# Space-time and physical fields inside a black hole

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Physical fields and the perturbations of the space-time metric inside a slowly rotating and weakly charged black hole are investigated. It is shown that in the Schwarzschild coordinates $r$ and $t$ for $r<r_g$ the radiatable multipoles of scalar fields vary in the limit $t\to\infty$ in accordance with the asymptotic law $\Phi = D_1 t^{-2(l+1)} + D_2 t^{-(2l+3)} \ln r$. Here, $D_1$ and $D_2$ are constants and $l$ is the multipole order. Fields with nonzero spin (including the gravitational field) increase as $r\to 0$ in proportion to a power of $r$, and not $\ln r$, as in the case of a scalar field. Thus, at a fixed distance $r=\text{const}$ from the singularity, the physical fields and all radiatable perturbations of the metric are damped in a power-law fashion with respect to the coordinate $t$, which is the radial coordinate inside the black hole. Fields of external sources inside the black hole are also considered.

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## 1. INTRODUCTION

The aim of this paper is to investigate the nature of the spacetime and physical fields (scalar, electromagnetic, etc) inside a black hole a long time after the hole has been formed as measured by the clock of a freely falling frame of reference. We shall assume that the collapsing body which formed the black hole had small deviations from sphericity and rotated slowly at the time when it crossed the surface of the gravitational radius, and that all fields at that time were sufficiently weak for one to be able to ignore their back reaction on the metric.

The solution of such a problem for the fields outside a black hole is well known. It was shown for the first time in[1,2] that all deviations from sphericity in the exterior spacetime are damped, and all that remains are the spherical metric and the "Kerr rotational components" of the metric, which are determined by the total angular momentum of the body. In[3-5] it was also shown that there is no magnetic field in the exterior space. Finally, the complete theory of fields with integral spin and zero rest mass in the spacetime outside a slowly rotating black hole was constructed in [6-12]. These investigations established a law according to which the fields are damped with time in the exterior space a long time after the formation of the black hole. This analysis was extended in[13-16] to fields with half-integral spin as well.

The above investigations were followed by numerous others that considered the behavior of fields in the exterior space of charged and rapidly rotating black holes.

The properties of the spacetime inside black holes is of fundamental importance for the problem of gravitational collapse and the nature of the singularity, although these regions are not accessible to direct investigation by an external observer. It has sometimes been asserted that below the black-hole horizon all dynamic perturbations of the gravitational field increase and that they cannot escape in the form of waves to infinity, so that the properties of the spacetime and the behavior of the physical fields in this region are extremely complicated and all perturbations must grow as the singularity is approached. We shall see that these conclusions need to be reviewed.

In the present paper, we show the following.

Suppose that the deviations from spherical collapse at the time the surface of the body crosses the event horizon are small and all fields weak, so that their back reaction on the metric can be ignored. We denote by $\tau$ the proper time of freely falling particles, and by $r$ the length of a circle, divided by $2\pi$, around the singularity. It can then be shown that for large $\tau$ (i.e., a long time after the formation of the black hole) and for any fixed $r$ (i.e., at a fixed distance from the singularity) all perturbations of the Schwarzschild metric inside the black hole and all fields are damped in a power-law fashion with respect to $\tau$. The actual power depends on the multipole order of the perturbation (and also on the initial conditions of the collapse).

This conclusion about the damping does not apply to the perturbations of the metric and fields which are determined by conserved (without allowance for radiation) integral properties of the collapsing body—its mass, angular momentum, and electric charge.

A precise formulation of the assertion and its proof will be given below, in Sec. 3.

Thus, the interior regions of a black hole are characterized by damping of perturbations and fields and a tending of spacetime to a "stationary" state, just as the exterior regions are. But there are important differences.

First, inside the black hole the Schwarzschild coordinate $r$ plays the role of the time, and one would more correctly say of the interior regions that they tend to a state which depends only on $r$ rather than that they tend to a stationary state.

Second, and this is more important, as the singularity is approached ($r$ decreases) for fixed $\tau$ perturbations of scalar type increase unboundedly in accordance with the logarithmic law $\propto \ln r$, and other fields and the perturbations of the metric increase in the general case as a power of $r$. Therefore, near the singularity the linearized theory of small perturbations ceases to be valid. The solution of the exact equations for the metric must be found from the general solution[17] near the singularity. However, it should be emphasized that the region in which the method of small perturbations is inapplicable contracts toward the singularity, becoming smaller and smaller with increasing $\tau$ (see Fig. 1). This region lies in the absolute future with respect to the regions with larger $r$ (further from the singularity) and naturally cannot influence them at all.

Price[7] has succinctly characterized the damping of all exterior fields of a black hole by the expression: "Anything that can be radiated will be radiated". For the interior regions of the black hole one can say: "Anything that can fall will fall (will be radiated into the singularity)."

Note that we consider the nonquantum problem, i.e., we take into account neither the Hawking evaporation of the black hole nor quantum processes near the true singularity. We shall say something about the quantum processes in the Conclusions. Finally, note the following. For both the exterior and the interior regions of a black hole, the conclusion about damping of radiatable nonspherical perturbations and fields applies only to fields generated by sources on the collapsing body. If a black hole is placed, for example, in the quadrupole gravitational field of external bodies, this field is not of course damped either outside or inside the black hole. We shall discuss perturbations of this kind in §5.

## 2. PROPAGATION OF RADIATION FIELDS INSIDE A BLACK HOLE

We shall consider weak fields and small perturbations of the metric on the background of the Schwarzschild metric near a black hole.[18] To do this, we shall use the mathematical formalism developed in [6–9]. Before we turn to the mathematical calculations, let us describe the general picture of the propagation of field variations in Schwarzschild spacetime. The general situation is

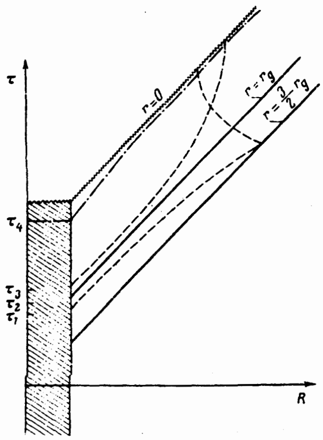


FIG. 1. Propagation of fields in the Schwarzschild spacetime outside and inside a black hole. The dashed curves represent null geodesics.

shown in Fig. 1. Inside the collapsing body (hatched region) a comoving coordinate system is used ($R$ is the comoving radial coordinate and $\tau$ is the proper time); outside the body, the coordinate system is extended by the Lemaître system[19] of freely falling test particles, whose proper time is used.

To be specific, we shall talk in this section about corrections $\delta g$ to the gravitational field[1)] associated with deviations from sphericity in the matter distribution $\delta\rho$ of the collapsing body. These deviations $\delta\rho$ increase during the collapse. They are still small when the surface of the body crosses $r_g$, at the time $\tau=\tau_2$ (see[2]), and become very large, destroying the picture of spherical collapse near the true singularity at $r=0$.

The mathematical analysis of Price *et al.*[6–9] refers to the change in the field outside the gravitational radius in the region to the right and below the line $r=r_g$. We shall make our analysis for the region inside the black hole, i.e., to the left and above the line $r=r_g$. We shall use the results of [6–9] for $r>r_g$.

During the contraction of the body, the increasing $\delta\rho$ generate a change in $\delta g$—they generate gravitational waves. As Price's analysis showed, these waves emitted by the body near (but somewhat earlier than) the moment $\tau_2$ (the crossing of $r_g$ by the surface) are reflected by the "potential barrier" in the region $r=3r_g/2$ and propagate inward toward the horizon $r=r_g$ (see Fig. 1). The closer the time of emission to the time $\tau_2$, the more complete is the reflection. The interference between the direct waves propagating to the right and the reflected waves has the consequence that outside $r_g$ the perturbations $\delta g$ are damped as one moves upward to the right (to large Schwarzschild $t$).

We shall also investigate $\delta g$ as one moves upward to the right, but for $r<r_g$. This region is reached by waves that leave the body after $\tau_2$, but very soon after this time (for example, at $\tau_3$ in Fig. 1), and also waves reflected from the potential barrier $r=3r_g/2$ that left the body just before $\tau_2$ (for example, at $\tau_1$). The interference between these waves determines $\delta g$ in the region $r<r_g$ as $\tau\to\infty$. This region is not reached by signals from the strong perturbations which develop in the body during the collapse near $r=0$. This is why we can use the method of small perturbations to solve our problem. Of course, in the region $\tau\to\infty, r<r_g$, the waves do grow as they approach the singularity $r=0$; for example, the energy density of the waves grows. However, as we shall show in §3, this growth near $r=0$ does not rule out applicability of our method since the curvature of spacetime also increases. The region of spacetime in which the deviations from sphericity are important is shown in Fig. 1 by the chain line. Inside the collapsing body, this region is determined by the time $\tau_4$ at which the increasing perturbations are no longer small, and outside the body this region is determined by Eq. (19) as $\tau\to\infty$.

Our task is to solve the equations for the fields in the region $r<r_g$ as $\tau\to\infty$, using as boundary conditions the conditions for $r=r_g$ obtained in [6,7] from the solution of the problem for $r>r_g$ and the conditions on the surface of the collapsing body at times in the neighborhood of $\tau_2$.

## 3. CALCULATION OF THE VARIATION OF RADIATION FIELDS INSIDE A BLACK HOLE

Here, we shall consider only radiatable multiples of the fields (i.e., $l \geqslant S$, where $l$ is the multipole order and $S$ is the spin of the field). It was shown in[6-10] that the properties of such modes for all fields are very similar and described by equations that differ in minor details. The same is true of the interior region. Therefore, we shall consider here the case of a scalar field. The generalization to other fields is made as in[6-10].

Thus, we shall consider the scalar field of massless particles:

$$\Phi_{;\alpha}^{;\alpha}=0. \tag{1}$$

We consider this field $\Phi$ outside the matter of a collapsing star on the background of the Schwarzschild metric inside the Schwarzschild sphere. As is shown in [15], one can here use the standard Schwarzschild coordinate system, except that $r$ is now the time coordinate and $t$ the spatial coordinate ($c=1, 2GM=1$):

$$ds^2=(1/r-1)^{-1}dr^2-(1/r-1)dt^2-r^2(d\theta^2+\sin^2\theta d\varphi^2), \tag{2}$$

$0<r<1$. The "time" $r$ flows in the direction of decreasing $r$, from 1 to 0.[2)] We expand $\Phi$ in scalar spherical harmonics with respect to the coordinate $r$ as a radial coordinate. That physically it plays the role of the time is for us immaterial. We write

$$\Phi=\sum_l \frac{1}{r}\Psi_l(t,r)Y_{lm}(\theta,\varphi). \tag{3}$$

We introduce the "tortoise coordinate"[6]:

$$x=-r-\ln(1-r),\quad 0<x<\infty. \tag{4}$$

Equation (1) then reduces to the following equation for $\Psi_l$ (in what follows, we omit the subscript $l$):

$$\frac{\partial^2\Psi}{\partial t^2}-\frac{\partial^2\Psi}{\partial x^2}=-\left(1-\frac{1}{r}\right)\left(\frac{1}{r^3}+\frac{l(l+1)}{r^2}\right)\Psi. \tag{5}$$

We shall solve this one-dimensional equation, whose coefficients do not depend on $t$, by the ordinary Fourier expansion method. The equation for the Fourier transform $\Psi_k$ is

$$\frac{d^2\Psi_k}{dx^2}=\left[-k^2+\left(1-\frac{1}{r}\right)\left(\frac{1}{r^3}+\frac{l(l+1)}{r^2}\right)\right]\Psi_k. \tag{6}$$

(We recall that here and below $r=r(x)$; see (4).)

We consider the lowest radiatable mode $l=0$ of the scalar field.

We denote by $\Psi_{[0]}$ the $t$-independent solution of (5) and the corresponding solution of (6) for $k=0$. This function has the form

$$\Psi_{[0]}=A_1 r\ln\frac{r}{e(1-r)}+A_2 r, \tag{7}$$

where $A_1$ and $A_2$ are arbitrary constants, and the $e$ is introduced into the denominator of the logarithm for convenience.

Before we go further, we note that the boundary of the collapsing body in the frame of reference [2] moves with a velocity that tends to the velocity of light when the boundary crosses $r_g$ (corresponds to the "time" $r=1$). The boundary is moving in the opposite direction to the propagation of the waves. Therefore, all waves have a large Doppler "red shift" as $r\to 1$ and the wavelengths tend to infinity (recall however that $r$ is the time coordinate!). It is these waves that reach the region $\tau\to\infty, r<r_g$. We shall therefore be interested in solutions of Eq. (6) for small $k$.

Bearing in mind what we have said, we shall seek the general solution of (6) in the form of the series[3)] (everywhere, as before, we assume $l=0$)

$$\Psi_k=\Psi_{[0]}+k^2\Psi_{[2]}+k^4\Psi_{[4]}+\dots . \tag{8}$$

Using (6), we can readily show that the following recursion relation holds for finding all the $\Psi_{2n}$:

$$\Psi_{2(n+1)}=\int_0^x\frac{y}{1-y}\Psi_{[2n]}(y)\ln\frac{y}{e(1-y)}dy-\ln\frac{r}{e(1-r)}\int_0^x\frac{y^2}{1-y}\Psi_{[2n]}(y)dy. \tag{9}$$

The general solution of (6) as $x\to\infty (kx\gg 1)$ has the form

$$\begin{aligned}\Psi_k&=C_1k^{-1}\sin kx+C_2\cos kx+O(e^{-x})\\&=C_1x(1-(kx)^2/6+\dots)+C_2(1-(kx)^2/2+\dots)+O(e^{-x}).\end{aligned} \tag{10}$$

Comparing now the series (10) with the series (8) and using (7) and (9), we find expressions for the coefficients $C_1$ and $C_2$ in the form of the series

$$\begin{aligned}C_1&=A_1(1+\alpha_2k^2+\mu_4k^4+\dots)+A_2(\gamma_2k^2+\delta_4k^4+\dots),\\C_2&=A_1(\beta_2k^2+\sigma_4k^4+\dots)+A_2(1+\delta_2k^2+\omega_4k^4+\dots),\end{aligned} \tag{11}$$

where the Greek letters denote constants that arise as a result of the integration (9). We require the expressions for $A_1$ and $A_2$ obtained from (11):

$$A_1=C_1-k^2(\alpha_2C_1+\gamma_2C_2)+\dots,\quad A_2=C_2-k^2(\beta_2C_1+\delta_2C_2)+\dots \tag{12}$$

We shall not particularize the values of the constants $\alpha, \beta, \gamma, \delta$ since all that is important for us is to establish that the differences between the constants $A$ and $C$ are small for small $k$ and that $A$ and $C$ are equal in the principal terms in $k$. Allowance for the following terms (including those $O(e^{-x})$) does not change this result. We now use the boundary conditions.

In accordance with the results of [6,9], as the gravitational radius is approached, $r\to r_g+0$, the solution for $\Psi_k$ has the form

$$\Psi_k=f_k[e^{ikx}-e^{-ikx}+B_ke^{-ikx}], \tag{13}$$

where $f_k$ is a constant which depends on the amplitude of the perturbation, and $B_k$ is the "coefficient of reflection" from the "potential barrier," and it is determined solely by the form of the second term in the square brackets in (6). As $k\to 0, B\to 0$. By continuity, the conditions (13) must also be satisfied as $r\to r_g-0$. Expressing now $C_1$ and $C_2$ in terms of $f_k, B_k$, and $k$:

$$C_1=2ik-ikB,\quad C_2=B, \tag{14}$$

and substituting the resulting expressions in (12), we find

$$\begin{aligned}A_1&=2if_kk\left\{1-\frac{B}{2}-k^2\left[\left(1-\frac{B}{2}\right)\alpha_2+\frac{B}{2ik}\gamma_2\right]+\dots\right\},\\A_2&=f_k\{B-k^2[2ik\beta_2-ikB\beta_2+\delta_2B]+\dots\}.\end{aligned} \tag{15}$$

The relations (7), (8), (14), and (15) enable us to conclude that as we advance into the region $\tau \to \infty$, where the decisive waves are those emitted ever nearer the time $\tau_2$ with, therefore, ever smaller $k \to 0$, the coefficients $C_1$ and $C_2$, which determine $\Psi_k$ as $x \to \infty$, and $A_1$ and $A_2$, which determine $\Psi_k$ as $x \to 0$, tend to zero, and the complete solution is damped.

Going over from the Fourier transform $\Psi_k$ to the function itself, as in [9], and then to $\Phi$, we find that $\Phi$ in order of magnitude must have the form (for $t \to \infty$)

$$\Phi \approx D_1 t^{-2} + D_2 t^{-3} \ln r. \tag{16}$$

Similar arguments apply to the case of $\underline{1} \neq 0$ multipoles. The result is analogous except that the power of $t$ in the damping of the multipoles depends on $\underline{1}$:

$$\Phi_l \approx D_1 t^{-2(l+1)} + D_2 t^{-(2l+3)} \ln r. \tag{17}$$

Thus, for any fixed $r$ and large $t$ the field is damped in a power-law fashion with respect to $t$. We recall that $t$ is here the spatial coordinate. On the other hand, for any fixed $t$ as $r \to 0$ the second term in (16) and (17) gives a logarithmic divergence at the true singularity.

To determine the back reaction of the field on the metric, it is necessary to compute the energy—momentum tensor of the field $\Phi$ and substitute it in the field equations. The calculation shows that as the true singularity $r=0$ is approached the tensor components $T_0^0$ and $T_1^1$ increase unboundedly:

$$T_0^0 \sim t^{-m} r^{-3} \quad (\text{as } r \to 0,\ t \to \infty); \tag{18}$$

here, $m$ depends on the multipole order. However, the individual terms in the tensor components $R_0^0$ and $R_1^1$ increase as $r \to 0$ only in proportion to $r^{-3}$. Therefore, for sufficiently large $t$ the components $T_i^k$ are always small compared with the individual terms in $R_i^k$ and so have little influence on the metric as $r \to 0$. Of course, allowance for the nonlinearity (self-gravitation) must change the result as $r \to 0$.

On the other hand, to calculate the behavior of fields with $S \neq 0$, including perturbations of the metric, it is necessary to make additional calculations with $\Phi$ (see[9]), which leads to a power-law (and not logarithmic) growth of the perturbations as $r \to 0$. The corrections to the metric become of order unity for

$$r^{-n} \approx |D_2^{-1}| t^{2l+3}. \tag{19}$$

Finally, we note that the coordinate $\tau$ can be expressed in terms of $t$ and $r$ in the form

$$\tau = t + f(r). \tag{20}$$

For fixed $r$ and $t \to \infty$,

$$\tau \approx t. \tag{21}$$

*Therefore,* the law of damping of $\Phi$ with respect to $\tau$ and $t$ as $t$, $\tau \to \infty$ and fixed $r$ has the same form.

## 4. NONRADIATABLE FIELDS ASSOCIATED WITH THE COLLAPSING BODY

We comment here briefly on the nonradiatable multipoles of the source. For the electric field we have, for example, the multipole $\underline{1}=0$ (the field of the electric charge of the collapsing body) and for the gravitational field $\underline{1}=0$ (the mass) and $\underline{1}=1$ (the field of the total angular momentum of the body).[4)] If the corrections to the metric associated with these fields are small on $r=r_g$ (small charge, slow rotation), then these fields can be continued in a known manner into the black hole (Reissner–Nordström and Kerr metrics) and they do not change at all with respect to $t$ and grow with respect to $r$. Near the singularity, they rearrange the metric.

Both types of field lead to the so-called future horizons (inner horizons) within the black hole. Such horizons are unstable and, probably, a true singularity arises on them because of perturbations.[20]

Thus, the fields of the nonradiatable multipoles in our problem must be regarded as additive terms until, at small $r$, they become large, modify the solution, and, probably, lead here to the occurrence of singularities.

## 5. FIELDS OF EXTERNAL SOURCES INSIDE THE BLACK HOLE

Hitherto, we have considered an isolated black hole, assuming that there is no influence of external bodies on it.

An external influence on a black hole can be of two kinds. First, there may be stationary fields of surrounding bodies. An example is the quadrupole gravitational field of external bodies. Second, radiation or matter can fall into a black hole, and this will also influence its internal structure.

Let us consider first the stationary fields of external sources.[5)] These fields penetrate through the gravitational radius into the black hole. We shall assume that they are weak on the radius $r=r_g$. Inside the black hole, as outside, they do not depend on the coordinate. As examples, let us consider the quadrupole gravitational field of external sources and a magnetic field which is homogeneous far from the black hole.

We begin with the gravitational field. In[1] it is shown that the exact expression for the metric of a black hole placed in an external quadrupole field has the form

$$ds^2 = \frac{\lambda-1}{\lambda+1} \exp\left[\frac{1}{2} q (3\lambda^2-1)(3\mu^2-1)\right] dt^2 - m^2(\lambda+1)^2$$
$$\times \exp\left\{-q\left[6\lambda(1-\mu^2) + \frac{9}{8} q (\lambda^2-1)(1-\mu^2)(9\mu^2\lambda^2-\lambda^2-\mu^2+1)\right]\right.$$
$$\left. + \frac{1}{2}(3\lambda^2-1)(3\mu^2-1)\right\}\left[\frac{d\lambda^2}{\lambda^2-1} + \frac{d\mu^2}{1-\mu^2}\right]$$
$$-m^2(\lambda+1)^2(1-\mu^2)\exp\left\{-\frac{1}{2} q (3\lambda^2-1)(3\mu^2-1)\right\} d\varphi^2. \tag{22}$$

Here, $m$ is the mass of the black hole ($q$ is the parameter which characterizes the quadrupole moment, $c=1$, $G=1$). If $q=0$, the field is spherical. The transition to the ordinary Schwarzschild coordinates is made by the substitution

$$\lambda = r/m - 1, \quad \mu = \cos\theta. \tag{23}$$

In the expression (22), all corrections associated with the external quadrupole field occur in the argument of an exponential. For small $q$, these corrections are small.

The metric (22) also applies inside the black hole for $(\lambda-1)/(\lambda+1)<0$. Here, the spatial coordinate is $t$ and the time coordinate $\lambda$. This metric describes the quadrupole field inside the black hole of external sources. It can be seen from the expression (22) that if the quadrupole corrections are small on the gravitational radius they remain small as the singularity is approached, $\lambda \to -1$.

We now consider a magnetic field. The solution for a magnetic field which is homogeneous at infinity is given in[4]. In this solution, the following components of the electromagnetic field tensor are nonzero:

$$F^{\theta\varphi}=\frac{B_0 \cot\theta}{r^2} \qquad F^{\varphi r}=-\frac{B_0}{r}\left(1-\frac{1}{r}\right). \tag{24}$$

This solution can be continued into the black hole. It is described in the frame of reference (2) by the same Eqs. (24). Inside the black hole, $r$ is the time coordinate, and in the frame of reference (2) there is, in addition to the radial magnetic field, an electric field with respect to the coordinate $\varphi$. Note that the components of $F^{ik}$ are variable with respect to the "time" $r$ and constant with respect to the radial coordinate $t$.

These electric and magnetic fields can be continued to the singularity $r=0$. The field energy density increases unboundedly as $r \to 0$.

We now make a remark about the perturbations made by matter falling into the black hole. Note that the singularity $r=0$ in the metric (2) is formally a Kasner singularity[21] with axial symmetry. For an investigation of perturbations near such a singularity in the general case, see, for example,[21,22]. We shall not consider them here in detail. It is only important to emphasize that, like the perturbations of the metric due to inhomogeneities of the collapsing body, and for the same reason, all perturbations of the metric due to matter falling into the hole are damped as $t \to \infty, r=\text{const}<1$ in the manner described in §3.

## 6. CONCLUSIONS

The main result of our analysis is the establishment of the rapid damping of the radiatable modes of all fields inside the black hole, just as happens in the exterior region. Generalizing Wheeler's well-known dictum: "A black hole has no hair," we can say: "A black hole has hair neither outside nor inside."

Let us make some remarks about the role of quantum processes. First, the quantum evaporation processes of black holes for holes with $m \gg m_{pl} \approx 10^{-5} g$ have characteristic times $T_{evap}$ much greater than the characteristic time $T_{bh}=r_g/c$ of the black hole:

$$T_{evap} \gg T_{bh}.$$

Therefore, for $t$ in the range

$$T_{bh} \ll t \ll T_{evap}$$

the laws we have obtained above inside the black hole are valid. For $t \approx T_{evap}$, of course, it is necessary to take into account the change in the metric due to the quantum processes in the entire region $0<r<1$.

On the other hand, as the singularity is approached the quantum processes grow (see[23]) and for $r \approx r_{pl} \approx 10^{-33}$ cm become decisive. In §3 we have seen that the perturbations of the metric associated with the growth of the radiatable modes become large when

$$\varphi(r_1) \approx |D_2^{-1}|\, t^{2l+3}, \tag{25}$$

where $\varphi(r)=r^{-n}$, $\ln r$.

If in (25) we replace $r_1$ by $r_{pl}$, we thereby determine a $t_{cr}$ such that for $t>t_{cr}$ for all $r>r_{pl}$ the metric is described by the expression (2), and there are no deviations from sphericity outside the quantum regions.

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1) The behavior of other fields is analogous, see §3.

2) The boundary of the matter of the collapsing star is described in the metric (2) by the equation

$$t=-\int \frac{r^{1/2}dr}{1-1/r},$$

and the constant is arbitrary (the pressure of the matter in the star can be ignored). The metric (2) applies only outside the matter of the star. It is assumed that at small $r$ the matter is compressed nearly parabolically.

3) The idea of seeking a solution of (6) in such a form is due to V. N. Lukash, to whom we are very grateful.

4) Of course, the spherical gravitational field ($l=0$) itself is not radiated; it is assumed that the coordinate origin is at the center of mass of the body, so that there is therefore no trivial dipole moment.

5) The stationarity condition means that the characteristic time of variation of the field is $\tau \gg r_g/c$.


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# Nuclear excitation during positron annihilation in the *K* shell of heavy atoms

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The excitation of a nucleus by a positron beam during the annihilation of the positrons with atomic electrons is discussed. Results are reported of a calculation of the excitation cross section for $^{115}$In and $^{235}$U. The calculations were performed in the transition current and charge scheme, using wave functions obtained by the relativistic Hartree-Fock-Slater method through a numerical integration of the Dirac equation. The Weisskopf single-particle nuclear transition matrix elements were used for $^{115}$In to estimate the cross section for the nuclear $E1$ transition induced by a monochromatic positron beam. The cross section at resonance is found to be $\sigma_{res}(E1) \sim 10^{-25}$ cm$^2$. A similar calculation for $^{235}$U yielded $\sigma_{res}(E1) \sim 5\times 10^{-26}$ cm$^2$. More accurate cross sections have been obtained for particular levels on the basis of existing experimental data on the nuclear-level spectrum.

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## INTRODUCTION

The annihilation of positrons with atomic electrons is one of the possible processes during the scattering of a positron beam by atoms. The annihilation process can be accompanied by the emission of one or more photons, or by the excitation of the nucleus. In this paper we report the results of an analysis of the cross section for nuclear excitation by a positron beam during the annihilation of positrons with atomic electrons.

If an atom intercepts a positron beam of energy $E_+$ and energy spread $\Delta E$, annihilation between a positron and an atomic electron with quantum numbers $nlj$ ($n$ is the principal quantum number and $j$ and $l$ are the resultant and orbital angular momenta of the electron) may be accompanied by resonant excitation of nuclear states with energies in the interval $\Delta E$ around $E_f = E_+ + E_{nlj}$ ($E_+$ and $E_{nlj}$ are, respectively, the total relativistic energies of the positron and the electron). It follows that, if a sufficiently narrow positron beam is available ($\Delta E$ less than the separation between the nuclear levels), one can scan the nuclear spectrum by varying the energy of the incident positrons. One would expect that the main contribution to the cross section would be that due to $K$-shell electrons. Our calculations have shown that, for the $L_I$ shell, the cross section is already smaller than the $K$-shell cross section by roughly an order of magnitude. We shall therefore confine our attention to $K$-shell electrons, i.e., we shall assume that only nuclear levels with energies $E_f = E_+ + E_{1s}$ are excited.

The above problem has already been treated theoretically by Present and Chen,[1] and the nuclear excitation cross section for positron annihilation was calculated in the Born approximation without taking into account the finite width of the $K$-shell hole. A more accurate theoretical analysis has become necessary following the work of Mukoyama and Shimizu,[2] who reported an experimental attempt to determine the cross section for the positron excitation of $^{115}$In, and who concluded that the cross section was greater by two orders of magnitude as compared with the theoretical prediction.[1]

In this paper, the nuclear excitation cross section will be calculated within the framework of the well-known transition current and charge scheme.[3] The wave functions of the electron and incident positron will be taken to be the solution of the Dirac equation for the average atomic field deduced by the Hartree-Fock-Slater method.

The cross sections have been calculated for $^{115}$In and $^{235}$U. The choice of $^{115}$In was dictated by the fact that the first estimates of the cross sections were made[2] for this nucleus, whereas $^{235}$U is an example of a heavy fissile nucleus for which it is interesting to investigate excitation during positron annihilation as a possible way of investigating nuclear fission in the subbarrier region. In both cases ($^{115}$In and $^{235}$U), there are modern experimental data on the nuclear excitation spectrum at energies $E_f$ below ~1.5 MeV. In this energy range, the excitation cross section can be calculated for individual levels by using experimental reduced nuclear transition probabilities or some accepted model. For example, the Nilsson model was used to identify the $^{235}$U levels. In the region of nuclear excitations where there are no

  