$\propto q$ and the damping length at a frequency of 1 GHz is equal to 1 cm, then at the frequency of 20 GHz needed to observe acoustic breakdown the damping length is of the order of 5×10^{-2} cm. The nonlinearity parameter $U\tau/\hbar$ may reach several hundreds. There is therefore reason to suppose that the phenomena we have examined may be observed experimentally in samples of reasonable dimensions. Let us note that in the case where the acoustic momentum $\hbar q$ is appreciably smaller than $2p_0$, it is not possible to observe acoustic breakdown in the literal sense of the word. It is, however, possible to observe the sound-stimulated magnetic breakdown phenomenon which consists in a reduction of the breakdown field under the action of a sound wave. This situation has the advantage from the experimental point of view that it requires sound of lower frequencies which undergo less damping.

1) We shall adhere to the following terminology. Allowed bands which arise as a result of the periodic field of the crystal lattice we shall term energy bands. Under the influence of the periodic field of a sound wave they break up into allowed and forbidden acoustic bands.

2) In the present work we shall not touch on the change in the electron spectrum in the field of a standing sound wave: this problem is more complicated than ours because it cannot be reduced to a stationary one by any transformation of coordinates.

3) We recall that the electron spectrum is periodic in **p**-space, with a period $\hbar\mathbf{q}$, and it is not therefore necessary to consider the two distant trajectories: their existence is a consequence of this periodicity.

4) This estimate is only applicable for the so called interband magnetic breakdown.[8] Only the latter is realized in our situation.

5) We have put the word "thermodynamic" in quotation marks because the state of a metal containing a traveling sound wave is not, strictly speaking, an equilibrium one.

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Surface impedance of cadmium in a magnetic field

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The behavior of the surface impedance of cadmium is investigated theoretically and experimentally in the range of Doppler-shifted cyclotron resonance. It is found that the reactance of the metal has minima in the neighborhood of the thresholds of various dopplerons. The experimental data agree with the theoretical results for the case of diffuse reflection of the electrons by the surface.

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Doppler-shifted cyclotron resonance (DSCR) leads to the appearance of singularities in the variation of the surface impedance of a metal with the value of the applied magnetic field **H**. The behavior of the surface resistance R (the real part of the impedance) of cadmium, for $\mathbf{H}\parallel[0001]$, has already been investigated.[1,2] A kink in the function $R(H)$ was observed experimentally in the neighborhood of a doppleron threshold. Theoretical investigation showed that the function $R(H)$ should in fact experience a kink for diffuse reflection of the electrons by the surface. Unfortunately, in the experiments cited[1,2] the imaginary part X of the impedance (the reactance) was not studied; and in regard to the resistance, the measurements gave only the functional dependence $R(H)-R(0)$ on an unknown scale.

In the present paper, the variation of the surface impedance of cadmium with magnetic field is investigated theoretically and experimentally. The experimental method used made possible measurement of the absolute changes both of the real and of the imaginary parts of the impedance. It was found that, in contrast to the resistance, the variation of the reactance with field has a nonmonotonic character: there are minima on the $X(H)$

curve. The analysis carried out showed that each of these minima corresponds to the threshold of a definite doppleron.

EXPERIMENT

The measurements of the surface impedance were made with circular polarizations of the radiofrequency field.[1] The imaginary part of the impedance was studied by means of an autodyne detector whose sensitivity was constant with respect to change of its oscillation frequency in a magnetic field.[3] Automatic maintenance of a prescribed value of the Q of the oscillatory circuit eliminated the influence of the change of surface resistance of the specimen in the magnetic field on the characteristic frequency of the circuit. For stabilization of the Q, instead of a photoresistor,[3] a thermoresistor was used, connected in series with the inductance. The thermoresistor was an incandescent electric lamp, type MN 0.22–6.3 B, which was the load of a feedback amplifier. Change of the output voltage of the amplifier over the range from 0.2 to 6.3 V led to a change of the resistance of the lamp from 2 to 28 Ω. Replacement of the photoresistor by the lamp eliminated shift of the generator frequency caused by change of the reactive component of the photoresistor impedance, which depended on its illumination. In the method used, the shift of the autodyne frequency was determined with good accuracy by the change of the specimen reactance alone. The function $X(H)$ and the derivatives dX/dH and d^2X/dH^2 were recorded by the traditional method. For this purpose, the signal from the autodyne was mixed with the alternating voltage of a reference generator, and the difference-frequency signal was fed to the input of a frequency converter, type ICh-6.

The surface resistance R of the cadmium in a magnetic field was measured by means of an amplitude bridge. A simplified functional diagram of it is shown in Fig. 1. The specimen under study was placed in the inductance coil of a parallel LC circuit. The resistance r connected in series with the inductance describes the value of all the losses in the circuit. The oscillatory circuit was connected through the resistance $\mathfrak{R}$ and the wide-band amplifier 2 to the output of the generator 1. The voltage U_0 on the circuit was maintained constant. This was insured by introduction of a constant-current negative-feedback circuit, which controlled the transfer constant of the amplifier 2. The circuit includes the detector 3 and the constant-current differential amplifier 4, at

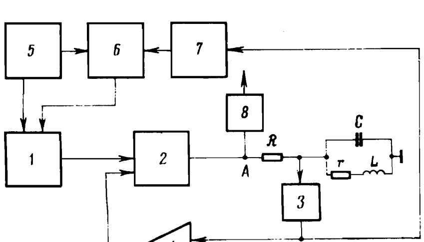

FIG. 1. Block diagram of the bridge for measurement of the surface resistance (for notation, see text).

whose second input the reference voltage enters. Adjustment of the transfer constant of amplifier 2 is made with one of the control grids of its input tube.

We shall suppose that at all values of the magnetic field, the frequency of the generator 1 coincides with the resonance frequency $\omega_r(H)$ of the circuit. Then it can be shown that the voltage U_A at the point A is related to the resistance $r(H)$ by the relation

$$U_A=U_0\frac{\mathfrak{R}}{[\omega_r(0)L]^2}\left[1-2\frac{\Delta\omega_r}{\omega_r(0)}\right]r(H), \tag{1}$$

where $\Delta\omega_r=\omega_r(H)-\omega_r(0)$. The factor in square brackets in (1) is only slightly dependent on the magnetic field, so that the change of the voltage U_A is with good accuracy proportional to the change of the surface resistance of the specimen.

For fine adjustment of the frequency of the generator 1 to the resonance frequency of the circuit, an additional negative-feedback circuit was used. For this purpose, the output signal of the generator 1 was frequency-modulated by means of the generator 5. The modulation was done at frequency 1000 Hz. In consequence of the frequency modulation and of the selective properties of the LC circuit, the output voltage of the detector 3 is amplitude-modulated. The amplitude of the modulation is proportional to the frequency derivative of the amplitude-frequency characteristic of the circuit. The signal produced by the difference between the characteristic frequencies of the circuit and of the generator 1 was separated out by the selective amplifier 7 and the synchronous detector 6. After power amplification, a controlling constant voltage was fed to a solenoid that biassed the ferrite core of the coil of the oscillatory circuit of the generator 1. As a result, its frequency was tuned to the resonance frequency of the circuit containing the specimen.

The voltage from the detector 8, which is proportional to the loss resistance in the circuit, was fed through a constant-current amplifier to the Y coordinate of an XY potentiometer. The voltage from a magnetic-field detector entered the X coordinate. Because of the fast action of the system for control of the transfer coefficient of amplifier 2, it was possible, simultaneously with the records of $R(H)$, to record the derivatives of the surface resistance with respect to magnetic field. In contrast to the traditional schemes, the amplitude-bridge scheme used makes it possible to separate out, with good accuracy, a signal due to the change of the surface resistance of the specimen alone, even when $\Delta r > r(0)$.

The cadmium specimens, with resistance ratio $\rho_{300}/\rho_{4.2}=3\cdot 10^4$, were monocrystalline plates of dimensions $10\times 3\times 0.57$ and $10\times 3\times 0.6$ mm. The hexagonal axis of the crystal was oriented along the normal to the plate surface.

Recorded in the experiment were the resistance $R(H)$ and the derivatives dR/dH and d^2R/dH^2, and also the reactance $X(H)$ and the derivatives dX/dH and d^2X/dH^2. Measurements were made in the magnetic field of an electromagnet, of intensity up to 18 kOe, over the frequency range 0.05–1.0 MHz and the temperature range

1.6–4.2 K. The magnetic field was directed along the hexagonal axis of the crystal.

The scale of the measurements of the real and imaginary parts of the surface impedance of cadmium was determined by an indirect calibration, which was carried out in a linearly polarized electromagnetic field at $T = 4.2$ K and $f = 1$ MHz. The principle of the calibration was replacement of the specimen under investigation by a specimen of the same geometry with a known value of the surface impedance. The change of specimens did not lead to a perceptible change in the distribution of the electromagnetic field in the measurement coil.

The calibration specimen was made from lead, with resistance $\rho_{10\,K} = 2.2 \cdot 10^{-7}$ Ω-cm, which practically coincides with the residual. The magnetoresistance of the specimen in a field $H < 1$ kOe was negligibly small. The depth of penetration of the electromagnetic field in the calibration specimen at frequency 1 MHz is $\delta_{Pb} = 2.3 \cdot 10^{-3}$ cm, which exceeds by almost two orders of magnitude the length of the free path of the electrons, calculated from the known value of the product $\rho l \approx 10^{-11}$ Ω cm^2 for lead. This means that for $f = 1$ MHz, the surface impedance of the calibration specimen can be calculated by the formula for the normal skin effect,

$$Z = (2\pi\omega/c^2\sigma)^{1/2}(1-i), \quad \omega = 2\pi f, \quad \sigma = \rho^{-1}. \tag{2}$$

The depth of penetration of the field for the cadmium specimen at $H = 0$ and $T = 4.2$ K has the value $\delta_{Cd} \approx 0.6 \cdot 10^{-3}$ cm.

The inductance coil used for calibration was of rectangular cross section and was wound on a complete form of plastic. The cadmium and lead specimens were fastened to a dielectric plate at distance 50 mm from each other. The plate could be moved freely within the coil by means of a rod that extended into the warm area through a gasket in the cover of the cryostat. First the calibration specimen was placed inside the inductance coil. In zero magnetic field, when the lead is in the superconducting state, its impedance may be taken to be zero. The change of surface impedance of the lead during the superconducting transition in a magnetic field parallel to the surface enables us to calibrate the scale of the record along the axis of ordinates on the *XY* potentiometer. After this, by moving the rod, the cadmium specimen was placed in the inductance coil, and $R(H)$ and $X(H)$ were recorded in a magnetic field perpendicular to the surface of the specimen. The final calibration was made with allowance for the dependence of the impedance of the coil on the magnetic field in the absence of the specimens.

The calibration obtained was used to determine the scale of the records of $R(H)$ and $X(H)$ made on the same specimen at $T = 4.2$ K and $f = 1$ MHz. With use of the autodyne detector with fixed Q and of the amplitude bridge with automatic frequency control, the relative error of the calibration was primarily determined by the difference of dimensions of the calibration specimen and of the one under study. The total error of determination of the scale amounted to about 10%.

Examples of records of the variation of the surface

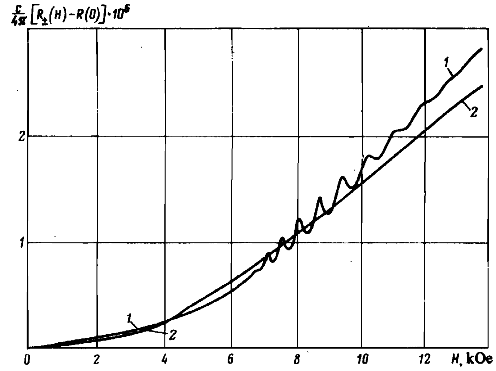

FIG. 2. Variation of the resistance $R_\pm$ (Guassian units) of cadmium with magnetic field: $f = 1$ MHz, $T = 1.6$ K, specimen thickness $d = 0.6$ mm. Curve 1 corresponds to minus polarization, 2 to plus polarization.

resistance of a plate with magnetic field intensity are given in Fig. 2. Along the axis of ordinates is plotted the change of resistance $R(H) - R(0)$ in a magnetic field. The absolute value of the resistance in zero magnetic field was not measured because of its small value. The value of the impedance of a thick plate of cadmium at $H = 0$ and $f = 1$ MHz can be estimated by the formula for the anomalous skin effect, by use of the ratio of the conductivity to the length of the free path, $\sigma/l = 0.55 \cdot 10^{11}$ Ω^{-1} cm^{-2} [4]:

$$Z(0) = \left(4\sqrt{3}\pi^3 \frac{fl}{\sigma c^4}\right)^{1/3}(1 - i\sqrt{3}) \approx 0.09 \cdot 10^{-6}(1 - i\sqrt{3})\frac{4\pi}{c}. \tag{3}$$

In the range of fields $H \sim 0.5$ kOe, the resistance $R_-(H)$ (curve 1) has a peculiarity in the form of a smoothed-out step. In larger fields, the function $R_-(H)$ varies approximately as H^2 up to the threshold field of an electronic doppleron, $H_L = 6.2$ kOe. Above the threshold, the resistance R_- oscillates in consequence of excitation of an electronic doppleron[5] in the plate and on the average increases linearly with the field. The maximum amplitude of the oscillating component exceeds by a factor of almost three the value of $R(0)$ given above. In the vicinity of the threshold H_L of an electronic doppleron, there is a smoothed-out kink on the $R_-(H)$ curve.

In the plus polarization, the resistance $R_+(H)$ varies approximately in proportion to the square of the magnetic field and has an anomaly in the form of a smoothed-out kink at field $H \approx 4.5$ kOe, corresponding to the threshold of a fundamental hole doppleron[5] (Fig. 2, curve 2). Above the threshold, $R_+(H)$ increases, with good accuracy, in proportion to H. The oscillations of a hole doppleron are small and not resolvable against the background of the strong variation of $R_+(H)$. For $H \gtrsim H_L$, the surface resistance in both polarizations depends noticeably on the temperature. In field 14 kOe, lowering of the temperature from 4.2 to 1.6 K leads to increase of R_- and R_+ approximately by a factor 1.5. The nearly linear nature of the function $R_+(H)$ in the high-field range indicates that in the temperature range $1.6\text{ K} \leq T \leq 4.2$ K, at $f = 1$ MHz, the thickness of the skin layer is small in comparison with the thickness of the plate, so that the smooth part of the impedance of the plate is twice

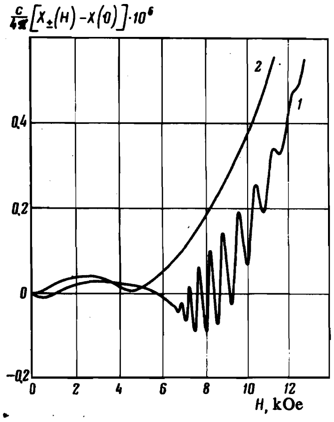

FIG. 3. Variation of the reactance $X_\pm$ of cadmium with magnetic field; $f=1$ MHz, $T=1.6$ K, $d=0.6$ mm. Curve 1, minus polarization; 2, plus.

the value of the impedance of the semiinfinite metal.

Figure 3 shows examples of the record of the reactance $X_\pm(H)$ of a cadmium plate. The origin and the scale have been chosen the same as for the surface resistance. We note first that the total change of reactance over the magnetic-field interval investigated, at frequency 1 MHz, is appreciably smaller than the change of resistance. On the curve $X_-(H)$ there are two minima, of which the first is located at field $H\approx 0.5$ kOe, and the second at a magnetic field slightly exceeding the threshold field H_L. In high fields, the reactance $X_-(H)$ increases rapidly, and against the background of its smooth variation large-amplitude oscillations of the electronic doppleron are evident. The amplitudes of oscillation of the reactance and of the resistance are, within the limits of error, the same.

In polarization plus, the reactance increases monotonically with increase of field up to $H\approx 2$ kOe. In the interval 2 kOe $<H<$ 4.5 kOe, the reactance decreases to a value close to the value $X(0)$. The location of this minimum with respect to magnetic field coincides with the threshold of the fundamental hole doppleron. In the high-field range $H>4.5$ kOe, the reactance increases rapidly with H.

The imaginary part $X_\pm(H)$ of the surface impedance, like the real part, changes noticeably on lowering of the temperature. This expresses itself in a deepening of the minima and an increase of the slope of the curves above the thresholds of the respective dopplerons.

Investigation of the impedance at various frequencies showed that all its characteristic features are shifted on the magnetic-field scale in proportion to the cube root of the frequency.

As is evident from Figs. 2 and 3, in the small-field range $H\sim 0.5$ kOe the surface impedance in minus polarization has anomalies similar to the anomalies in the vicinity of the thresholds of electronic and hole dopplerons. In order to investigate the functions $Z_\pm(H)$ in more detail in the weak-field region, the variation of the derivatives of the impedance with magnetic field was studied. Figure 4 shows examples of the records of the second derivative d^2X/dH^2 at frequency 400 kHz. In minus polarization, three groups of oscillations are observed. The period of oscillation ΔH is practically constant within each group. The values of ΔH for the individual groups are 36, 65, and 250 Oe respectively. The periods of the oscillations are, with good accuracy, in the ratios $1/7:1/4:1$. The oscillations with period 250 Oe have been observed earlier.[6] They are due to excitation in the plate of a Gantmakher-Kaner wave, caused by holes of the "monster." The oscillations with the smallest period begin practically at zero field, but their amplitude increases sharply at $H\approx 350$ Oe. The oscillations with periods 36 and 65 Oe occur within narrow ranges of magnetic field. Their amplitudes are greatest in the interiors of the corresponding ranges.

In plus polarization, the oscillations with period 36 Oe are also observed. They are evident in weak magnetic fields and become practically unresolvable in the same field range as do the corresponding oscillations in minus polarization. The oscillations with period 65 Oe are practically absent in the hole polarization, and the oscillations with period 250 Oe are at least orders of magnitude weaker than in minus polarization. The amplitude of oscillation of each group depends strongly on temperature. They become distinguishable against the background of the slow variations of the derivative at temperatures below 3 K.

The features that have been mentioned of the short-period oscillations allow us to suppose that they are due to excitation in the plate of multiple dopplerons.[7] According to theory, multiple dopplerons may exist in metals whose Fermi surfaces do not possess axial symmetry. If the direction of the field is an axis of symmetry of the Fermi surface of order s, then dopplerons of multiplicity $n=ms\pm 1$ can be propagated, where m is an integer. For $\mathbf{H}\parallel[0001]$, the hole monster in cadmium has an axis of symmetry of the third order, and dopplerons of multiplicity 1, 2, 4, 5, 7, etc., can be propagated in the metal. The second and fifth harmonics are not observed ex-

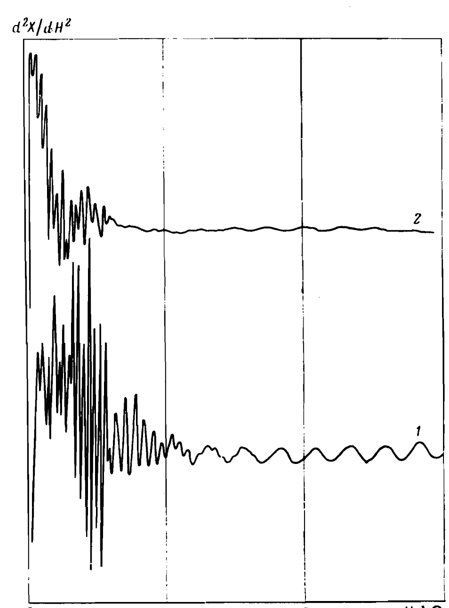

FIG. 4. Records of the derivatives $d^2X_\pm/dH^2$ as functions of the magnetic field: $f=0.4$ MHz, $T=1.6$ K, $d=0.6$ mm. Curve 1, minus polarization; 2, plus.

perimentally. The observed short-period oscillations are apparently caused by excitation of the fourth and seventh multiple dopplerons. It is in the range of existence of these oscillations that a kink is observed in the function $R_-(H)$ and a minimum in $X_-(H)$.

THEORY AND DISCUSSION OF RESULTS

The properties of a doppleron are qualitatively only slightly sensitive to the nature of the singularity of the nonlocal conductivity.[5] Therefore Refs. 1 and 8 considered a Fermi-surface model that had the form of a parabolic lens. This model is attractive in its simplicity, because the nonlocal conductivity corresponding to it has a resonance singularity of pole type and no branch points. As a result, one can obtain analytic expressions for the surface impedance and the field distribution in a specimen, regardless of the character of the reflection of the electrons by the surface.[8] But the parabolic-lens model also has a shortcoming, consisting in the absence of collisionless cyclotron absorption of the field in the short-wavelength range. The absence of such absorption has no significant effect on the behavior of the real part of the impedance; but it is *a priori* difficult to expect that a change of the imaginary part, which is due to attenuation, is obtained correctly in this case.

In other words, in order to determine the function $X(H)$ it is necessary to consider a model of the Fermi surface in which there is cyclotronic absorption. The simplest model of this type, having the form of a corrugated cylinder, was proposed in Ref. 9. Here we shall use a modification of this model considered in Ref. 6 in application to a compensated metal. The corresponding nonlocal conductivity for plus and minus circular polarizations has the form

$$\sigma_\pm(q)=\pm i\frac{nec}{H}\{[(1-i\gamma)^2-q^2]^{-1/2}-1\}, \tag{4}$$

where

$$q=\frac{ku}{2\pi},\quad u=\frac{c}{eH}\left(\frac{\partial S}{\partial p_z}\right)_{ext}, \tag{5}$$

k is the wave vector of the electromagnetic field, n is the concentration of the electrons, u is their extremal displacement during a cyclotron period, $S(p_z)$ is the area of a cross section p_z = const of the Fermi surface, and γ is the ratio of the frequency of collision of the electrons with the scatterers to the cyclotron frequency.

The surface impedance of a bulk specimen, for diffuse reflection of the carriers, is determined by the expressions (see, for example, Ref. 6)

$$\frac{c^2Z_\pm}{4\pi\omega}=\left[\frac{i}{2\pi}\int_{-\infty}^{\infty}dq\ln\frac{D_\pm(q)}{q^2}\right]^{-1}, \tag{6}$$

$$D_\pm(q)=q^2-\frac{4\pi i\omega u^2}{c^2(2\pi)^2}\sigma_\pm(q), \tag{7}$$

where

$$D_\pm(q)=0 \tag{8}$$

is the dispersion equation for the characteristic modes of the electromagnetic field in an infinite metal.

The method of calculating the integral (6) is described in Ref. 6. We shall consider the impedance for minus polarization. On substituting (4) in (6) and integrating, we obtain

$$\left(\frac{c^2Z_-}{2\pi\omega u}\right)^{-1}=\pi\left(q_s+q_D+\frac{1-i\gamma}{2}\right.$$
$$\left.+(1-i\gamma)\{(1-r^2)^{1/2}\arccos r+[r(r+2)]^{1/2}\ln[1+r+[r(r+2)]^{1/2}]\}\right), \tag{9}$$

$$r=(1/4+\xi)^{1/2}-1/2,\quad \xi=\omega neu^2/\pi cH(1-i\gamma)^2, \tag{10}$$

where q_s and q_D are the solutions of the dispersion equation (8) corresponding to the skin and doppler components of the field and determined by the expressions

$$q_s^2=2i\gamma\xi/(2-\xi),\qquad q_D^2=(1-r^2)[1-2i\gamma/(1-r)^2]. \tag{11}$$

Formulas (9) and (11) are correct everywhere except in a small neighborhood of the doppler threshold, where $|2-\xi|\sim\gamma^{1/2}$. We shall not give the expression for Z_+, because in the model being considered there are no waves in plus polarization and the impedance has no singularities.

The curves $R_-(H)$ and $X_-(H)$ are shown in Fig. 5. The parameters n and $(\partial S/\partial p_z)_{ext}$ were so chosen that the position of the doppleron threshold and the period of the oscillations coincided with those observed experimentally. The length of the free path of the electrons was taken as 1 mm.

The surface resistance R_- increases with H approximately quadratically up to the doppleron threshold, which at frequency 1 MHz is located at H = 6.2 kOe. In the vicinity of the threshold, the R_- graph has a kink, smoothed out in consequence of the finite length of the free path of the electrons. In the range of existence of the doppleron, R_- varies approximately linearly. The more rapid growth of R_- in this region is due to excitation of a doppleron and increase of the depth of the skin layer. Qualitatively, the behavior of $R_-(H)$ is the same as in the parabolic-lens model considered in Ref. 1. This shows that the presence of a kink on the $R_-(H)$ curve is independent of the model of the Fermi surface and characteristic of diffuse reflection of the electrons.

The behavior of the resistance $R_-(H)$ is in qualitative agreement with the experimental curve 1 in Fig. 2. The presence of oscillations on the curve in the latter case is due to the effect of the plate. In addition, on

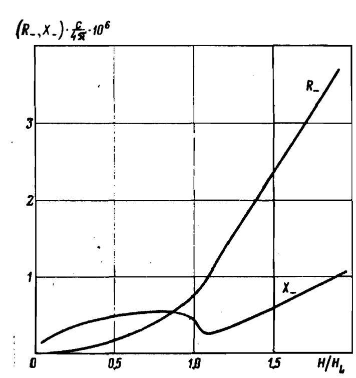

FIG. 5. Calculated curves of resistance and reactance; H_L = 6.2 kOe for f = 1 MHz.

curve 1 of Fig. 2 there is a section of more rapid rise at $H \sim 0.5$ kOe. This rise and the associated kink of the curve are due to the existence of multiple hole dopplerons. In the theoretical model, hole copplerons are absent, because nonlocal effects in the hole conductivity were not taken into account.

In contrast to the surface resistance, the reactance $X_-(H)$ varies nonmonotonically. In the range $H < H_L$, the reactance has a broad maximum. In the vicinity of the doppleron threshold, X_- decreases and reaches a minimum. On further increase of the field, X_- increases rapidly. Such a change of reactance with field is in qualitative agreement with the experimental curve 1 in Fig. 3. The presence of a minimum on the $X_-(H)$ curve for $H \gtrsim H_L$ is due to the appearance of a propagating electromagnetic mode—a doppleron. Excitation of a wave in a metal should cause a drop of its reactance near the threshold of this wave. It is easy to show this if we recall that the reactance is proportional to the value of the total magnetic flux of the wave field across the metal.[10] Under skin-effect conditions, the magnetic flux Φ is proportional to the skin depth δ:

$$\Phi = \int_0^\infty H_s(z)\,dz \sim H(0)\,\delta, \tag{12}$$

where $H_s(z)$ is the field in the skin layer, and $H(0)$ is the exciting magnetic field. But in the vicinity of the threshold, the magnetic field of the skin component decreases in consequence of the excitation of the wave:

$$H_s(0) = H(0) - H_D(0); \tag{13}$$

here $H_s(0)$ and $H_D(0)$ are the magnetic fields of the skin and doppleron components at the surface of the metal. The total magnetic flux Φ is equal to $\Phi_s + \Phi_D$, where

$$\Phi_s = \int_0^\infty H_s(z)\,dz \sim H_s(0)\,\delta, \tag{14}$$

$$\Phi_D = \int_0^\infty H_D(z)\,dz = H_D(0) \int_0^\infty \cos(k_D' z)\exp(-k_D'' z)\,dz = \frac{H_D(0)\,k_D''}{k_D'^2 + k_D''^2} \sim H_D(0)\,\frac{k_D''}{k_D'^2}. \tag{15}$$

Here k_D' and k_D'' are the real and imaginary parts of $k_D = 2\pi q_D/u$.

Near the threshold of a doppleron, $k_D' \sim 1/\delta$, while $k_D'' < k_D''$. As a result, Φ_D is found to be considerably less than $H_D(0)\delta$; that is, the total magnetic flux Φ is determined mainly by the skin component. In the range of larger fields, the skin depth δ increases:

$$\delta = 1/k_s'' = u/2\pi q_s'' \propto H. \tag{16}$$

Therefore the value of the flux

$$\Phi \approx \Phi_s \approx H_s(0)\,\delta$$

initially decreases, reaches a minimum, and then increases with increase of H.

Thus we may conclude that near the threshold of a wave, the surface reactance should have a minimum. Accordingly, a natural explanation is obtained of the small minimum on the experimental curve 1 in Fig. 3 at fields $H \sim 0.5$ kOe. In fact, it occurs exactly in the field range where multiple hole dopplerons are observed and where there is a smoothed-out step on the $R_-(H)$ curve (Fig. 2). In similar fashion one explains the presence of a minimum on curve 2 of Fig. 3 at $H \approx 4.5$ kOe; this minimum is located near the threshold of the fundamental hole doppleron, which exists in plus polarization.

We assumed above that the reflection of the electrons by the surface was diffuse. For comparison, we shall discuss the behavior of the surface impedance for specular reflection of the electrons. In this case the expression for Z_- has the form

$$Z_- = \frac{4i\omega u}{c^2} \int_{-\infty}^{\infty} dq \{q^2 - \xi[((1-i\gamma)^2 - q^2)^{-1/2} - 1]\}^{-1}. \tag{17}$$

The integral is the sum of the residues at the poles of the integrand and the integral along the sides of the cut. The residues make a contribution both to R_- and to X_-; the integral along the sides of the cut, only to R_-.

The variation of R_- with H for specular reflection of the electrons was investigated earlier.[11,12] It was shown that in this case the surface resistance has a resonance maximum at the threshold of the wave. Therefore we shall consider here only the reactance

$$X_- = -4\pi c^{-2}\omega u\,\mathrm{Im}\{q_s^{-1}[1 - {}^1\!/_2\xi((1-i\gamma)^2 - q_s^2)^{-3/2}]^{-1} + q_D^{-1}[1 - {}^1\!/_2\xi((1-i\gamma)^2 - q_D^2)^{-3/2}]^{-1}\}. \tag{18}$$

Since the value of q_s is small, whereas q_D is comparable with unity, the second term in the curly brackets is small in comparison with the first everywhere except in a small neighborhood of the doppleron threshold $\xi \approx 2$. Therefore the expression for X_- can be expressed approximately in the form

$$X_- \approx 2(2-\xi)^{-1}\,\mathrm{Im}\,q_s^{-1}. \tag{19}$$

It is evident that X_- changes sign at the threshold of the wave. Thus the behavior of the reactance is significantly different for diffuse and for specular reflection of the electrons. Alig[13] remarked earlier that with specular reflection of the electrons in a noncompensated metal, the reactance X_- changes sign at the helicon threshold.

We have considered the dependence of the impedance of a semiinfinite metal on magnetic field in the cases of diffuse and of specular reflection of the electrons. It is of interest to study the transition from one limiting case to the other; that is, to consider the behavior of the impedance for values of the coefficient of specularity p different from 0 and 1. The solution of this problem for the corrugated-cylinder model is very complicated. Therefore we shall restrict ourselves to discussion of the dependence of the impedance on the value of p in the simplest model, the parabolic lens. The behavior of the surface resistance in such a model, for various values of p, was studied by Medvedev.[14] It was shown that even for a small departure of p from unity the height of the resonance maximum in R_- at the doppleron threshold decreases sharply, and that for $p \approx 0.7$ the dependence of R_- on H becomes practically the same as for $p = 0$.

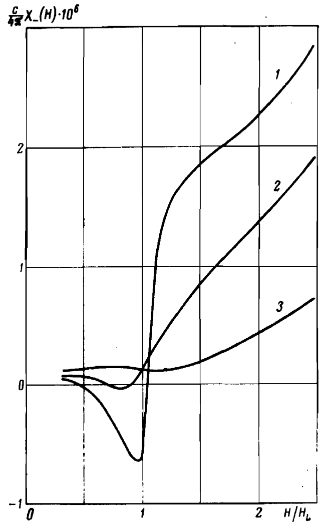

FIG. 6. Calculated $X_{-}(H)$ curves for three values of the coefficient of specularity: Curve 1, $p=1$; Curve 2, $p=0.8$; Curve 3, $p=0$.

Therefore it remains for us here to consider only the behavior of the reactance X_{-}.

An expression for the impedance of a plate in the parabolic-lens model was obtained in Ref. 8 (see formulas (29) and (30)). The impedance of a semiinfinite metal is obtained from these formulas by letting $T^{\pm}_{D,s}$ approach unity and introducing a factor $\frac{1}{2}$:

$$Z_{-}=4\pi c^{-2}\omega u\{q_s[(1-i\gamma)^2-q_D^2] -q_D[(1-i\gamma)^2-q_s^2] -\lambda(1-i\gamma)(q_s^2-q_D^2)\}\{q_Dq_s(q_s^2-q_D^2) -\lambda q_D[(1-i\gamma)^2-q_D^2](1-i\gamma) +\lambda q_s[(1-i\gamma)^2-q_s^2](1-i\gamma)\}^{-1}, \qquad (20)$$

where q_s and q_D are the solutions of the dispersion equation

$$q^2=\xi\{(1-i\gamma)[(1-i\gamma)^2 -q^2]^{-1}-1\}; \qquad (21)$$

the parameter ξ, as before, is determined by formula (7).

Graphs of the function $X_{-}(H)$ for various values of p are shown in Fig. 6. It is evident that even at a small difference of p from unity, the region of negative values of X_{-} is decreasing rapidly. For $p=0.75$ the value of X_{-} remains positive over the whole range of fields. Above the doppleron threshold, the value of the reactance depends strongly on the coefficient of specularity p. Such sensitivity of the reactance to the value of p in principle enables us to obtain information about the nature of the reflection of electrons.

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