $\Omega' = \Omega_0 - \Omega$, where Ω_0 is the frequency of the wave incident on the plasma and Ω is the frequency of the sound wave. We consider for simplicity backward scattering, when $\Omega = 2k_0 s$. In the laboratory frame, the frequencies of the incident and scattered waves are respectively $\omega_0 = \Omega_0 - k_0 u$ and $\omega' = \Omega' + k_0 u$. Substituting these relations in the expression for Ω', we obtain $\omega' = \omega_0 + 2k_0(u - s)$. We see therefore that at $u > s$ the frequency of the scattered radiation in the laboratory frame is larger than the frequency of the incident radiation.

The same conclusions can be obtained by regarding the wave that determines the anti-Stokes scattering as a wave with negative energy.[6,7]

2. THE AMPLIFICATION COEFFICIENTS

In an inhomogeneous laser plasma, one of the main obstacles to the growth of the initial perturbations and to the stabilization of the SMBS is the violation of the decay conditions for the wave vectors.[8,9] The growing waves interact therefore with the pump wave in abounded region of space, and it is there that they are amplified (drift instability). If the plasma properties vary little in the wave resonant-interaction region, then the gain can be obtained from the dispersion equation. As is customary in nonlinear optics,[10] the dispersion equation must be solved for $\mathbf{k}$ (boundary-value problem), and not with respect to ω as in Sec. 1.

The first and second terms in the right-hand side of (1.1) determine the Stokes and anti-Stokes scattering, respectively. When the signs of the frequency and of the wave vector k are simultaneously reversed, the second term goes over into the first. We can therefore seek for the dispersion equation a single solution that is valid for both Stokes and anti-Stokes scattering, if it is assumed that the latter corresponds to negative ω.

We use again perturbation theory and obtain from (1.1) in the zeroth approximation ($\mathbf{k} = \mathbf{k}^{(0)}$)

$$\omega - \mathbf{k}^{(0)}\mathbf{u} = k^{(0)} s, \quad k^{(0)2} = 2\mathbf{k}^{(0)}\mathbf{k}_0, \tag{2.1}$$

where $\mathbf{u}$ and $\mathbf{k}_0$ are slow functions of the coordinates.

Consider a plasma that is inhomogeneous only in the x direction, which coincides with the expansion direction ($\mathbf{u} = \{u(x), u, 0\}$), and a pump wave propagating counter to the plasma motion ($\mathbf{k}_0 = \{-k_0(x), 0, 0\}$). To simplify the calculations we assume a constant plasma temperature (the temperature changes little because of the high electronic thermal conductivity in the laser-plasma corona). Then relation (2.1) takes the form

$$\omega - k_\parallel^{(0)} u = s(k_\perp^2 + k_\parallel^{(0)2})^{1/2}, \quad k_\perp^2 + k_\parallel^{(0)2} = -2k_\parallel^{(0)} k_0, \tag{2.2}$$

where we have omitted the superscript (0) of $k_\perp$.

In an inhomogeneous plasma relations (2.2) determine the longitudinal component of the wave vector of the sound wave $k_\parallel^{(0)}$ and, implicitly, the point x_0 at which a sound wave of frequency ω and transverse wave-vector projection $\mathbf{k}_\perp$ interacts resonantly with the pump wave. It follows from (2.2), in particular, that

$$k_\parallel^{(0)} = -k_0(1 \pm \eta), \quad k = k_0[2(1 \pm \eta)]^{1/2}, \tag{2.3}$$

where $\eta = (1 - k_\perp^2/k_0^2(x))^{1/2}$. The plus and minus signs in (2.3) correspond to two different sound wave with the same value of $\mathbf{k}_\perp$, from which the scattering is either forward or backward.

We consider now the solution of the dispersion equation in first-order approximation. Owing to the plasma inhomogeneity and the interaction with the pump wave, the longitudinal component of the wave vector of the resonant sound wave changes. A correction should be found only for that component ($k_\parallel = k_\parallel^{(0)} + k_1$). Substituting in (1.1) relations (2.2) and expanding all the coordinate-dependent quantities in series about the point x_0, we obtain the following equation for the determination of k_1:

$$\left[k_1^2(u^2 - s^2) - 2k_1 s(k_\parallel s + ku) - 2\Delta x \frac{uk_\parallel ks}{L_u}\right] \times \left[k_1^2 + 2k_1(k_0 + k_\parallel) - \frac{\Delta x k_\parallel \omega_p^2}{c^2 k_0 L_N}\right] = -\frac{v_E^2 k^2 \omega_{Li}^2}{4c^2}\sin^2\theta, \tag{2.4}$$

where

$$k_\parallel \equiv k_\parallel^{(0)}, \quad k = (k_\perp^2 + k_\parallel^2)^{1/2}, \quad L_u = u(du/dx)^{-1}, \quad L_N = -N(dN/dx)^{-1},$$
$$\Delta x = x - x_0, \quad \sin^2\theta = [\mathbf{n}_0(\mathbf{k} - \mathbf{k}_0)]^2/(\mathbf{k} - \mathbf{k}_0)^2.$$

It should be noted that the terms proportional to k_1^2 in the square brackets of (2.4) must be taken into account if the coefficients of the corresponding terms linear in k_1 are small. These coefficients are determined by the x-projections of the group velocities of the sound and scattered waves. In fact,

$$v_s = \frac{\partial\omega}{\partial k_\parallel} = \frac{k_\parallel}{k}s + u, \quad v_c = -\frac{\partial(\omega_0 - \omega)}{\partial(k_0 + k_\parallel)} = \frac{c^2(k_\parallel + k_0)}{\omega - \omega_0}. \tag{2.5}$$

If any of the quantities in (2.5) vanishes, then the corresponding wave has a turning point in the interaction region. In this case the geometric-optics approximation is generally speaking not valid. However, as was illustrated with stimulated Raman scattering as an example, the difference between the results of the exact calculation[11] and the calculation in the geometric-optics approximation[12] reduces to a numerical coefficient of the order of unity. This justifies the use of Eq. (2.4) in those cases when the terms linear in k_1 in the square brackets are small and it is necessary to take into account terms proportional to k_1^2.

Hydrodynamic calculations show that at a sufficiently high intensity of the pump wave the laser-plasma corona expands at supersonic velocity. We confine ourselves to just this case. Then, according to (2.5), $v_s \neq 0$ and there are no turning points or the sound waves in the interaction region.

If the point x_0 is close to a turning point for the scattered wave, so that the inequality

$$|k_1| > 2|k_\parallel + k_0| \tag{2.6}$$

is satisfied, the Eq. (2.4) takes the form

$$\left[k_1^2 + \frac{\Delta x \omega_p^2}{L_N c^2}\right]\left[k_1(M2^{1/2}-1) - \frac{\Delta x}{L_u} 2^{1/2} k_0 M\right] = \frac{v_E^2 k_0 \omega_{Li}^2 \sin^2\theta}{4c^2 s^2} \tag{2.7}$$

where $M = u/s$ is the Mach number. To find the gain, the region of variation of Δx was broken up into two parts. The first term of the second square bracket of (2.7) was assumed small in the first part, and the second term was assumed small in the other part. The solution of Eq. (2.7) in these two regions was used to determine the local gain $\mathrm{Im} k_1(\Delta x)$, and integration over Δx yielded the total gain

$$\varkappa_i = \frac{\pi k_0 L_N v_E^2 \sin^2\theta}{12 v_{Te}^2 (2^{1/2}M-1)}\left[1 + \frac{\pi M^{1/2}(v_E|\sin\theta|/v_{Te})^{1/2}}{6^{3/4} 2^{1/2}(L_u/L_N)^{3/4}(\omega_p/k_0 c)(2^{1/2}M-1)}\right]^{-1}. \tag{2.8}$$

Here $v_{Te}^2 = T_e/m$.

If an inequality inverse to (2.6) is satisfied, neither of the growing waves has turning points in the interaction region. Equation (2.4) is then quadratic and it follows from its solution that only the backscattered wave, corresponding in (2.3) to $k_\parallel = -k_0(1+\eta)$, is amplified. The amplification is in a region of half-width Δx_0 about the point x_0, where

$$\Delta x_0^2 = \frac{v_E^2 \omega_p^2 \sin^2\theta\,\eta}{2k_0^2 c^2 v_{Te}^2 (1+\eta)^{1/2}}[2^{1/2}M - (1+\eta)^{1/2}] \times\left[\frac{2^{1/2}M\eta}{L_u} + \frac{\omega_p^2}{2k_0^2 c^2 L_N}[2^{1/2}M - (1+\eta)^{1/2}]\right]^{-2}. \tag{2.9}$$

The total gain is in this case

$$\varkappa_0 = \int_{-\Delta x_0}^{\Delta x_0} d\Delta x\, \mathrm{Im}\, k_1(\Delta x) = \frac{\pi k_0 L_N v_E^2 \sin^2\theta}{4 v_{Te}^2 (1+\eta)^{1/2}} \times\left|2^{1/2}M - (1+\eta)^{1/2} + 2^{1/2}M\frac{L_N k_0^2 c^2}{L_u \omega_p^2}\eta\right|^{-1}. \tag{2.10}$$

Expression (2.10) coincides with the gain obtained in[1] by solving the system of equations for the amplitudes of coupled waves (see also[3]). According to the inequality inverse to (2.6), formula (2.10) is valid if

$$1 > \eta^{3/2} > \eta_0^{3/2} = \frac{v_E \omega_p |\sin\theta|}{2^{5/2} v_{Te} k_0 c (2^{1/2}M-1)^{1/2}}. \tag{2.11}$$

We note that in our analysis the only waves that are amplified are those with like signs of the projections of the group velocities on the x axis (co-moving waves). In,[1,2] where the solution of the equations for the amplitudes of the unstable waves with sources was investigated, it was shown that a similar gain is obtained also for waves with oppositely directed group-velocity projections (opposing waves).

3. EQUATION FOR PUMP WAVE

An equation for the dependence of the pump-wave energy flux density q_0 on the coordinate can be derived from the law of conservation of the total energy flux of the interacting waves:

$$\frac{dq_0}{dx} = -\frac{d}{dx}\int_{-\infty}^{\infty} d\omega \int d\mathbf{k}(q_{\omega,\mathbf{k}} + q_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}), \tag{3.1}$$

where $q_{\omega,\mathbf{k}}$ and $q_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}$ are the longitudinal components of the energy fluxes of the waves into which the pump wave breaks up. In the stationary state these fluxes statisfy the following relations:

$$\frac{dq_{\omega,\mathbf{k}}}{dx} = \frac{\partial W_{\omega,\mathbf{k}}}{\partial t}, \quad \frac{dq_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}}{dx} = \frac{\partial W_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}}{\partial t},$$

where $W_{\omega,\mathbf{k}}$ is the density of the energy released by the pump wave in the form of waves of frequencies ω and wave vectors $\mathbf{k}$. Since the number of produced quanta is equal to the number of decaying quanta ($N_{\omega,\mathbf{k}} = N_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0} = N_{\omega_0,\mathbf{k}_0}$), it follows that $W_{\omega,\mathbf{k}}/W_{\omega,\omega_0,\mathbf{k}-\mathbf{k}_0} = \omega/(\omega_0-\omega)$ and the energy fluxes of the produced waves are connected by the relation

$$\frac{1}{\omega}\frac{dq_{\omega,\mathbf{k}}}{dx} = \frac{1}{\omega_0-\omega}\frac{dq_{\omega-\omega_0,\mathbf{k}-\mathbf{k}_0}}{dx},$$

which is the consequence of the well-known Manley-Rowe[13] relations. Using this relation and recognizing that the longitudinal component k of the wave vector is expressed from the dispersion equation in terms of ω and $\mathbf{k}_\perp$ (see (2.3)), we rewrite (3.1) in the form

$$\frac{dq_0}{dx} = -\omega_0\int_{-\infty}^{\infty} d\omega \int \frac{d\mathbf{k}_\perp}{\omega}\frac{dq_{\omega,\mathbf{k}_\perp}}{dx},$$

where the spectral density of the energy flux $q_{\omega,\mathbf{k}_\perp}$ is connected with the spectral density of the energy $U_{\omega,\mathbf{k}_\perp}$. For longitudinal waves, this connection is given by

$$q_{\omega,\mathbf{k}_\perp} = \frac{\partial\omega}{\partial k_\parallel}U_{\omega,\mathbf{k}_\perp}.$$

It was shown in Sec. 2 that a wave with specified values of ω and $\mathbf{k}_\perp$ interacts with the pump wave in a small vicinity of the point x_0 and that its energy in the interaction region increases exponentially. Therefore

$$U_{\omega,\mathbf{k}_\perp} = U^{(0)}_{\omega,\mathbf{k}_\perp}\exp\left[2\int_{-\Delta x_1}^{x-x_0} d(\Delta x)\,\mathrm{Im}\,k_1(\Delta x)\right],$$

where $x_1 = x_0 + \Delta x_1$ is the boundary through which the wave enters the interaction region, and $U^{(0)}_{\omega,\mathbf{k}_\perp}$ is the spectral density of the energy of the non-amplified waves; we shall assume this energy to be determined by the thermal-fluctuation level $U^{(0)}_{\omega,\mathbf{k}_\perp} = [T_e/(2\pi)^2](\partial k_\parallel/\partial\omega)$. Using the relations given above, we write down the equation for the pump wave in the form

$$\frac{dq_0}{dx}=-\frac{\omega_0 T_e}{(2\pi)^3}\int\limits_{-\infty}^{\infty} d\omega\int d\mathbf{k}_\perp\frac{1}{\omega}\,\mathrm{Im}\,k_1(x-x_0)\exp\left[2\int\limits_{-\Delta x_1}^{x-x_0} d(\Delta x)\;\mathrm{Im}\,k_1(\Delta x)\right]. \tag{3.2}$$

This equation was obtained earlier[12] from the general relations of nonlinear electrodynamics.

At each point x, the pump wave interacts (at a fixed value of $\mathbf{k}_\perp$) with sound waves from a narrow frequency interval, and it is possible to change over from integration with respect to ω to integration over the interaction region.[12] As a result we get

$$\frac{dq_0}{dx}=-\frac{\omega_0 T_e}{(2\pi)^3}\int d\mathbf{k}_\perp\frac{d\omega}{dx}(e^{2\varkappa}-1)\frac{1}{ks}, \tag{3.3}$$

where $\varkappa$ is the total gain.

In the moving-plasma case of interest to us, the low-frequency dielectric constant is of the form $\epsilon=(kr_{De})^{-2}-\omega_{Li}^2(\omega-\mathbf{k}'\mathbf{u})^{-2}$. Calculating $d\omega/dx$ with the aid of this expression and introducing a polar coordinate frame in wave-vector space in the plane perpendicular to the pump-wave propagation direction we obtain from (3.3) at $M>1$

$$\frac{dW}{dx}=\frac{\omega_0 M T_e k_0^2}{2^{1/2}(2\pi)^3 L_u c}\int\limits_0^{2\pi} d\varphi\int\limits_0^1 d\eta\,\eta(1+\eta)^{1/2}(\exp[2\varkappa(\eta,\varphi)]-1), \tag{3.4}$$

where $q_0=-cW$, $W=E_0^2/8\pi$ is the pump-wave energy density, φ is the angle reckoned from the direction of the polarization vector $\mathbf{n}_0$. The quantity $\varkappa$ in different regions of variation of φ and η is determined by formulas (2.8) and (2.10), with $\sin^2\theta=\sin^2\varphi+\eta^2\cos^2\varphi$. Formula (2.8), as follows from (2.11) is valid at $\eta<\eta_0$, and formula (2.10) at $\eta>\eta_0$. Accordingly, the integral in (3.4) can be written in the form

$$4\int\limits_0^{\pi/2} d\varphi\left\{2\int\limits_0^{\eta_0} d\eta\,\eta e^{2\varkappa_1}+\int\limits_{\eta_0}^1 d\eta\,\eta(1+\eta)^{1/2}e^{2\varkappa_2}\right\}-\frac{8\pi}{15}(2^{1/2}+1).$$

In the first term, after elementary integration with respect to η by the known methods,[14] we can easily estimate the integral with respect to φ if we take account of the fact that the main contribution is made by the region $\varphi\sim\pi/2$. In the second term, the main contribution is made by the region $\eta\gtrsim\eta_0$ and $\varphi\sim\pi/2$, and the integral can also be estimated if the following inequalities are assumed to hold:

$$\frac{\pi k_0 L_N v_E^2}{4v_{Te}^2(2^{1/2}M-1)}>1,\quad 2\frac{L_N(\omega_0^2-\omega_p^2)}{L_u\omega_p^2}>1. \tag{3.5}$$

As a result, Eq. (3.4) is transformed into

$$\frac{dy}{d\xi}=\frac{k_0 r_0\cdot 3^{1/2}M^{1/2}\nu^2}{2^3 2^{1/2}\pi^{1/2}\lambda^2 l_u(l_N y)^{1/3}}\left\{\frac{2Q^2(1+P)}{(4+3P)^{1/2}}\exp\left[\frac{2D}{3(1+P)}\right]\right.$$
$$\left.+\frac{(1+Q)^3(Q+1/2D)}{D(2+3Q)^{1/2}}\exp\left[\frac{2D}{1+Q}\right]\right\}, \tag{3.6}$$

where

$$\xi=\frac{\omega_0 x}{c},\quad y=\frac{\pi v_E^2}{4v_{Te}^2}=\frac{\pi^2 r_0 c}{T_e\omega_0^2}|q_0|,\quad r_0=\frac{e^2}{mc^2}=2.8\cdot10^{-13}\text{ cm},$$
$$l_N=k_0L_N,\quad l_u=k_0L_u,\quad \lambda=L_N/L_u,\quad \nu=\omega_p^2/\omega_0^2=N/N_c,\quad N_c=m\omega_0^2/4\pi e^2$$

is the critical electron density, and

$$Q=\left(\frac{4\lambda^3 y}{2\pi M\nu^2}\right)^{1/3},\quad P=\frac{\pi Q^{3/4}}{6^{3/4}2^{1/4}},\quad D=\frac{l_N y}{2^{1/2}M}. \tag{3.7}$$

The first and second terms in the curly bracket of (3.6) stem respectively from the integration region $\eta<\eta_0$ (which corresponds to scattering at angles close to 90°) and from the integration region $\eta\gtrsim\eta_0$ (backscattering).

4. SOLUTION OF THE EQUATION FOR THE PUMP WAVE

We have assumed so far that the hydrodynamic characteristics of the plasma (the Mach number M and the particle density N) depend only on the coordinates, and that the plasma expansion is stationary. Equation (3.6), however, can also be used for nonstationary expansion if the quantities M and N vary little during the time when the slower sound wave traverse the region of resonant interaction and the pattern of spatial amplification of the waves is established. This condition can be written in the form

$$\frac{M}{\partial M/\partial t},\frac{N}{\partial N/\partial t}>\frac{\Delta x_{0,\max}}{s}, \tag{4.1}$$

where $\Delta x_{0,\max}$ is the maximum width of the interaction region. When the condition (4.1) is satisfied, the quantities M and N need not be connected by the flux-conservation law.

We consider by way of example the following hydrodynamic functions:

$$M=u/s=M_c+M_1(x/x_c-1),\quad N=N_0\exp(-\gamma x^2), \tag{4.2}$$

where M_c is the Mach number at the point x_c where the density is equal to the critical value[1)] N_c, N_0 is the electron density at $x=0$; the quantities γ and M_1 are certain constants which, like the quantities M_c and N_0, can vary slowly with time. Expression (4.2) corresponds to one of the simplest models of spherical expansion of a laser plasma at constant temperature and to a linear variation of the velocity with coordinate.[15,16]

Since the concentration (4.2) does not vanish with increasing coordinate, it is necessary to specify in some manner the point $x=x_b$ at which the intensity of the incident wave can be regarded as constant. We have chosen by way of example a point at which the density equals $0.01N_c$.

Equation (3.6) was solved with a computer using the following parameters: $T_e=10^3$ eV, $(v_{Te}/c)^2=2\cdot10^{-3}$, $k_0=6\cdot10^4$ cm^{-1} (neodymium laser) $(N_0/N_c)=5$, $x_c=2\cdot10^{-2}$ cm, $x_b=4\cdot10^{-2}$ cm, $(x_b-x_c)=200$ μm. The results of the calculation for two values of the incident-wave energy flux q_0 (6×10^{14} and 3×10^{14} W/cm^2) and for two laws governing the variation of the Mach number are shown in Fig. 1. It is seen that up to a definite value of the coordinate the intensity of the pump wave remains unchanged, and subsequently decreases as the critical density is approached. The attenuation

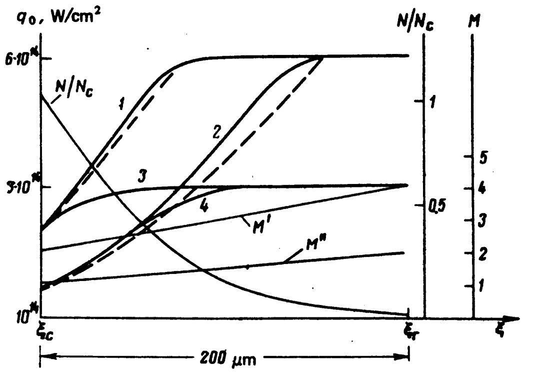

FIG. 1. Pump-wave energy flux density q_0, plasma concentration N/N_c, and Mach number $M=u/s$ as functions of the coordinate $\xi=x\omega_0/c$. Plots 1 and 3 correspond to the line M', while plots 2 and 4 to the line M''. The dashed lines are the results of calculations by the approximate formulas (4.4) and (4.5).

of the pump wave starts in the denser plasma and is smaller the faster the plasma flow and the lower the intensity of the incident wave. It is seen from the figure that independently of the intensity of the incident wave at given functions M and N, the same energy flux reaches the critical density.

Calculations were made also for a flux 3×10^{14} W/cm², when the plasma concentration decreased from the critical value by a factor of one hundred over a length of 50 μm. In this case the SMBS has practically no effect on the wave propagation.

At the employed parameters, which are typical of many experiments with laser plasma, the quantities in (3.6) satisfy the conditions $Q<1$ and $Q<1/2D$. The principal term is then the second one in the curly bracket of (3.6), and we can write the approximate equation

$$\frac{dy}{d\xi}=\frac{k_0r_0M'^{1/2}v^2}{2^3 2^{3/4}\pi^{1/2}\lambda^2 l_u l_N'^{5/2} y^{5/2}}\exp[2^{1/2}l_N y/M]. \qquad (4.3)$$

Just as in the studies of stimulated Raman scattering[12,17], we can construct an approximate analytic solution of (4.3). The reason is that the pre-exponential factors in (4.3) are much less than unity and to obtain an appreciable decrease of the function y over a scale on the order of ξ_c the argument of the exponential must be large enough. This means that the function y is e to its limiting value y_0 up to the coordinate ξ_1 at which $(dy/d\xi)\sim y_0/\xi_c$. This yields

$$\xi_1^2\approx\frac{\xi_c^3 y_0}{2^{1/2}M_c\ln(N_0/N_c)\mathscr{L}},$$
$$\mathscr{L}=\ln\left\{\frac{2^{1/4}\pi^{1/4}y_0^{7/2}\xi_c^{5/2}}{k_0r_0M_c'^{1/2}[\ln(N_0/N_c)]^{5/2}}\right\}, \qquad (4.4)$$

where $\xi_c=\omega_0x_c/c$ and it is assumed that $\xi_1\sim\xi_c$ in the logarithm $\mathscr{L}$. At $\xi<\xi_1$ the function y begins to decrease, and with it also the argument of the exponential in (4.3), and this slows down the decrease of y. As a result y takes on a value corresponding to the condition $(dy/d\xi)\approx y_0/\xi_c$, or

$$y\approx\xi^2\frac{2^{1/2}M_c\ln(N_0/N_c)}{\xi_c^3}\left[\mathscr{L}+\left(\frac{\xi^2}{\xi_c^2}-1\right)\ln\frac{N_0}{N_c}\right]. \qquad (4.5)$$

The dashed lines in the figure show the solution obtained from formulas (4.4) and (4.5).

If it turns out that $\xi_1\leqslant\xi_c$, then all the radiation reaches the critical density, and the attenuation of the pump wave by the SMBS can be neglected. From the condition $\xi_1=\xi_c$ we obtain the value of y_0 starting with which the pump-wave attenuation due to the SMBS comes into play:

$$y_0=\frac{2^{1/2}M_c\ln(N_0/N_c)}{\xi_c}\ln\left(\frac{10^{-3}\xi_c^{5/2}}{k_0r_0M_c'^{1/2}[\ln(N_0/N_c)]^{5/2}}\right), \qquad (4.6)$$

where $y_0\approx0.1$ is assumed under the logarithm sign. It is seen from (4.6) that the energy flux density that reaches the critical density without appreciable change is larger the faster the plasma flow and the larger the density gradient.

We must dwell on the conditions under which the results are applicable.

1. We have assumed that the dimension of the wave interaction region is much less than the plasma-inhomogeneity scale. Since the main contribution to Eq. (3.6) comes from the gain (2.10) at $\eta>1$ and $\varphi\sim\pi/2$, it follows from (2.9) that $\Delta x_{0,\max}=(v_E/8v_{Te})(L_NL_u/M)^{1/2}$, and $\Delta x_{0,\max}<L_N$ can be represented in the form

$$\frac{v_E^2}{v_{Te}^2}<8M\frac{L_N}{L_u}. \qquad (4.7)$$

For the functions (4.2), the inequality (4.7) is transformed into $y<\pi M_1\xi_c/\xi\ln(N_0/N_c)$ and is satisfied in the examples considered by us.

2. The fact that the plasma flow is quasistationary implies satisfaction of the conditions (4.1), which can be rewritten with the aid of the expression given above for $\Delta x_{0,\max}$ in the form

$$\frac{\partial M/\partial t}{M},\ \frac{\partial N/\partial t}{N}<\frac{4Mv_{Te}u}{v_E(L_NL_u)^{1/2}}. \qquad (4.8)$$

3. In the derivation of the gains and in the estimate of the integrals of Eq. (3.4) it was assumed that inequalities (2.11) and (3.5) are valid. It is easy to verify that in our calculation the inequality (2.11) and the first inequality of (3.5) held, but the second inequality of (3.5) was violated as the critical density was approached (at $\xi/\xi_c\lesssim1.1$ or $N/N_c\gtrsim1.5$). Our results in the vicinity of the critical density must therefore be regarded only as estimates.

CONCLUSION

It follows from the foregoing analysis that the spectral composition and the intensity of the SMBS depend substantially on the dynamics of the laser plasma. Thus, in supersonic expansion the spectrum of the scattered radiation broaden in the blue direction (anti-Stokes scattering). The distribution of the intensity over the spectrum is determined by the coordinate dependences of the density and of the velocity of the plasma.

Another factor very sensitive to the laser-plasma dynamics is the SMBS-induced exhaustion of the pump wave. This follows from the fact that small changes of the argument of the exponential in (4.3) affect strongly the character of the penetration of the pump wave into the plasma. According to the definition (3.7), this exponent is directly proportional to the intensity of the wave and to the plasma-inhomogeneity scale is inversely proportional to the plasma velocity. With increasing inhomogeneity scale we therefore have an increased scattering intensity, in qualitative agreement with the ideas concerning the influence of the contrast. Conversely, with increasing plasma expansion velocity, the scattering intensity decreases and it can be stated that the plasma motion suppresses the SMBS. This conclusion agrees with the results of a numerical solution of the gasdynamics equations.[18] The physical cause of the suppression of the SMBS is that the acoustic waves drift together with the plasma and pass more rapidly through the region of the resonant interaction in the inhomogeneous plasma, and consequently have a smaller growth.

It should be noted that our analysis is not quite consistent, since we did not take into account the fact that the plasma hydrodynamic characteristics (the density and the expansion velocity) are themselves dependent on the incident-wave intensity. A consistent allowance for this dependence is possible if the equations hydrodynamics and the equation for the pump waves are solved simultaneously.

We are grateful to V. P. Silin and R. R. Ramazashvili for a number of useful remarks.

[1)] We note that the quantity M_c determines the plasma velocity at the point x_c, but not the velocity of the point x_c itself.

[1]C. S. Liu, M. N. Rosenbluth, and R. B. White, Phys. Fluids **17**, 1211 (1974).
[2]D. W. Forslund, J. M. Kindel, and E. L. Lindman, Phys. Fluids **18**, 1002 (1975).
[3]C. S. Liu, Adv. Plasma Phys. **6**, 121 (1975).
[4]A. A. Galeev, G. Laval', T. O. Neil, M. Rozenblyut, and R. Z. Sagdeev, Zh. Eksp. Teor. Fiz. **65**, 973 (1973) [Sov. Phys. JETP **38**, 482 (1974)].
[5]L. M. Gorobunov, Usp. Fiz. Nauk **109**, 631 (1973) [Sov. Phys. Usp. **16**, 217 (1973)].
[6]A. A. Galeev and R. Z. Sagdeev, Voprosy teorii plazmy (Problems of Plasma Theory), Vol. 7, Atomizdat, 1973, p. 39.
[7]V. I. Bespalov and G. A. Pasmanik, Radiofizika **19**, 634 (1976).
[8]A. D. Piliya, Tenth Intern. Conf. on Phenomena in Ionized Gases, Paper No. 4.3.8.1, Oxford, 1971.
[9]M. N. Rosenbluth, Phys. Rev. Lett. **29**, 565 (1972).
[10]N. Bloembergen, Nonlinear Optics, Benjamin, 1965.
[11]C. S. Liu, M. N. Rosenbluth, and R. B. White, Phys. Rev. Lett. **31**, 697 (1973).
[12]L. M. Gorbunov, V. I. Domrin, and R. R. Ramazashvili, Zh. Eksp. Teor. Fiz. **70**, 2161 (1976) [Sov. Phys. JETP **43**, 1128 (1976)].
[13]S. A. Akhmanov and R. V. Khokhlov, Problemy nelineĭnoĭ optiki (Problems of Nonlinear Optics), VINITI, 1964, pp. 91, 92.
[14]A. B. Migdal, Kachestvennye metody v kvantovoĭ teorii (Qualitative Methods in Quantum Theory), Nauka, 1975, p. 14.
[15]W. Fader, Phys. Fluids **11**, 2200 (1968).
[16]A. N. Polyanichev, Izv. Vyssh. Uchebn. Zaved Fiz. **11**, 79 (1976).
[17]L. M. Gorbunov, V. I. Domrin, and R. R. Ramazashvili, Pis'ma Zh. Tekh. Fiz. **2**, 690 (1976) [Sov. Tech. Phys. Lett. **2**, 270 (1976)].
[18]B. Bezzerides, D. F. Du Bois, D. W. Forslund, J. M. Kindel, K. Lee, and E. L. Lindman, Preprint LA-UR-76-1580 (Los Alamos).

Translated by J. G. Adashko

Experimental investigation of the emission of a mercury plasma near the photorecombination thresholds at high pressure

Yu. K. Kurilenkov and P. V. Minaev

Institute of High Temperatures, USSR Academy of Sciences
(Submitted 26 June 1977)
Zh. Eksp. Teor. Fiz. **74**, 563–569 (February 1978)

The spectra of the line and continuous emission of a mercury plasma were investigated in the frequency interval $(0.4–1.25)\times 10^{15}$ sec^{-1} at electron densities 5×10^{15} and 4×10^{17} cm^{-3}. At high charged-particle densities it was observed that the spectral lines vanish near the photorecombination thresholds, but the thresholds themselves are hardly displaced. In a less dense plasma, in the near-threshold regions of the spectrum, a coalescence of the spectral lines was observed in accordance with the Inglis-Teller model, leading to an apparent shift of the photorecombination thresholds towards lower frequencies.

PACS numbers: 52.25.Ps

A well known density effect in a plasma is the coalescence of the higher terms of the spectral series near the photorecombination (photoionization) thresholds.[1,2] To describe the transition of the line spectrum into a continuum it is customary to use the assumption that the oscillator-strength density df/dE is not perturbed.[2,3] It was shown[4] that on going to a nonideal plasma a qualitatively new effect may appear,

 0038-5646/78/4702-0295$02.40 ©1978 American Institute of Physics