identified for the first time. The significance of the obtained results goes beyond the limits of the present paper; the magnetic multipole moments can manifest themselves in the interaction of any rare-earth ion with the ligands and the external fields, and can, consequently, be observed by the methods of the electron-nuclear double resonance and optical and γ-ray spectroscopies.

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[1)] The possibility of an indirect interaction of the nuclear moments via the exchange-coupled electron shells was pointed out in the work of one of the present authors.[10]

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Spin waves in a medium with nonequilibrium spin orientation

A. G. Aronov

B. P. Konstantinov Nuclear Physics Institute, USSR Academy of Sciences, Leningrad

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The small exchange interaction which exists in any material leads, in the presence of nonequilibrium polarization of the electrons, to the appearance of spin waves with a quadratic dispersion law at small momenta. The spin waves exist for either sign of the exchange interaction and for any degree of degeneracy of the electron gas. The Landau damping of these waves is small compared with their frequency.

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In the discussions of the phenomena that arise during spin injection[1] and optical orientation of spins[2] in semiconductors and metals the exchange interaction has been disregarded on the grounds that it is small. In this article we shall show that allowance for the small exchange interaction leads to a qualitatively new phenomenon—the existence of spin waves in a medium with spins oriented in a nonequilibrium manner. Moreover, we shall show that these waves exist for either sign of the exchange interaction and for any degree of degeneracy, and that their damping is small compared with their frequency.

The simplest way to obtain the dispersion law of the spin waves is to calculate the spin part of the magnetic susceptibility, just as is done, e.g., in the Stoner model of ferromagnets (cf., e.g.,[3]), and investigate its poles. In the self-consistent field model the exchange interaction between the electrons leads to the result that the energy of particles with a given spin depends on their distribution function.

For simple parabolic bands,

$$\varepsilon_\alpha(\mathbf{k})=k^2/2m-\sum_{\mathbf{k}'}V_{\mathbf{k}-\mathbf{k}'}n_\alpha(\mathbf{k}'). \tag{1}$$

Here, $V_{\mathbf{k}-\mathbf{k}'}$ is a Fourier component of the potential of the exchange interaction. In Landau's Fermi-liquid theory, $V_{\mathbf{k}-\mathbf{k}'}$ is related to the antisymmetric scattering amplitude. For a nondegenerate electron gas the simplest approximation for the potential $V_{\mathbf{k}-\mathbf{k}'}$ is a screened Coulomb potential. However, despite the fact that it is possible to establish the dispersion law at small momenta for any form of the potential, it has not been possible to calculate the constants, even for the screened Coulomb potential. Therefore, in the following we take it that

$$\varepsilon_\alpha(\mathbf{k})=\frac{k^2}{2m}-\frac{I}{N}\sum_{\mathbf{k}'}n_\alpha(\mathbf{k}'). \tag{2}$$

We observe immediately that the constant $I\sim Ne^2/mT \ll T$ for a nondegenerate gas and $I\sim e^2N^{1/3}<\mu$ in the case of degeneracy.

Calculating the response to an external magnetic field

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$$H_+(\mathbf{r}, t)=(H_x+iH_y)e^{-i\omega t+i\mathbf{q}\mathbf{r}}, \quad (3)$$

we obtain for the corresponding susceptibility the following expression[3]:

$$\chi(\omega, \mathbf{q})=2\mu_B^2\frac{\Gamma(\omega, q)}{1-I\Gamma(\omega, \mathbf{q})/N}, \quad (4)$$

$$\sum_{\mathbf{k}}\frac{n_{\mathbf{k}+\mathbf{q}/2,\downarrow}-n_{\mathbf{k}-\mathbf{q}/2,\uparrow}}{\omega-\varepsilon_{\mathbf{k}+\mathbf{q}/2,\downarrow}+\varepsilon_{\mathbf{k}-\mathbf{q}/2,\uparrow}+i\delta}=\Gamma(\omega, q). \quad (5)$$

Here, $n_{\mathbf{k},\alpha}$ is the distribution function of the electrons for which the z-component of the spin is α, and N is the total electron concentration.

The dispersion equation for the spin waves is obtained by equating the denominator of $\chi(\omega, \mathbf{q})$ to zero and has the form

$$1=\frac{I}{N}\sum_{\mathbf{k}}\frac{n_{\mathbf{k}+\mathbf{q}/2,\downarrow}-n_{\mathbf{k}-\mathbf{q}/2,\uparrow}}{\omega-\mathbf{q}\mathbf{v}-IR+i\delta}, \quad (6)$$

where we have used the fact that, to within an unimportant constant,

$$\varepsilon_{\mathbf{k},\downarrow}=\frac{k^2}{2m}-\frac{IR}{2}, \quad \varepsilon_{\mathbf{k},\uparrow}=\frac{k^2}{2m}+\frac{IR}{2}; \quad (7)$$

here,

$$R=\frac{1}{N}\sum_{\mathbf{k}}(n_{\mathbf{k}\uparrow}-n_{\mathbf{k}\downarrow}) \quad (8)$$

is the degree of polarization of the electrons, determined by the pumping. It is convenient to bring the dispersion equation to the form

$$\omega=\frac{I}{N}\sum_{\mathbf{k}}\frac{\mathbf{q}\mathbf{v}}{\omega-\mathbf{q}\mathbf{v}-IR+i\delta}(n_{\mathbf{k}+\mathbf{q}/2,\downarrow}-n_{\mathbf{k}-\mathbf{q}/2,\uparrow}). \quad (9)$$

We notice immediately that the dispersion equation for waves of the other polarization will differ from (9) only in the sign of $\omega+i\delta$. This means that the real parts of the frequencies of waves with opposite polarizations differ in sign while their imaginary parts coincide.

A. The nondegenerate case

If the electron gas is not degenerate and the energy relaxation of the nonequilibrium electrons proceeds sufficiently rapidly, the distribution functions $n_\alpha(\mathbf{k})$ are equilibrium functions and

$$n_\alpha(\mathbf{k})=\exp\{(\mu_\alpha-\varepsilon_{\mathbf{k},\alpha})/T\}. \quad (10)$$

Introducing $\mu_\uparrow=\mu+\delta\mu/2$ and $\mu_\downarrow=\mu-\delta\mu/2$, we obtain for the degree of polarization R the equation

$$R=\text{th}\frac{\delta\mu+IR}{2T}, \quad (11)$$

which is, in fact, an equation for $\delta\mu$ for given R. If the exchange interaction is sufficiently small, so that $R=0$ when $\delta\mu=0$ (this is the condition $I<2T$), the sign of $\delta\mu$ coincides with the sign of R.

Assuming the frequency ω to be real and separating out the real part of Eq. (9), for ω, $qv_T \ll IR$ we obtain the dispersion equation

$$\omega=\frac{1}{IR^2N}\sum_{\mathbf{k}}(\mathbf{q}\mathbf{v})^2(n_{\mathbf{k}\downarrow}-n_{\mathbf{k}\uparrow})-\frac{1}{2RN}\sum_{\mathbf{k}}(\mathbf{q}\mathbf{v})^2\frac{\partial}{\partial\varepsilon_{\mathbf{k}}}(n_{\mathbf{k}\downarrow}+n_{\mathbf{k}\uparrow}). \quad (12)$$

Substituting (10) into (12) and taking (11) into account, after simple transformations we find

$$\omega=-\frac{q^2}{m}\frac{T}{IR}+\frac{q^2}{2mR}. \quad (13)$$

Since, usually, $T\gg IR$, we have

$$\omega\approx-q^2T/mIR, \quad (13a)$$

i.e., the sign of the frequency is determined by the sign of IR. For a ferromagnetic interaction ($I>0$), $\omega<0$.

For comparison we give the results for the dispersion law of the spin waves in a ferromagnetic metal:

$$\omega=q^2I^2R/64m\mu^2. \quad (14)$$

It can be seen from (13a) and (14) that in the nonequilibrium case the effective mass of the spin waves is proportional to the degree of polarization and to the magnitude of the exchange interaction, whereas in equilibrium it is inversely proportional to the polarization and to the square of the exchange-interaction energy. In the treatment of the degenerate case below we shall return to a qualitative discussion of this fact.

We shall calculate the collisionless damping of the spin waves. Taking the imaginary part of (12) we obtain

$$\text{Im}\,\omega=-\pi\frac{I}{N}\sum_{\mathbf{k}}\mathbf{q}\mathbf{v}\delta(\omega-IR-\mathbf{q}\mathbf{v})\{n_{\mathbf{k}+\mathbf{q}/2,\downarrow}-n_{\mathbf{k}-\mathbf{q}/2,\uparrow}\}. \quad (15)$$

Using (10) and performing simple calculations, we find that, for arbitrary $\mathbf{q}$ and $\omega\ll T$,

$$\text{Im}\,\omega=\frac{m^2TI}{2\pi qN}(\omega-IR)\,\text{sh}\frac{\delta\mu}{2T}\exp\left\{\frac{\mu-q^2/8m}{T}-\frac{m(\omega-IR)^2}{2q^2T}\right\}. \quad (16)$$

As we have already said, sign $\delta\mu$ = sign R, and, therefore, sign Im$\omega=-1$, i.e., the spin waves are damped. In the region of small q their damping is exponentially small, and, therefore, the spin waves behave like sharply defined quasi-particles. Of course, there is always the damping due to spin relaxation and characterized by a time τ_s, and the collision damping, which, for $ql\gg 1$, is small compared with the collisionless damping.

B. Degenerate statistics

The study of the behavior of the spin waves in the case of degeneracy is of special interest for metals, in which practically the only method of producing nonequilibrium orientation of the spins is spin injection. In the degenerate case it is easy to convince oneself that the connection between R and $\delta\mu$ is determined by the relation

$$R=\frac{\delta\mu+IR}{2}\frac{1}{N}\frac{\partial N}{\partial\mu}=\frac{3}{4}\frac{\delta\mu+IR}{\mu}. \quad (17)$$

The condition for the appearance of spontaneous magnetization (the Stoner criterion) is

$$\frac{I}{2}\frac{1}{N}\frac{\partial N}{\partial\mu}=1.$$

We assume that $I<2N\partial\mu/\partial N$, and so the sign of R coincides with the sign of $\delta\mu$.

For $q\ll p_F$ the dispersion equation can be represented in the form

$$\omega=\frac{I}{N}\sum_{\mathbf{k}}\frac{\mathbf{qv}}{\omega-\mathbf{qv}-IR+i\delta}(\delta\mu+IR+\mathbf{qv})\frac{\partial n_0}{\partial\varepsilon} \tag{18}$$

or

$$\omega=\frac{I}{N}(\delta\mu+\omega)\sum_{\mathbf{k}}\frac{\mathbf{qv}}{\omega-\mathbf{qv}-IR+i\delta}\frac{\partial n_0}{\partial\varepsilon}. \tag{18a}$$

Equation (18a) can be analyzed in general form without using the assumption of small $qv_F\ll IR$ and $\omega\ll IR$. Assuming ω to be real, we obtain after simple transformations the following dispersion equation:

$$qv_F=(IR-\omega)\,\mathrm{th}\frac{qv_F}{IR}\frac{\delta\mu}{\delta\mu+\omega}. \tag{19}$$

It follows from (19) that, since sign $\delta\mu=$ sign R, there are only two nontrivial cases: $I>0$ and $I<0$ for $R>0$. It is obvious that we need not analyze the cases with $R<0$, since, like the wave of the opposite polarization, they differ only in the sign of the frequency.

In the case of a ferromagnetic interaction ($I>0$), Eq. (19) has a solution only when $\omega<0$, as in the nondegenerate case. In fact, for $\omega>0$ the derivative of the right-hand side with respect to q is always smaller than or equal to v_F.

The spin-wave spectrum terminates at $\omega_t=-\delta\mu$, which corresponds to

$$q_t=(IR+\delta\mu)/v_F={}^2/_3Rp_F\ll p_F. \tag{20}$$

For small values of q the dispersion law is quadratic:

$$\omega=-(qv_F)^2/3IR. \tag{21}$$

The $\omega(q)$ dependence is given in this case in Fig. 1. The fact that the frequency has the opposite sign to that

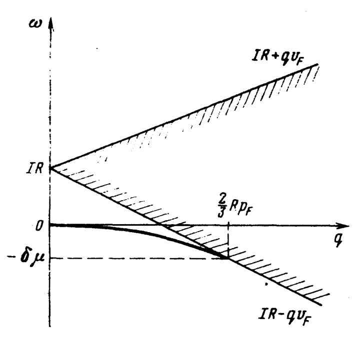

FIG. 1. Spin-wave dispersion law for a ferromagnetic exchange interaction $I>0$.

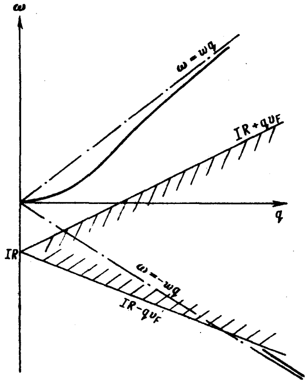

FIG. 2. Spin-wave dispersion law for an antiferromagnetic exchange interaction $I<0$.

in the case of the equilibrium ferromagnet is connected with the following circumstance. In equilibrium, the loss in kinetic energy of the electrons with the onset of spontaneous polarization is compensated by the large gain in exchange energy, and, therefore, a spin flip (which corresponds to the appearance of a spin wave) is associated with the expenditure of energy. In the nonequilibrium case, the loss in kinetic energy is not compensated by the gain in exchange energy, because of the small magnitude of the latter. Therefore, the excitation of spin waves is energetically favorable, and their frequency has the opposite sign. The same arguments are also valid in the case of nondegenerate statistics, but in this case the entropic contribution $T\Delta S$ appears in the role of the kinetic energy.

For an antiferromagnetic interaction ($I<0$) the dispersion equation (19) is conveniently represented in the form

$$qv_F=(|I|R+\omega)\,\mathrm{th}\frac{qv_F}{|I|R}\frac{\delta\mu}{\delta\mu+\omega}. \tag{22}$$

It follows from (22) that a solution exists for $\omega>0$ and for $\omega<-\delta\mu$, if $\delta\mu>|I|R$. However, as we shall see below, the oscillations with $\omega<-\delta\mu$ turn out to be strongly damped, and we shall not consider them. The solutions with $\omega>0$ also only exist when $\delta\mu>|I|R$. Because of (17), this condition is always fulfilled, since

$$\delta\mu=({}^4/_3\mu+|I|)R>|I|R. \tag{23}$$

For $\omega\gg\delta\mu$, from (22) we have, using (23),

$$\frac{1}{w}=\mathrm{th}\frac{1}{w}\left(1+\frac{4}{3}\frac{\mu}{|I|}\right), \tag{24}$$

where $w=\omega/q$. Thus, the spectrum coincides with the spin-wave spectrum in a normal metal.[3] But if $\omega\ll\delta\mu$, then

$$\omega=(qv_F)^2/3IR. \tag{25}$$

A plot of $\omega(q)$ for this case is presented in Fig. 2.

In the presence of pumping of the spins there is a sharp change in the spin-wave spectrum in the region of small wave vectors, i.e., in precisely the region in which experiments to detect spin waves in normal metals have been carried out.[4]

We now calculate the damping of the spin waves. It is obvious that, at $T=0$, so long as the frequency satisfies the condition $-qv_F<\omega-IR<qv_F$, the waves are undamped. It can be seen from analysis of Eqs. (19) and (22) that the frequencies of the waves investigated satisfy this condition, and, therefore, the waves are undamped. At finite temperatures damping appears:

$$\mathrm{Im}\,\omega=\frac{\pi}{4}\frac{\partial N}{\partial\mu}\frac{1}{N}\frac{\delta\mu+\omega}{qv_F}(\omega-IR)In_0\left\{\frac{m(\omega-qv_F)^2}{2q^2}-\mu\right\}, \quad (26)$$

where $n_0(x)$ is the Fermi function. Using (19), we can convince ourselves that $\mathrm{Im}\omega<0$ for all the oscillations considered.

We shall now discuss why, despite the inverted spin populations, the spin waves are damped. The point is that the exchange interaction conserves the total spin. Therefore, it cannot itself lead to the creation of waves. The Landau damping is loss of kinetic energy of a wave to kinetic energy of random motion of the individual particles, with conservation of the moment. Thus, after the disappearnce of a wave all that remains is the flipped spin of one electron. Of course, other interactions, e.g., interaction with a phonon via spin spin-orbit coupling, will lead to the creation of spin waves and to the generation of these phonons. If further damping mechanisms were absent we would have a laser situation. However, the usual mechanisms of damping of phonons are much stronger than the amplification associated with the creation of spin waves, and, therefore, a state with nonequilibrium orientation of the spins is stable.

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Bound states of fast electrons axially channeled in the single crystals

V. V. Kaplin and S. A. Vorob'ev

Nuclear Physics Institute of the Tomsk Polytechnic Institute
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A model of bound states of channeled fast electrons in crystals is developed. The case of axial channeling of particles with $E=0.5$–15 MeV along the principal crystallographic directions of silicon is considered in detail. The essential role of the high-lying states in anomalous passage through the crystals is demonstrated. A criterion is determined for separating deep-lying electronic states that determine the orientation dependence of the anomalous strong elastic scattering by atomic chains. It is shown that the deep-lying states are transformed into strongly absorbed states when the electron energy is increased. The quantitative data are used to discuss the angular distributions obtained by electron diffraction with an *ESG*-2.5 MeV accelerator. The bound-state model is used to explain the orientation dependences of the anomalous Rutherford scattering of electrons in single crystals. The orientational passage of relativistic electrons through thin crystals is also discussed. The energy dependence of the formation of bound states and the influence of neighboring atomic chains on the electron motion are considered. The possibility of realizing bound states of the molecular type in single crystals is discussed.

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Under certain conditions of the passage of charged particles through single crystals, the correlation in the scattering of periodically disposed atoms becomes appreciable, thereby changing the character of the elastic and inelastic collision processes. New phenomena such as channeling and blocking of fast heavy positively charged particles and the ensuing secondary orientational effects have been investigated in sufficient detail and have found practical applications.[1–3] To some degree of approximation, the principal regularities and physical premises developed for the case of heavy charged particles can be extended to orientational effects that take place when fast electrons and positrons pass through single crystals.[3,4] However, many features, such as the small particle mass, the effects of positron annihilation, or the negative sign of the electron charge, call for an independent study of the orientational phenomena involving electrons and positrons.

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