

# Cross sections of transitions between highly excited levels as a result of collisions with charged particles

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A quasiclassical description of a highly excited atomic electron is used to find the transition cross section in terms of the Born amplitudes in the impact parameter representation. The amplitudes are found in the momentum transfer representation by means of asymptotically accurate (in respect of the principal quantum number  $n$ ) Born amplitudes. The results are given of numerical calculations for transitions from the  $n = 10$  and  $n = 100$  states. The feasibility of comparison of these results with experiments is considered.

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## 1. INTRODUCTION

Cross sections of transitions between highly excited atomic and ionic levels are of great interest in many aspects of the physics of very-low-density plasma. Experimental and theoretical investigations of these cross sections meet with serious difficulties. The cross sections of transitions between highly excited levels as a result of collisions with electrons have been calculated<sup>[1–3]</sup> in the Born approximation. Calculations using the “normalized” theory of perturbations with a dipole potential are reported elsewhere.<sup>[4]</sup> The general approach which can be used to deal with highly excited states outside the perturbation theory framework has been formulated recently.<sup>[5–7]</sup> The paper by Beĭgman *et al.*<sup>[5]</sup> develops a correct classical approach to the problem of an inelastic collision between a charged particle and an excited atom, in which use is made of the action function of an atomic electron. Presnyakov and Urnov<sup>[6]</sup> propose the application of the quantum model of equidistant levels. Percival and Richards<sup>[7]</sup> describe a classical approach based on the correspondence principles. The equivalence of all three approaches in the  $|n - n'| \ll n$  case ( $n$  and  $n'$  are the principal quantum numbers of the states between which the transition takes place) was demonstrated by Richards.<sup>[8]</sup>

The main result of these investigations<sup>[5–7]</sup> is an expression for the amplitude of a transition in terms of the Born amplitudes based on the impact parameter representation. As shown by Beĭgman and Urnov,<sup>[3]</sup> the dipole approximation is valid for transitions between neighboring levels and, therefore, the calculation of the Born amplitudes then presents no difficulties. Calculations of the cross sections for transitions between neighboring levels, carried out within the framework of the approach developed in the cited papers,<sup>[5,6]</sup> are reported by Beĭgman *et al.*<sup>[9]</sup> For transitions with  $|n - n'| > 1$  the momentum interaction plays the main role and until now it has not been possible to describe this interaction within the impact parameter method used earlier.<sup>[5–7]</sup> We shall obtain the Born amplitude corresponding to the momentum part of the interaction and we shall use the relationships between the exact scattering amplitudes in the momentum and impact parameter representations<sup>[10,11]</sup> and

calculate the cross sections using the approach described above<sup>[5–7]</sup> in the most interesting cases.

The cross sections of transitions involving neutral atoms are found to decrease severalfold in the range of velocities of practical interest. In the case of ions the discrepancy between these cross sections and the Born values decreases rapidly with the ion charge. The contribution of the transitions with  $|n - n'| > 1$  to the total inelastic broadening cross section for incident electrons of  $\sim 1$  eV energy is  $\sim 30\%$  (for  $n = 10$ ) and it is considerably greater in the processes of electron diffusion between highly excited levels. The role of the transitions with  $|n - n'| > 1$  rises with increasing velocity of the incident particle.

We shall assume that a charged particle moves along a rectilinear trajectory. In the case of collisions with electrons this requires that the condition  $E \gg E_n$  is satisfied ( $E$  is the energy of an external electron and  $E_n$  is the ionization potential of an excited atomic electron).

We shall use mainly the formulas from the paper by Beĭgman *et al.*<sup>[5]</sup> For brevity, we shall call these quasiclassical formulas<sup>[5]</sup> in view of the quasiclassical description used there for a highly excited atomic electron.

## 2. BORN AMPLITUDES IN THE IMPACT PARAMETER REPRESENTATION

The Born amplitudes of inelastic transitions in the momentum transfer representation were calculated by Beĭgman and Urnov.<sup>[3]</sup> They showed that the probability of momentum transfer can be divided in a natural manner into two terms. The first term, which dominates in the case of low momenta, is related to the dipole interaction of an external particle and the atom in question. The second term predominates in the range of momenta which satisfy the classical momentum approximation in which the role of the atomic nucleus in the collision process is ignored. For convenience, we shall call the latter the “momentum” or “nondipole” part of the interaction. Our problem will be to find expressions for both terms in the impact parameter representation.

The dipole part of the Born amplitude of a transition

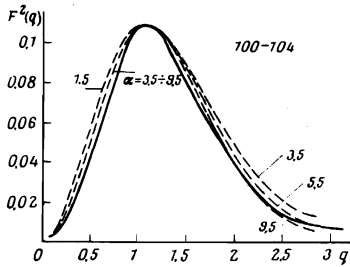


FIG. 1. Approximation of the momentum part of the amplitude of the 100–104 transition by Eq. (8); the continuous curve represents Eq. (7) and the dashed curve—Eq. (8).

from a state  $n$  to a state  $n'$  can be written in the form<sup>[4,9]</sup>

$$|A_d(k)|^2 = \frac{4}{Z^2 k^4 (r^2 + r_0^2) v^2} \left[ \frac{n^2 f_{n \rightarrow n'} k^2}{(n+n')(nn')} \right]^{R^2} \times [K_0^2(\beta) + K_1^2(\beta)], \quad (1)$$

$$r = Z\rho/a_0 n^2, \quad r_0 = Z\rho_0/a_0 n^2, \quad v = V/Zv_0, \quad n' = n+k, \\ \beta = k \frac{p_0}{n} (r^2 + r_0^2)^{1/2}, \quad p_0 = \frac{1}{2} \left( \frac{1}{n} + \frac{1}{n'} \right),$$

where  $a_0 = 0.53 \times 10^{-8}$  cm and  $v_0 = 2.18 \times 10^8$  cm/sec are the atomic units of length and velocity;  $Z$  is the spectroscopic symbol of the ion;  $\rho$  is the impact parameter;  $V$  is the velocity of an external charged particle;  $f_{n \rightarrow n'}$  is the oscillator strength of an  $n \rightarrow n'$  transition;  $K_0$  and  $K_1$  are the Macdonald functions;  $\rho_0$  is the regularization parameters.<sup>1)</sup> The expression containing the Macdonald functions can be approximated, with an error not exceeding 10%, by the formula

$$\beta^2 [K_0^2(\beta) + K_1^2(\beta)] \approx (1 + \pi\beta) e^{-2\beta}. \quad (2)$$

Using the Kramers approximation for the oscillator strength and applying the approximation (2), we find that Eq. (1) gives

$$A_d(k) = \frac{L_d e^{-\beta}}{Zv (r^2 + r_0^2)^{1/2} k^2} (1 + \pi\beta)^{1/2}, \quad L_d = 0.7. \quad (3)$$

We have ignored the difference between  $n$  and  $n'$  in Eq. (3). In general, Eq. (3) is valid if  $k \gg 1$ . A calculation carried out using a more rigorous formula<sup>[12]</sup> gives  $L_d = 0.6$  for  $k=1$ .

We shall describe the nondipole part of the interaction in terms of the impact parameter representation emphasizing the relationship between the amplitudes in the momentum and impact parameter representations. The nondipole part of cross section of an  $n \rightarrow n'$  transition will be represented in the form

$$\sigma_{n \rightarrow n'} = \frac{8\pi a_0^2}{v^2} \frac{1}{n^2} \left( \frac{nn'}{k} \right)^2 \int_{q_{\min}}^{q_{\max}} \frac{dq}{q} F^2(q), \quad (4)$$

where  $q = (nn')p/Zp_0$ ;  $p_0 = 2 \times 10^{-19}$  g · cm · sec<sup>-1</sup>;  $p$  is the momentum transferred to an atom in a collision.

Following Vinogradov and Vainshtein,<sup>[10]</sup> we shall describe the amplitude  $A_i(r)$  by

$$A_i(r) = \frac{2}{2\pi v} \frac{1}{k^2} \int d^2q e^{-iqr} \frac{F(q)}{q}, \quad (5)$$

where the integral is taken over the plane in which the vector  $q$  satisfies the condition  $\omega + Z(q \cdot v)/(nn') = 0$  ( $\omega$  is the frequency of the  $n \rightarrow n'$  transition). We shall use  $q_{\parallel}$  and  $q_{\perp}$  for the components of the vector  $q$  parallel and perpendicular to the vector  $v$ . Integrating Eq. (5) over the angles between the vectors  $q$  and  $r$ , we obtain

$$A_i(r) = \frac{2}{v} \frac{1}{k^2} \int_0^{\infty} \frac{J_0(q_{\perp} r) F(q)}{q} q_{\perp} dq_{\perp}, \quad (6) \\ q^2 = q_{\parallel}^2 + q_{\perp}^2, \quad q_{\parallel} = \frac{\omega}{v} \left( \frac{nn'}{Z} \right).$$

Following Beigman and Urnov,<sup>[3]</sup> we find that the function  $F(q)$  is described by

$$F^2(q) = \frac{2}{3} \frac{1}{Z^2} \frac{k}{q^2} \int_0^q dq' \frac{k(q/k)^2}{[1 + (q/k)^2]^2} \\ \times \left\{ \frac{q^2}{q^2 + k^2} \left( 1 + \frac{12k^2}{n^2 + k^2} \right) J_0^2 \left( k \sqrt{1 + \left( \frac{q}{k} \right)^2} \right) - \left( 1 - \frac{12k^2}{k^2 + q^2} \right) \right. \\ \left. \times \left[ J_0' \left( k \sqrt{1 + \left( \frac{q}{k} \right)^2} \right) \right]^2 \right\}. \quad (7)$$

The expression (7) is too complex to calculate the integral (6). However, an analysis of this expression shows that (7) has a maximum at  $q \sim k$  and the main “functional” part of this expression is  $q^2 + k^2$ . Therefore, we shall approximate  $F(q)$  by

$$F(q) = f \left( \frac{q}{kq_m} \right) \left[ \frac{2\alpha}{(2\alpha-1) + (q/q_m k)^2} \right]^{\alpha}. \quad (8)$$

The expression (8) has a maximum at  $q = kq_m$  and its value at the maximum is  $f_0$ ;  $\alpha$  is a free parameter. The quality of this approximation is illustrated in Fig. 1 for the example of the amplitude of the 100–104 transition. Using Eq. (8), we find that the amplitude  $A_i(r)$  is given by

$$A_i(r) = \frac{2f_0 (2\alpha q_m^2)^{\alpha}}{v Z k^2 q_m} \\ \times \frac{(q_0 k r)^{\alpha-1} K_{\alpha-1}(q_0 k r)}{(q_0)^{2(\alpha-1)} 2^{\alpha-1} \Gamma(\alpha-1)}, \quad (9) \\ q_0^2 = q_m^2 (2\alpha-1) + (p_0/v)^2.$$

For a half-integer value of  $\alpha$  the expression (9) can be represented in terms of elementary functions

$$A_i(r) = \frac{2f_0}{vZ} \left( \frac{2\alpha q_m^2}{q_0^2} \right)^{\alpha} \frac{q_0^2}{q_m} \\ \times \frac{e^{-z}}{2(\alpha-1)} \sum_{j=0}^{\infty} \frac{(2z)^j (2j-1)!}{j! (2j)! (j-1)!} \frac{j!}{(j-1)!} \\ j = \alpha - 1/2, \quad z = q_0 k r. \quad (10)$$

It follows from Fig. 1 that  $f_0 = 0.33$  and  $q_m = 1.1$ ; it is also clear from Fig. 1 that the value of  $\alpha$  has little effect on the results. We shall assume that  $\alpha = 3.5$ . Using Eqs. (3) and (9), we can calculate the Born cross sections and compare them with the calculations of Beigman and Urnov<sup>[3]</sup> carried out in the momentum approximation. For the parameter  $r_0 = 3$ , the results agree to within 10–20%.

### 3. QUASICLASSICAL TRANSITION AMPLITUDES

We shall consider an atom with a set of quantum numbers  $\gamma = (n, l, m)$  which undergoes a transition  $\gamma \rightarrow \gamma'$  as

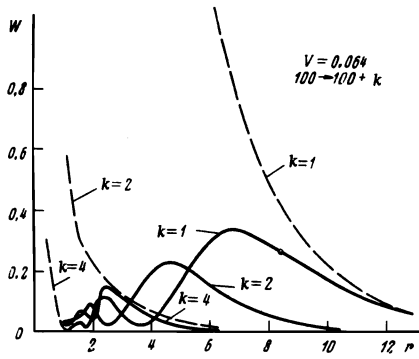


FIG. 2. Probabilities of the 100-101, 100-102, and 100-104 transitions. The continuous curves represent a quasiclassical approximation and the dashed curves the perturbation theory; the impact parameter is in units of  $a_0 n^2$  ( $a_0 = 0.53 \times 10^{-8}$  cm).

a result of a collision. For high quantum numbers the transition amplitude is<sup>[5]</sup>

$$a_{n'n'} = \frac{1}{(2\pi)^3} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} d^3u \exp[-ik \cdot u - iS(\gamma, u)], \quad (11)$$

where the vector is given by  $\mathbf{k} = (n' - n, l' - l, m' - m)$ ; for  $S \ll n$ , we have  $S(\gamma, u) = \sum A_k \exp(-ik \cdot u)$ .

The function  $S$  in Eq. (11) is a modified form of the classical action function for a collision expressed in terms of the action variables and the corresponding phase variables  $\mathbf{u} = (u_1, u_2, u_3)$ . If the perturbation theory is valid (i. e., if the increase in the action is small), it follows from Eq. (11) that

$$a_{n \rightarrow n'} = -iA_k.$$

Thus, the coefficients  $A_k$  are equal to the Born transition amplitudes apart from the phase factor.

The probability of an  $n \rightarrow n'$  transition, summed over the orbital quantum numbers of the initial and final states [ $(n^2 W_{n \rightarrow n'})$ ], is readily found from Eq. (11):

$$n^2 W_{n \rightarrow n'} = \sum_{\substack{l, m \\ l', m'}} |a_{n \rightarrow n'}|^2 = \sum_{l, m} \frac{1}{(2\pi)^3} \left| \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} du_1 du_2 du_3 \exp[-ik_1 u_1 - iS(\gamma, u)] \right|^2. \quad (12)$$

If the function  $S$  is independent of the variables  $u_2$  and  $u_3$  (and, consequently, also independent of  $l$  and  $m$ ), Eq. (12) reduces to the one-dimensional expression<sup>2)</sup>

$$W_{n \rightarrow n'} = \left| \frac{1}{2\pi} \int_0^{2\pi} du \exp[iku - iS(\gamma, u)] \right|^2. \quad (13)$$

We shall now use Eq. (13). Since, in general, the problem is not one-dimensional, the use of Eq. (13) automatically implies some averaging procedure. We shall define this procedure as follows:

$$\left. \begin{aligned} S(n, u) &= \sum A_k e^{-iku}, \\ A_k^2 &= \frac{1}{n^2} \sum_{l, m} |A_k|^2 = A_d^2 + A_i^2, \end{aligned} \right\} \quad (14)$$

where the amplitudes  $A_d$  and  $A_i$  are given by Eqs. (3) and (9). We can show that Eqs. (13) and (14) are exact in any one of the following three cases: 1) if the perturbation theory is valid; 2) if the matrix elements of the interaction potential vanish in all cases except for  $l=l'$  and  $m=m'$ ; 3) if the matrix elements vanish in all cases with the exception of  $n'=n \pm r_1$ ,  $l'=l \pm r_2$ ,  $m'=m \pm r_3$ , where  $r_1$ ,  $r_2$ , and  $r_3$  are positive integers.

By way of illustration, Fig. 2 gives the probabilities  $W(r)$  for the 100-101, 100-102, and 100-104 transitions as a function of the impact parameter  $r$ . The probability of a transition between neighboring levels ( $k=1$ ) has a wide maximum in the range of impact parameters which are between six and eight times as large as the characteristic size of an atomic orbit. The maximum probability is  $\sim 0.3$ . For transitions with  $k>1$ , the principal maximum becomes flatter and shifts toward lower values of  $r$ . In the case of these transitions with  $k>1$  we find, in contrast to the usual situation, that  $W(r)$  is greater than the Born value in a wide range of the impact parameters. This means that at these velocities the "step" excitation plays an important role. In the case of small impact parameters the value of  $W(r)$  begins to oscillate. The question whether these oscillations are associated with the adopted averaging procedure requires further study. The oscillation region makes a relatively small contribution to the total transition cross section. In contrast to the oscillations, the shift of the principal maximum follows<sup>[5]</sup> from the general structure of Eqs. (11)-(14) and may, in principle, be detected experimentally by analyzing the corresponding differential cross sections.

#### 4. TRANSITION CROSS SECTIONS

The cross section is found from Eq. (13) by integration over the impact parameter. The Born cross section is proportional to the factor  $Z^{-4} n^{-2} (nm'/k)^3$  so that we obtain the following expression for the cross section:

$$\left. \begin{aligned} n^2 \sigma_{n \rightarrow n'} &= \frac{\pi a_0^2}{Z^4} \left( \frac{nn'}{k} \right)^3 \bar{\sigma}_{n \rightarrow n'}, \\ \bar{\sigma}_{n \rightarrow n'} &= 2k^2 Z^2 \int_0^\infty W_{n \rightarrow n'}(r) r dr, \end{aligned} \right\} \quad (15)$$

where the probability  $W_{n \rightarrow n'}(r)$  is given by Eq. (13).

The value of  $\bar{\sigma}$  is plotted in Fig. 3 as a function of the velocity for the transitions 10-11, 10-12, and 10-14.

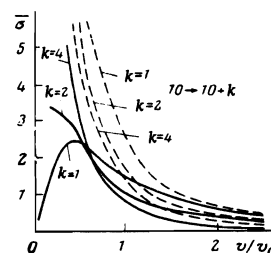


FIG. 3. Cross sections of the 10-11, 10-12, and 10-14 transitions. The continuous curves represent the quasiclassical approximation and the dashed curves give the perturbation theory results ( $v_0 = 2.18 \times 10^8$  cm/sec).

TABLE I. Values of  $K_n$  for calculation of inelastic width [see Eq. (20)].

$kT/Z^2 Ry$	$n=10$			$n=100$		
	$Z=1$	$Z=2$	$Z=\infty^*$	$Z=1$	$Z=2$	$Z=\infty^*$
0.02	1.91	3.55	19.8	21.4	28.4	76.43
0.04	2.61	4.7	20.83	23.14	30.15	65.15
0.08	3.45	5.92	20.11	23.04	29.39	52.62
0.16	4.34	6.99	18.02	21.3	26.64	40.91
0.32	5.13	7.63	15.25	18.8	22.84	31.06
0.64	5.62	7.67	12.41	16.0	18.27	23.36
1.28	5.69	7.18	9.85	13.36	15.24	17.59

\*The case  $Z=\infty$  corresponds to the Born approximation.

For the transitions with  $k=1$  the cross section has a maximum at the velocities  $v \sim 0.3$  and it is much smaller than the Born value. The results obtained for  $k=1$  differ little from those reported earlier<sup>[4]</sup> where the perturbation theory is used for the large impact parameters and the transition probability for small values of  $r$  is assumed to be  $1/2$ . This provides a further confirmation that in the case of the  $k=1$  transitions the main contribution is due to the large impact parameters to which the dipole approximation can be applied. The situation is very different for the transitions with  $k>1$ . The main contribution to the cross section is then made by the momentum part of the interaction. In the range of velocities under consideration the cross section rises monotonically on reduction of the velocity but remains smaller than the Born value. It should be noted that quantities of the  $k^2 \sigma_{n \rightarrow n+k}$  type govern the rate of diffusion of atomic electrons between highly excited levels. It is clear from Fig. 3 that at low energies of the incident particles the diffusion processes are dominated by the transitions with  $k>1$ . In the case of the  $100 \rightarrow 101$ ,  $100 \rightarrow 102$ , and  $100 \rightarrow 103$  transitions the calculated cross sections agree to within  $\sim 15-20\%$  with the "control" points obtained by Gee *et al.*<sup>[13]</sup> from "classical" calculations.

### 5. INELASTIC BROADENING CROSS SECTION

The Stark broadening of  $n+k \rightarrow n$  lines is governed, for sufficiently large values of  $n$ , by inelastic collisions between excited atoms and electrons.<sup>[14,15]</sup> In accordance with these results, the Stark width of an  $n+k \rightarrow n$  line with  $k \ll n$  is given by

$$\delta\omega = N_e [\langle V\sigma_n \rangle + \langle V\sigma_{n+k} \rangle], \quad \langle V\sigma_n \rangle = \sum_{k \neq 0} \langle V\sigma_{n \rightarrow n+k} \rangle. \quad (16)$$

The angular brackets in Eq. (16) denote averaging over the Maxwellian electron velocity distribution and  $N_e$  is the density of electrons which should be summed over all the possible values of  $k \neq 0$ . In fact, the sum is dominated by the terms with small values of  $k$ . We shall now obtain an expression for the inelastic broadening cross section and we shall give some numerical results.

It follows from Eq. (15) that the total cross section  $\sigma_n$  is

$$\sigma_n = \sum_{k \neq 0} \frac{\bar{\sigma}_{n \rightarrow n+k}}{|k|^2} = 2Z^2 \int_0^\infty r dr \sum_{k \neq 0} W_{n \rightarrow n+k}(r). \quad (17)$$

If we use the property

$$\sum_{k=-\infty}^{+\infty} e^{ik(u-u')} = \delta(u-u')$$

in the interval  $0-2\pi$ , we find from Eqs. (13) and (17) that

$$\sum_{k \neq 0} W_{n \rightarrow n+k} = 1 - \frac{1}{2\pi} \int_0^{2\pi} du e^{-i\epsilon(u)}. \quad (18)$$

The expression (18) eases greatly the calculation of  $\langle V\sigma_n \rangle$ , which can be expressed conveniently in the form

$$\langle V\sigma_n \rangle = 10^{-8} n^4 Z^{-3} K_n \text{ (cm}^3/\text{sec)}, \quad K_n = \frac{2.18}{\Theta^{3/2}} \int_0^\infty e^{-E/kT} \left( \frac{E}{Z^2 Ry} \right) \bar{\sigma}_n \frac{dE}{Z^2 Ry}, \quad \Theta = \frac{kT}{Z^2 Ry}. \quad (19)$$

The quantities  $K_n$  are weak functions of the temperature and quantum number. The data for the charges  $Z=1, 2$ , and  $\infty$  and for the quantum numbers  $n=10$  and  $100$  are given in Table I.

The relative width of an  $n+1 \rightarrow n$  line is found from Eqs. (16) and (19):

$$\frac{\delta\omega}{\omega} = 0.48 \cdot 10^{-21} \frac{(n+1/2)^7}{Z^3} N_e K_n. \quad (20)$$

Exact data on the broadening of  $n+1 \rightarrow n$  lines in the  $n \sim 100$  range are of exceptional interest for the diagnostic of plasmas in planetary nebulae.<sup>[16,17]</sup>

Direct experimental information on the cross sections of transitions between highly excited levels are lacking and it would be very difficult to obtain them at present. Indirect data, which are the inelastic scattering cross sections averaged over the Maxwellian distribution, can be obtained for fairly high values of  $n$  from the laboratory data on the widths of  $n+1 \rightarrow n$  lines. The first such experiment on a decaying hydrogen plasma was described by LaSalle *et al.*<sup>[18]</sup> According to this experiment, the relative width of the  $13 \rightarrow 12$  line is  $\delta\omega/\omega = 0.75 \cdot 10^{-16} N_e$ . The electron temperature deduced from the  $H_\beta, H_\gamma$ , and  $H_\delta$  Balmer lines is very low:  $\sim 0.15-0.17$  eV. At this very low temperature the contribution of the inelastic transitions with  $k \geq 1$  is negligible. According to LaSalle *et al.*,<sup>[18]</sup> elastic transitions make a contribution to the line width which is 2.5 times smaller than the experimental value. However, the ratio of the line intensities to the intensity of the continuous spectrum does not correspond to  $T_e \sim 0.15$  eV. LaSalle *et al.*<sup>[18]</sup> concluded that the excess of the radiation in the continuous spectrum may be due to bremsstrahlung in a field of neutral atoms. It should be pointed out that the assumption of a local thermodynamic equilibrium, used in the determination of the temperature from the Balmer lines, is not fully justified for a nonstationary plasma. The ratio of the line intensities to the continuous spectrum corresponds to  $\sim 1$  eV, which seems to be very reasonable for this type of plasma. In this case the expression (20) and the theory given above for the calculation of the cross sections give a width which is half the experimental value. The Born cross sections overestimate the result by a factor of 2.

In view of this situation it would be interesting to determine experimentally the widths of  $n+1 \rightarrow n$  lines in hydrogen and hydrogen-like plasmas and at the same time to determine independently the temperature and density. It would be very desirable to measure the widths of several lines in the same experiment which would make it possible to separate the contribution of elastic transitions to the width, which may still be large for  $n \sim 10$ .

<sup>1</sup>The pole potential is used in this case. <sup>[4]</sup> The regularization parameter introduced in Eq. (1) will be defined later so that the cross sections calculated using Eq. (1) are identical with the Born values. <sup>[3]</sup>

<sup>2</sup>We shall omit the index 1 of the components  $k_1$  and  $u_1$  of the vectors  $\mathbf{k}$  and  $\mathbf{u}$ , denoting them simply by  $k$  and  $u$ . As in the preceding sections, we shall use again one-dimensional expressions and, therefore, such simplification of the notation should cause no confusion.

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## Coherence transfer in metastability exchange in the mixture of the isotopes $\text{He}^3$ and $\text{He}^4$

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We consider coherence transfer in the exchange of metastability in a mixture of  $\text{He}^3$  and  $\text{He}^4$ . It is shown that this process results in appreciable shifts of the magnetic-resonance frequency in the  $2^3S_1$  states of the  $\text{He}^3$  and  $\text{He}^4$  atoms. The appearance of these shifts, which has been predicted theoretically, is confirmed by experiments on the optical orientation and magnetic resonance of  $2^3S_1$ -metastable helium atoms. The dependences of these shifts on the temperature, pressure, and concentration of the helium isotopes in the  $\text{He}^3$ - $\text{He}^4$  mixture are determined.

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### INTRODUCTION

In experiments on optical orientation of atoms, the circulation of the coherence between the ground and resonantly excited states leads to the well known optical shift of the magnetic-resonance frequency on account of real optical transitions.<sup>[1]</sup> The magnitude of this shift is small and is much less than the magnetic-resonance line width, since the light intensity is usually such that the atoms remain in the excited state only a negligible part of the time in comparison with the time of their stay in the ground state.

A similar situation obtains for the helium isotope  $\text{He}^3$  when exchange of metastability takes place between two

atoms, one of which is in the ground state  $1^1S_0$  and the other in the metastable  $2^3S_1$ . In this case the atom goes from the ground to the metastable state, and returns to the ground state after a time  $\tau$ . Coherence is transferred thereby from the ground to the metastable state and back. Since the precession in the metastable state is much faster (the gyromagnetic states for the  $2^3S_1$  and  $1^1S_0$  levels,  $\gamma_m$  and  $\gamma_f$ , differ by three orders of magnitude), it follows that an increase of the resonance frequency, comparable in magnitude with the resonance-line width, takes place in the ground state. In view of the practical importance of this question, for example for quantum magnetometers and gyroscopes, this frequency shift had been discussed in many papers,<sup>[2-4]</sup>