

# Nuclear level shift in the ( $p\bar{p}$ ) atom

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It is shown that the attractive nuclear forces between  $p$  and  $\bar{p}$  push out the Coulomb levels if the depth of the nuclear potential well is large enough for a bound state to appear. The case of a complete atomic-spectrum restructuring which occurs at a critical value of the attractive nuclear potential (threshold for the appearance of the nuclear bound state) is examined. It is shown that the region of critical values is very narrow and is determined by the small parameter  $R/a$ , where  $a$  is the Bohr radius and  $R$  the range of nuclear forces. In the post-critical region, the shift is again small and the Coulomb spectrum is restored. The results do not depend on the detailed shape of the nuclear potential.

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## §1. INTRODUCTION

The nucleon-antinucleon interaction at low energies has recently attracted considerable attention among both theoreticians and experimentalists. Despite the presence of the annihilation interaction between the nucleon and the antinucleon, the potential approach can be used to analyze the properties of the  $N\bar{N}$  system because the size of the  $N\bar{N}$  annihilation region is much smaller than the characteristic size of the nucleon-antinucleon systems under consideration.<sup>[1]</sup> Detailed theoretical analyses show that, because of the presence of the strong attractive interaction between the nucleon and the antinucleon, the  $N\bar{N}$  system has a rich spectrum of bound and resonant quasinuclear states with mass defects up to a few hundred MeV.<sup>[1]</sup> These theoretical predictions have been confirmed experimentally in recent years as a result of studies of the annihilation of slow antiprotons in hydrogen and deuterium.<sup>[2]</sup>

The properties of the antiproton atoms ( $\bar{p}p$ ) and ( $\bar{p}d$ ) are particularly interesting. Studies of cascade processes in  $\bar{p}$  atoms ( $x$ -ray transitions between atomic levels and the annihilation of antiprotons in bound atomic states) may provide valuable information on the nucleon-antinucleon interaction.<sup>[3]</sup> The immediate problem is to determine for the ( $\bar{p}p$ ) and ( $\bar{p}d$ ) atoms the level shifts and widths due to the strong interaction.

When only the Coulomb interaction is taken into account, the ( $\bar{p}p$ ) atom has a ground-state binding energy ( $-E_{1S}^c = m\alpha^2/4 = 12.5$  keV and first Bohr radius  $a = 57.5$  fm. The levels of this atom are shifted relative to the values given by the well-known formulas of quantum mechanics (including the fine and hyperfine structures) because of the presence of the strong interaction between  $p$  and  $\bar{p}$  and the corrections due to quantum electrodynamics (QED). The main QED correction is vacuum polarization, due to the creation of the virtual pairs  $e^+e^-$ , which appears separately from the Lamb shift because the radius of the first Bohr orbit of the atom is less than the Compton wavelength of the electron, so that the particles lie in the region where screening due to vacuum polarization is, in general, important. However, the shift of the 1S state due to vacuum polarization is numerically small:  $\Delta E_{1S} = E_{1S} - E_{1S}^c = -0.038$  keV. The Lamb shift and the hyperfine

structure are negligible in the 1S state.

Theoretical calculations of the ( $p\bar{p}$ ) level shifts due to the nuclear interaction have been reported by Dal'karov and Samoïlov.<sup>[4]</sup> The level shifts were found by solving the Schrödinger equation for the  $p\bar{p}$  system with the Coulomb and nuclear interactions included (the latter were described with the aid of the one-boson exchange potential OBEP). The 1S level shifts obtained in this way were found to lie between +0.5 and +1.1 keV (i.e., the level was "pushed out"). The atomic level shifts and widths were also calculated by Caser and Omnes,<sup>[5]</sup> who started with the  $\bar{p}p$  amplitude corresponding to the complex optical-model potential. This also resulted in a positive 1S level shift of the order of 1 keV. It is, however, important to emphasize that the positive value obtained for  $\Delta E_{1S}$  by these workers<sup>[5]</sup> is of completely different origin and is a trivial consequence of the fact that all the levels are pushed out in the optical model (because of the highly non-Hermitian nature of the Hamiltonian which is introduced to take annihilation into account).

It has been shown<sup>[6]</sup> that the optical model which, in this case, is a semiempirical model describing the  $p\bar{p}$  interaction cross sections, is not generally suitable for calculations of discrete levels. The annihilation-interaction range  $r_a$  is roughly one-tenth of the range  $R$  of nuclear forces due to light-meson exchange in the  $t$ -channel.<sup>[1,6]</sup> The level shift due to the annihilation interaction must, therefore, contain an additional small factor of the order of  $(r_a/R)^3 \sim 10^{-3}$ . Annihilation corrections will be neglected in our calculations of atomic level shifts.

We shall give a theoretical analysis of the spectrum of bound states in the system in which both Coulomb forces and short-range attractive forces (with range much smaller than the Bohr radius) are present. Our main aim is to elucidate the mechanism responsible for the shifts of atomic levels due to the presence of the strong short-range attraction between the particles. It turns out that this problem will also involve the analysis of the complete rearrangement of the Coulomb spectrum when the atom becomes "unrecognizable" because of the presence of additional attractive forces with range  $R$  much smaller than the Bohr radius  $a$ .

The parameter  $R/a$  has, however, the effect that the "critical" values of the depth of the nuclear potential well lie within very narrow bands (they are proportional to  $R/a$ ). Provided these bands are avoided, the atom remains practically unaffected, however strong the nuclear interaction, and the level shifts due to the short-range forces turn out to be small. The critical points themselves, on the other hand, correspond to the appearance of  $S$  levels in the short-range potential well (with different numbers of radial nodes).

It was found in the course of the solution of the problem that these results are valid not only for the atomic-level shifts but are general physical results in the sense that they are encountered whenever a two-particle quantum-mechanical system has two types of attractive force with different ranges. Accordingly, to bring to light the physical features of the problem, we begin by considering some general relationships and illustrate them by simple examples.

In Sec. 2, we consider the general relationship between the energy-level shift and the mass operator for short-range forces. In Sec. 3, we consider the simple example of two rectangular wells to demonstrate the effect of the strong interaction on the energies of bound states in a potential well of large radius. In Secs. 4 and 5, we give the results of calculations of the  $(p\bar{p})$ -atom level shifts for two strong interaction potentials, namely, the separable potential and the rectangular potential well.

## §2. MASS OPERATOR AND LEVEL SHIFT

Let  $f(\mathbf{k}, \mathbf{k}', E)$  be the scattering amplitude due to the short-range nuclear potential. This amplitude is taken outside the energy surface ( $\mathbf{k}^2 \neq \mathbf{k}'^2 \neq mE$ , where  $\mathbf{k}, \mathbf{k}'$  are the momenta of one of the particles in the C.M.S.). The mass operator  $\hat{M}(E)$  is related to  $f$  as follows:

$$\hat{M}_{\nu\nu}(E) = -\frac{4\pi}{m} \int \varphi_{\nu}(\mathbf{k}) f(\mathbf{k}, \mathbf{k}', E) \varphi_{\nu}(\mathbf{k}') \frac{d\mathbf{k} d\mathbf{k}'}{(2\pi)^6}, \quad (1)$$

where  $E$  is the kinetic energy of  $p$  and  $\bar{p}$  in the CM system,  $m$  is the mass of the nucleon, and  $\varphi_{\nu}(k)$  is the wave function for the discrete state with energy  $E_{\nu}^{(0)}$ .

The wave functions  $\varphi_{\nu}$  correspond to the Hamiltonian for the "long-range" forces. In the case of the  $(p\bar{p})$  atom (or other hadron atom), the wave functions  $\varphi_{\nu}$  correspond to the Coulomb problem unperturbed by the nuclear interaction. We note further that, strictly speaking, the amplitude  $f$  will also contain terms due to the Coulomb interaction between the particles when they are within the range of the nuclear forces. The amplitude  $f$  does not coincide with the scattering amplitude for three particles interacting through electromagnetic and nuclear forces even if it is taken on the energy surface. This difference occurs because the Coulomb interaction is turned on in  $f$  only in virtual states between nuclear interactions, but not in the initial and final states of the entire scattering process. This is illustrated in Fig. 1a, where the rectangular block corresponds to the pure nuclear scattering am-

plitude and the oval block to the Coulomb amplitude. The Coulomb corrections to  $f$  are proportional to  $\alpha$  and are small in comparison with the contribution due to the pure nuclear interactions. We may therefore suppose that  $f$  is practically equal to the amplitude for pure nuclear scattering.

The mass operator (1) corresponds to the diagram shown in Fig. 1b to within the factor  $i(2\pi)^{-4}$ . The shaded square in this figure is the amplitude  $(4\pi/m)f(\mathbf{k}, \mathbf{k}', E)$ . The vertex parts  $\gamma_{\nu}(k)$  in this diagram are related to  $\varphi_{\nu}(\mathbf{k})$  by

$$\gamma_{\nu}(k) = \frac{k^2 - mE_{\nu}^{(0)}}{m} \varphi_{\nu}(k), \quad E_{\nu}^{(0)} < 0. \quad (2)$$

Since the amplitude  $f$  is a scalar, the integral in (1) is nonzero only when the states  $\varphi_{\mu}$  and  $\varphi_{\nu}$  have equal quantum numbers. In the case of the  $p\bar{p}$  atom, this means that the mass operator may mix states  $(n, l)$  with different principal quantum numbers  $n$  but equal orbital angular momenta  $l$ . Henceforth, we shall confine our attention mainly to the nuclear shifts of the  $S$  levels. We shall therefore consider matrix elements of the mass operator between the  $S$  states. With this in mind, we begin by noting that, because of the presence of the short-range nuclear forces, the scattering amplitude  $f$ , expressed as a function of the momenta  $k$  and  $k'$ , will, in general, vary more slowly than the wave functions  $\varphi$ . In point of fact, the nuclear amplitude  $f$  can vary appreciably over momentum intervals of the order of  $R^{-1}$  whereas, for the atomic wave functions, the characteristic interval is of the order of  $a^{-1} \ll R^{-1}$ .

The exception to this rule is the case where the amplitude  $f$  has a pole in  $E$  corresponding to nuclear levels. When this is so, then, in addition to the range  $R$ , there is one further parameter with the dimensions of length, namely, the state radius  $r_{\epsilon} = (m |\epsilon_N|)^{-1/2}$ , where  $\epsilon_N$  is the nuclear level energy. When the latter is small enough in absolute magnitude, so that the state radius  $r_{\epsilon}$  is comparable with the Bohr radius, the amplitude  $f$  will vary appreciably over the same momentum interval as the atomic wave functions  $\varphi$ .

It is precisely this case that will be considered below. Here, however, we shall assume that the poles of  $f$  lie well away from atomic levels ( $r_{\epsilon} \approx R \ll a$ ). In view of the foregoing, the slowly-varying amplitude  $f$  can be taken outside the integral sign in (1) and, if we note that

$$\int \varphi(\mathbf{k}) d\mathbf{k} = (2\pi)^{-3} \Psi(0)$$

where  $\Psi_{\nu}(\mathbf{r})$  are the wave functions in the coordinate representation, we obtain

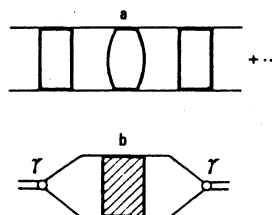


FIG. 1.

$$\hat{M}_{\mu\nu}(E) = -\frac{4\pi}{m} f(k_0, k_0', E) \Psi_{\mu}(0) \Psi_{\nu}^*(0), \quad (3)$$

where  $k_0, k_0'$  are certain values of the momenta  $k, k'$  and lie in the effective integration interval ( $0 \leq k(k') < a^{-1}$ ).

Equations (1) and (3) are not based on any potential model for the scattering amplitude. The understanding of this point is particularly important for applications to the theory of nucleon-antinucleon systems which are characterized by annihilation into hadrons. The influence of annihilation on the spectrum of discrete states, on the other hand, cannot be taken into account with the aid of a potential well (such as the optical potential). We therefore emphasize once again that the only condition for the validity of (3) is that the amplitude  $f$  should be a slowly-varying function of the momenta as compared with the wave functions  $\varphi$  (this is the only condition expressing the short-range character of the forces).

Let us now estimate the order of magnitude of the matrix elements  $M_{\mu\nu}$  in the case of the ( $p\bar{p}$ ) atom. Since we are interested in negative energies  $E$  close to the atomic-level energies, we may suppose, in very approximate calculations, that  $E=0$ , so that the interval in which  $f$  changes appreciably as a function of  $E$  is determined by the range  $R$  of the forces and by the position of the nuclear levels (which we assume to be sufficiently distant from the atomic levels). On the same basis, we may suppose that  $k_0 = k_0' = 0$ . The amplitude  $f(k_0, k_0', E)$  in (3) is therefore close to the nuclear  $p\bar{p}$  scattering length. To within an order of magnitude, we may suppose that  $f \approx R \approx 1$  fm. Next, for the 1S state, we have  $\Psi_{1S}(0) = (\pi a^3)^{1/2}$  and, if we substitute these values in (2), we obtain the following approximate result for the diagonal matrix element of the mass operator for the 1S state of the ( $p\bar{p}$ ) atom:  $|M_{1S,1S}| \approx 4.2 \times 10^{-6} (\text{fm})^{-1} = 0.84$  keV. This amounts to about 10% of the separation between the 2S and 1S levels. The nondiagonal matrix elements are smaller still:

$$|M_{1S,nS}| \approx \frac{0.84}{n^3} \text{ keV}.$$

Hence, it follows that

$$E_{nS} - E_{1S} \gg |M_{1S,nS}|. \quad (4)$$

This inequality simplifies the determination of the nuclear shift of the 1S level from the given scattering amplitude  $f$ . The mass operator determines the Green matrix  $\hat{D}(E)$ :

$$\hat{D}(E) = (1 - \hat{d}\hat{M})^{-1}\hat{d}, \quad (5)$$

where  $\hat{d}$  is the diagonal matrix

$$\hat{d}_{\mu\nu} = \delta_{\mu\nu} (E - E_{\mu}^{(0)}). \quad (6)$$

The nuclear-shifted levels are the poles of the eigenvalues of the matrix  $\hat{D}(E)$ . It is readily shown that, when (4) is satisfied, the nondiagonal matrix elements in (6) can be neglected. The matrix  $\hat{D}(E)$  therefore be-

comes diagonal and its matrix elements are given by

$$D = d / (1 - dM) \quad (7)$$

where, for brevity, we have omitted the subscripts.

It follows from (6) and (7) that the equation for the shifted levels is

$$E - E_{\mu}^{(0)} = M(E). \quad (8)$$

The condition for the validity of (8) is that, as indicated above, the shifts must be small in comparison with the S-level separation. Moreover, we have seen that  $M(E)$  is a very slowly-varying function of  $E$  on intervals of the order of the separation between the atomic levels, so that we may substitute  $M(E) \approx M(E_{\mu}^{(0)}) \approx M(0)$  in (8). If we then use (3), we obtain the following expression for the energy of the 1S state:

$$W_{1S} - E_{1S}^{(0)} = -\frac{4\pi}{m} f |\Psi_{1S}(0)|^2, \quad (9)$$

where  $E_{1S}$  is the complex energy of the 1S state.

From (9), we obtain the level shift and width:

$$\Delta E(1S) = \text{Re } W_{1S} - E_{1S}^{(0)} = -\frac{4\pi}{m} |\Psi_{1S}(0)|^2 \text{Re } f, \quad (10)$$

$$\Gamma_{1S} = -2 \text{Im } W_{1S} = (v\sigma_a)_{v \rightarrow 0} |\Psi_{1S}(0)|^2, \quad (11)$$

where  $v$  is the relative velocity of the annihilating particles and  $\sigma_a$  is the cross section for the annihilation of free  $p$  and  $\bar{p}$ .

Equation (9) has been known and used, for example, in the theory of "exotic" atoms (see, for example, the paper by Caser and Omnes).<sup>[5]</sup> However, the conditions for the validity of these formulas were not always properly understood. Firstly, the fact that (9)-(11) are valid in the case of small shifts was frequently interpreted as a consequence of the use of perturbation theory in the interaction generating the scattering amplitude  $f$ . Secondly, (9) was usually obtained within the framework of a model potential for  $f$ .<sup>[7]</sup> The first of these misunderstandings is closely related to the widely held view that the short-range forces of attraction will necessarily lead to a negative shift  $\Delta E$  in those cases where (9) can be used, i.e., that these forces increase the binding energy relative to the unperturbed value. The validity of perturbation theory in the case of short-range forces mean that the scattering length  $f$  and the potential  $V$  of these forces are related by the Born formula

$$f = -\frac{m}{2\pi} \int_0^{\infty} V(r) r^2 dr. \quad (12)$$

Hence, and from (11), it follows that the sign of  $\Delta E$  is the same as that of  $V$  (i.e., it is negative for attractive forces). On the other hand, the idea that the potential approximation must be used for  $f$  when (9) is employed suggests the use of the optical model (complex potential) in order that the influence of inelastic channels, for example, annihilation in the case of ( $p\bar{p}$ ) atoms,

can be taken into account in (10). In reality, however, it is clear from the foregoing that (9) is in no way connected with perturbation theory in the strength of the short-range forces, nor with the potential approach to the evaluation of the scattering length  $f$ . The formula given by (9) and the equivalent expressions given by (10) and (11) are valid whenever the shifts are small and when the scattering amplitude  $f$  varies very slowly as a function of momenta as compared with the wave functions  $\varphi$  or, what amounts to the same, in comparison with the vertex parts  $\gamma$  [formula (2)] in Fig. 1b.

It follows from the foregoing that the levels are pushed out, as mentioned in Sec. 1, even in the case of a strong-range attraction, or in the absence of inelastic processes (i.e., for real scattering length  $f$ ). In point of fact, it is clear from (10) that the sign of the shift depends on the sign of  $\text{Re}f$ , and this is not determined by the sign of the potential when the Born approximation (12) is not valid. Moreover, it is easily seen that, if the short-range potential well is sufficiently deep, so that a bound nuclear state appears, the scattering length  $f$  may be negative and, correspondingly, the shift  $\Delta E$  is positive. A good example of this is the neutron-proton scattering length. In this case, the triplet scattering length is negative because there is the bound  ${}^3S_1$  state (the deuteron). The singlet length, on the other hand, is positive because the corresponding potential well is too shallow and the  ${}^1S_0$  state does not appear in the neutron-proton system. It is readily shown analytically that the atomic level will be pushed out as a result of the presence of the nuclear bound state. Let us suppose that the amplitude  $f(E)$  has a pole at  $E = E_N - i\Gamma_a/2$ , which corresponds to the nuclear bound state of  $p\bar{p}$  with an annihilation width  $\Gamma_a$ .

Next, we assume that the pole part of the amplitude predominates, in which case

$$f(E_{1S}^{(0)}) \approx -\frac{g}{E_{1S}^{(0)} - E_N + i\Gamma_a/2}, \quad E_{1S}^{(0)}, E_N < 0, \quad |E_N| \gg |E_{1S}^{(0)}|. \quad (13)$$

Here  $g$  is a real positive quantity (this follows from the fact that  $\text{Im}f > 0$  in the presence of inelastic processes even for  $E < 0$ ). Substituting (13) in (10), we obtain

$$\Delta E(1S) = \frac{4g}{ma^3} \frac{E_{1S}^{(0)} - E_N}{(E_{1S}^{(0)} - E_N)^2 + \Gamma_a^2/4} \approx \frac{4g}{ma^3} \frac{|E_N|}{E_N^2 + \Gamma_a^2/4}. \quad (14)$$

It is also clear from this expression that the fact that the nuclear level has an annihilation width does not increase, and may even reduce, the shift of the atomic level (contrary to the results obtained with the aid of the optical model,<sup>[5]</sup> according to which absorption facilitates the pushing out of the level). We note, however, that the presence of the nuclear level does not in itself ensure that the shift  $\Delta E(1S)$  will be positive. Equation (14) was, in fact, obtained on the assumption that the pole term (13) predominates in the amplitude  $f$  for  $E = E_{1S} < 0$ . If the level  $E_N$  lies too far from  $E_{1S}$ , this assumption may not correspond to reality, and the sign of the shift may, in principle, be different.

The above discussion has shown that sufficiently

strong nuclear attraction may lead to the pushing out of the atomic level. On the other hand, it follows from (10) and (12) that a weak nuclear attraction will depress the atomic level. The question therefore is: what is the mechanism responsible for the change in the sign of  $\Delta E(1S)$  as the nuclear attraction is reduced? Formally, it may be concluded from (14) that  $\text{sign} \Delta E(1S) = \text{sign}(E_{1S}^{(0)} - E_N)$ . In reality, however, Eq. (14) ceases to be valid if the shift  $\Delta E(1S)$  becomes comparable in order of magnitude with the  $2S$ - $1S$  level separation. If the annihilation width  $\Gamma_a$  were to be zero, or very small, this would follow unavoidably for sufficiently small  $|E_N|$ . The annihilation widths of nuclear levels of the  $p\bar{p}$  system are definitely nonzero, but they decrease with increasing state radius (in inverse proportion to the cube of this radius<sup>[13]</sup>). This is why, when  $|E_N|$  is small, the width  $\Gamma_a$  may also turn out to be small. Hence, it follows that, to settle the level shift question for  $E_N \approx E_{1S}$ , we must use the general formulas (1) and (5) for the matrices  $\hat{M}$  and  $\hat{D}$ . Since annihilation effects in the atomic level-shift problem play, as we have seen, a secondary role, it is reasonable to neglect annihilation altogether when the essence of the situation is to be elucidated, and use a suitable potential model to calculate the nuclear scattering amplitude  $f$ . However, the evaluation of the matrices  $\hat{M}$  and  $\hat{D}$  in this case is no simpler than the solution of the Schrödinger eigenvalue problem with interaction Hamiltonian in the form of the sum of the Coulomb and nuclear (short-range) potentials.

### §3. ENERGY LEVELS FOR RECTANGULAR WELLS

It should be clear from the foregoing that the level shift due to the influence of short-range forces is not connected specifically with the Coulomb interaction. The characteristic features of this effect should be seen in all cases, whenever there are two potentials that have different ranges and are not too singular at short distances (so that the center cannot be reached).

The simplest example of the effect of a short-range potential on the energies of bound states due to longer-range forces is the level spectrum associated with the potential in the form of two rectangular wells with different radii:  $V(r) = V_1(r) + V_2(r)$ , where

$$V_1(r) = \begin{cases} -V_{10}, & r < r_1 \\ 0, & r \geq r_1 \end{cases}, \quad V_2(r) = \begin{cases} -V_{20}, & r < r_2 \\ 0, & r \geq r_2 \end{cases}. \quad (15)$$

We shall assume that the potential  $V_2(r)$  is deep enough for several bound states with energies  $E_{n0}$  ( $n = 1, \dots, N$ ) to appear for  $V_{10} = 0$ . Moreover, we assume that  $r_1 \ll r_2$  and  $V_{20} \ll 1/mr_1^2$ . The latter condition is introduced to eliminate bound states with radial nodes at distances of the order of the range  $r_1$  of the short-range forces for  $V_{10} = 0$ .

The set of equations whose solution gives the position of the levels  $E$  in the potential  $V(r)$  has the following form for  $-V_{20} < E < 0$ :

$$k \text{ctg} kr_1 = q \text{ctg} (qr_1 + \delta), \quad q \text{ctg} (qr_2 + \delta) = -(-mE)^{1/2}, \\ q = [m(V_{20} + E)]^{1/2}, \quad k = [m(V_{20} + V_{10} + E)]^{1/2}, \quad (16)$$

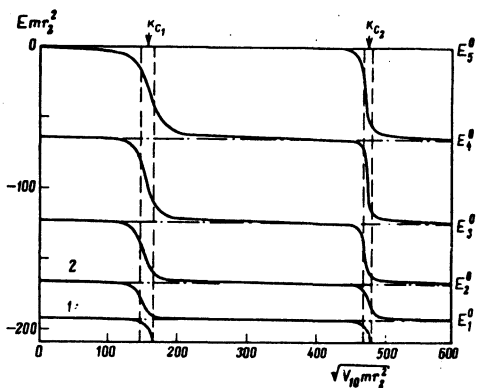


FIG. 2.

where  $\delta$  is the unknown phase.

Figure 2 shows the bound-state energies in the potential  $V(r)$  as functions of the depth of  $V_1(r)$  for  $r_1/r_2 = 0.01$  (solid line). The broken lines show the values of the depth of  $V_1(r)$  for which the next levels appear. The rearrangement of the spectrum occurs for  $V_{10} = V_{10}^{(c)}$ , which corresponds to the appearance of eigenstates in the potential  $V_1(r)$ . The former ground state in  $V_2(r)$  is then localized in the narrow and deep well  $V_1(r)$ , and the corresponding energy level shifts downward to a much deeper position (curve 1). It is replaced by the first excited state (curve 2), and so on. After the rearrangement, the lowest energy state localized in the wide well now has a radial node. Outside the rearrangement region (the corridor defined by the broken lines in Fig. 2), the spectrum is always similar to the level spectrum in the potential  $V_2(r)$ , and the level shift  $\Delta E$  can be positive or negative.

The important feature of the above rearrangement of the spectrum is that it occurs in a narrow, near-critical region of values of  $V_{10}$ . The reason for this is that the range of  $V_1(r)$  is small in comparison with the "size" of states localized in  $V_2(r)$ .

The behavior of the levels in a potential such as (14) has been analyzed by Koch *et al.*,<sup>[8]</sup> also in connection with the problem of the spectrum of hadron atoms (they were concerned with the levels of  $K$ -mesic atoms). In contrast to the formulation of the problem given here, Koch *et al.*<sup>[8]</sup> used the complex short-range potential. The presence of the imaginary part of the potential led to the appearance of a trivial level shift due to absorption. The complete rearrangement of the spectrum, which occurs when discrete levels appear in the short-range potential, has an effect on the results obtained by Koch *et al.* in that the shifts obtained by them exhibit oscillations (these shifts are always positive because of strong absorption). Similar oscillations in the  $K$ -mesic level shifts were reported by Ericson.<sup>[9]</sup>

We have already mentioned that the model involving a complex potential is hardly valid for the evaluation of level shifts when strong absorption is taken into account. What is required in such cases is the solution of the coupled-channels problem, which leads to results quite different from those obtained on the basis

of the optical model.<sup>[1]</sup>

#### §4. EVALUATION OF THE ATOMIC LEVEL SHIFTS IN THE CASE OF A SEPARABLE SHORT-RANGE POTENTIAL

We shall now show that the shift and rearrangement of the spectrum of levels of the  $(p\bar{p})$  atom due to the presence of short-range nuclear forces is of exactly the same nature as in the above example of two rectangular wells. To show this analytically, we use the separable model for the short-range forces. We assume that the interaction Hamiltonian corresponding to these forces is

$$\hat{V}\Psi(r) = \int V(r, r') \Psi(r') dr', \quad V = \lambda \xi(r) \xi(r'), \quad \int |\xi|^2 dr = 1. \quad (17)$$

Since  $\hat{V}$  is a short-range interaction, we must have  $|\xi(r)| < e^{-r/R}$  for  $r \rightarrow \infty$ . Since we are interested in the  $S$ -level shifts, we must assume<sup>[2]</sup> that  $\xi$  is a function of only  $|r| = r$ .

Our aim is to find the eigenvalues of the Hamiltonian

$$H = H_c + \hat{V}, \quad H_c = H_0 + V_c, \quad (18)$$

where  $\hat{H}_0$  is the Hamiltonian for the free particles and  $V_c$  is the Coulomb interaction energy. The operator  $\hat{H}_c$  is the Hamiltonian for the "usual" Coulomb problem (without the nuclear interaction between  $p$  and  $\bar{p}$ ). The Schrödinger equation corresponding to the Hamiltonian given by (18), whose solution gives the exact wave function  $|\Psi\rangle$  of a bound state of energy  $E$ , will be written in the form

$$(E - \hat{H}_c)^{-1} \lambda \xi(r) \langle \xi | \Psi \rangle = |\Psi\rangle, \quad \langle \xi | \Psi \rangle = \int \xi \cdot \Psi dr. \quad (19)$$

Since  $V$  is separable, the wave function  $|\Psi\rangle$  is unknown and is eliminated by multiplying (19) from the left by  $\xi(r)$  and then integrating with respect to  $r$ :

$$\langle \xi | (E - H_c)^{-1} \lambda \xi \rangle = 1/\lambda. \quad (20)$$

We shall use the spectral representation of the Coulomb Green function

$$(E - H_c)^{-1} = \sum_{n,l,m} \frac{|\varphi_{nlm}\rangle \langle \varphi_{nlm}|}{E - E_n^c} + \int dk \frac{|\varphi_k\rangle \langle \varphi_k|}{E - k^2/2m + i0}, \quad (21)$$

where  $\varphi_{nlm}$  and  $\varphi_k$  are the Coulomb functions for the discrete and continuous spectra, respectively, and  $E_n^c$  are the energies of the unperturbed atomic states. Substituting (21) in (20), we obtain

$$\sum_{n,l,m} \frac{|\langle \xi | \varphi_{nlm} \rangle|^2}{E - E_n^c} + \int dk \frac{|\langle \xi | \varphi_k \rangle|^2}{E - k^2/2m + i0} = \frac{1}{\lambda}. \quad (22)$$

Since the range  $R$  of the potential  $V$  is assumed small in comparison with the Bohr radius  $a$ , we may use the approximate result

$$|\langle \xi | \varphi_n \rangle|^2 = \frac{\beta_n}{n^2} \left(\frac{R}{a}\right)^3 \approx n^{-3} \left(\frac{R}{a}\right)^3. \quad (23)$$

The constants  $\beta_n \approx 1$  are slowly-varying functions of  $n$ . Next, we have

$$\int dk \frac{|\langle \xi | \varphi_k \rangle|^2}{E - k^2/2m + i0} \approx -2m \int \frac{dk}{k^2} \frac{|\langle \xi | e^{ikr} \rangle|^2}{(2\pi)^3} \approx -2mR^2 = -\frac{1}{\lambda_0}, \quad (24)$$

which is valid because the main contribution to the integral with respect to  $k$  is provided by the region with  $k \approx R^{-1}$ . For such values of  $k$ , the wave functions corresponding to the continuous spectrum,  $\Psi_{\mathbf{k}}$ , are almost plane waves (the condition for the validity of the Bohr approximation  $\alpha^2/v \approx R/a \ll 1$  is satisfied). In deriving (23) and (24), it was assumed that  $\xi(r) = r^{-1}(2/R)^{1/2} e^{-r/R}$ . The integral in (24) with this form of  $\xi(r)$  can be evaluated exactly and differs from the estimates given by (24) by the presence of a slowly-varying logarithmic term which can be replaced by a constant for  $E \sim E_n^c$ .

When (23) and (24) are taken into account, Eq. (22) simplifies to

$$\sum_n \frac{1}{n^3(E - E_n)} = \left(\frac{a}{R}\right)^3 \left\{ \frac{1}{\lambda} + \frac{1}{\lambda_0} \right\}. \quad (25)$$

Since the right-hand side of this equation contains the large coefficient  $(a/R)^3$ , the spectrum of the exact Hamiltonian (18) differs appreciably from the Coulomb spectrum only in a small neighborhood of the point  $\lambda = -\lambda_0$  (the size of the rearrangement region  $\delta\lambda/\lambda \sim R/a$ ). The dependence of the energy spectrum of the  $(p\bar{p})$  atom on  $\lambda$  is illustrated in Fig. 3 for  $a/R = 50$ .

The value  $\lambda = -\lambda_0$  corresponds to the appearance of an eigenstate in the potential. When  $-\lambda_0 < \lambda < 0$ , the level energies are close to the Coulomb levels and are shifted downward:

$$E \approx E_n + \frac{\lambda}{n^2} \left(\frac{R}{a}\right)^3. \quad (26)$$

After the rearrangement of the spectrum, the atomic levels are shifted upward:

$$E_{|n| \rightarrow \infty} = E_n + \frac{\lambda_0}{n^2} \left(\frac{R}{a}\right)^3, \quad (27)$$

and the previous 1S level is localized within the range of the potential  $V$  and is replaced by the previous 2S level, and so on (Fig. 3). Since there is a unique

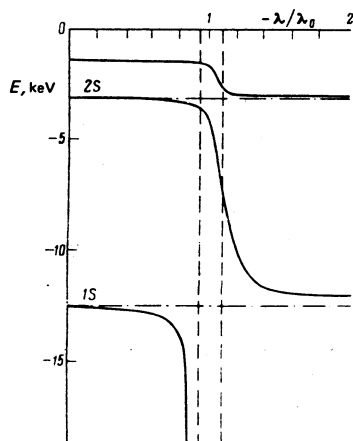


FIG. 3.

bound state in the separable potential for  $\lambda < -\lambda_0$ , the rearrangement of the spectrum will occur only once as  $\lambda$  increases.

## §5. ATOMIC LEVEL SHIFT IN THE CASE OF A SQUARE-WELL NUCLEAR POTENTIAL

We shall now consider the influence of short-range forces on the level spectrum of the  $(p\bar{p})$  atom for a more realistic model of the nuclear interaction in the  $p\bar{p}$  system. In particular, we shall assume that the nuclear potential can be represented by a rectangular potential well. Our aim will, therefore, be to obtain the bound-state spectrum for the potential

$$U(r) = \begin{cases} -V_0, & r \leq R \\ -e^2/r, & r > R \end{cases}. \quad (28)$$

The bound S-state wave function in the internal ( $r < R$ ) and external ( $r > R$ ) regions is

$$\Psi(r) = \begin{cases} A j_0([2m(V_0 + E)]^{1/2} r/\hbar), & r \leq R, \\ B \exp\{-r(-2mE)^{1/2}/\hbar\} G(1-n, 2, 2r(-2mE)^{1/2}/\hbar), & r > R, \end{cases}$$

where  $E$  is the required level energy and  $n = (-me^4/2\hbar^2 E)^{1/2}$ . The function  $G(\alpha, \beta, \rho)$  is a confluent hypergeometric function of the second kind which decreases exponentially (for  $E < 0$ ) as  $\rho \rightarrow \infty$  and is singular as  $\rho \rightarrow 0$ .

The condition of continuity of the logarithmic derivative of  $\Psi(r)$  at  $r = R$  is the equation for the eigenenergies  $E$ . This equation can be solved numerically. Figure 4 shows the results of the calculation for  $R = 1.2$  fm. It has been shown<sup>[1]</sup> that one quasinuclear S-state should appear in the nucleon-antinucleon potential (for a fixed spin and isospin) with a binding energy of at least 10 MeV. This corresponds to well depths  $V_0 > 90$  MeV. It follows from Fig. 4 that, for  $k_0 = 307$  MeV/sec ( $V_0 = 100$  MeV), the shift is  $\Delta E_{1S} = +2$  keV. The order of magnitude and the sign of this shift are in agreement with the work of Dal'karov and Samoilov.<sup>[4]</sup> At the same time, Fig. 4 clearly exhibits the presence and the nature of the "expulsion" of the 1S level of the  $(p\bar{p})$  atom by the attractive nuclear forces. As expected from the general theory (Sec. 2), the reason for this is

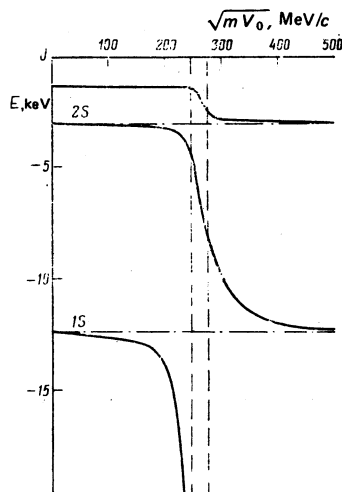


FIG. 4.

the presence of a bound quasinuclear state of the  $N\bar{N}$  system, i.e., precisely the fact that the nuclear attraction between  $p$  and  $\bar{p}$  is strong enough. There is also the striking fact that, as the nuclear attraction increases, the shift  $\Delta E_{1S}$  may fall and its sign may be reversed for a certain value of  $V_0$ . At the same time, the probability of a complete rearrangement of the Coulomb spectrum due to the presence of the nuclear forces becomes very small (but finite). For this to be so, the depth  $V_0$  of the nuclear well must lie in a very narrow interval of critical values. A deviation of  $V_0$  from the critical value  $V_0 = 36$  MeV by 2% restores the overall pattern of the atomic spectrum.

## §6. CONCLUSIONS

In conclusion, it may be useful to emphasize the general character of the phenomenon which we have discovered and of its leading features.

The restructuring of the spectrum of discrete levels due by forces whose range is small in comparison with the size of orbits executed in finite motion will always occur whenever the strength of the short-range attractive forces is in itself sufficient for the appearance of a bound state. The interval of critical values of the depth of the short-range potential for which the above spectrum rearrangement takes place is very narrow and is proportional to the ratio of the range of these forces to the radius of the finite-motion orbit in the main ("broad") potential well. The level shifts due to the short-range forces will therefore be small, independently of the strength of these forces, with the exception of a finite number of small intervals around the critical values. Nevertheless, the small shifts contain a "trace" of the rearrangement of the spectrum. This trace is the sign of the shift which should be positive at moderate distances from the critical region (i.e., it corresponds to the pushing out of levels despite the attractive nature of the short-range forces).

The above problem in classical nonrelativistic quantum mechanics could have been formulated and solved many years ago. The reason why this was not done was probably that, until quite recently, one did not encounter real physical objects whose properties could be understood only by solving this problem.

Such physical systems are now being investigated experimentally. They include, above all, the two classes of hadron atoms, namely, the  $K$ -mesic atom and, especially, the  $\bar{p}$  atom. The strong short-range forces in this case are the nuclear forces. Measurements of the atomic level shifts and of their signs can, as shown above, be a source of important information on the nuclear interaction between hadrons.

Other atomic systems in which an effect analogous to that discussed above may be important are ordinary atoms in which the nuclear charge is close to the critical value (so that the 1S level enters the lower continuum). Atoms of this kind have recently been considered by Popov, who showed, in particular, that the atomic spectrum could undergo a complete rear-

angement for certain critical values of the nuclear charge.<sup>[10]</sup> The characteristic small length in this case is the Compton length of the electron, i.e., the distance over which relativistic effects associated with the interaction between the electron and the external electrostatic field of the nucleus become important. The interval of critical values of the nuclear charge corresponding to this rearrangement of the spectrum is, however, proportional to the logarithm of the ratio of the Compton length to the Bohr radius and not to the ratio itself. This is most likely due to the relativistic character of the problem (in particular, the presence in the interaction of terms that are proportional to  $r^{-2}$  at the origin and are too singular). Nevertheless, the overall physical picture of the effect is undoubtedly identical with the phenomena investigated here.

In addition to the atoms discussed above, the quasinuclear  $B\bar{B}$  systems ( $B$  = baryon) are further physical objects for which two types of force of comparable strength but different range are important. The "long-range" forces in this case are the usual nuclear forces producing nonrelativistic bound states of the  $B\bar{B}$  system with orbit radii of the order of 1 fm. The short-range forces, on the other hand, are those due to the annihilation processes (the range of these forces is of the order of the Compton length of the baryon, i.e.,  $\approx 0.2$  fm). The fact that the width of the region in which the rearrangement of the spectrum due to the presence of short-range forces takes place is small is important for the physics of quasinuclear systems. In relation to the quasinuclear system, this fact implies that the probability of an appreciable change in the spectrum of quasinuclear levels due to the contribution of the annihilation interaction is small: the annihilation shifts and widths of such levels should, as a rule, be small.

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<sup>1</sup>The solution of the multichannel problem with level shifts due to the short-range annihilation forces will be given elsewhere.

<sup>2</sup>We recall that states with different orbital angular momenta correspond to different potentials in the separable approximation.

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