Thermodynamics of nonideal low-temperature plasma

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An expression for the Coulomb part of the thermodynamic potential of a plasma is obtained, on the basis of a quantum physical model of the plasma and a diagram technique, in the form of an expansion of the chiral type in the parameter $\zeta=\lambda^{-3}\exp(\beta\mu)$. Besides the known terms proportional to $\zeta^{3/2}$, $\zeta^2\ln\zeta$, and ζ^2, terms proportional to $\sim\zeta^{5/2}\ln\zeta$ and $\zeta^{5/2}$ are also calculated. The latter terms contain contributions from the states of both the continuum and the discrete energy spectrum. The results are used for a plasma in which exchange effects are small but the direct interaction plays an essential role.

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1. INTRODUCTION

It is known that a low-temperature plasma ($T\sim 10^5 - 10^5$ K) is a quantum system, since its physical properties depend, first, on the quantum character of the interaction of the charges that have a continuous energy spectrum, and second on the bound states of the discrete energy spectrum. To calculate the thermodynamic function of a plasma it is customary in the literature to use two approaches, dubbed the "chemical" and the "physical" models. In most cases the simpler chemical model is considered, where the presence of particles of definite species, having a continuous energy spectrum, is postulated beforehand.

The influence of the internal structure of the particles (bound states) on the thermodynamic functions is taken into account by introducing the partition functions for isolated particles. The latter assumption gives rise to certain difficulties connected with the divergence of the aforementioned sums. In practice of the calculations of the thermodynamic quantities, these divergences are eliminated by various kinds of "cutoffs" of the partition functions, based on one physical consideration or another. Thus, the introduction of the concept of an isolated particle with an internal structure is in the chemical model an artificial procedure, that leads to fundamental difficulties in the theory.

A consistent approach to the determination of the thermodynamic functions of a plasma is based on the physical model, where the nuclei and electrons are considered, while the Coulomb interaction between them is taken into account on the basis of the quantum-statistical theory. The contributions to the thermodynamic functions of the continuous and discrete spectra arise simultaneously and are finite. The results obtained from this point of view in the equilibrium theory of a nonideal plasma reduce to an expression for the Coulomb part of the thermodynamic potential $\Delta\Omega$ in the form of an expansion in powers of the quantity $\zeta=\lambda^{-3}\times e^{\beta\mu}$ [1–4]:

$$-\beta\Delta\Omega/V=A\zeta^{3/2}+B\zeta^2\ln\zeta+C\zeta^2, \qquad (1)$$

where $\beta=1/kT$; V is the volume of the system; $\lambda=(2\pi\beta\hbar^2/m)^{1/2}$ is the de Broglie wavelength of the particle; μ is the chemical potential of the particle; A, B, and C are known temperature-dependent coefficients. The first two terms of this expansion describe the contribution made to $\Delta\Omega$ by the continuous spectrum. The third term contains the contribution of the discrete spectrum in the Planck-Larkin form.[2,5] Strictly speaking, this contribution was calculated in[2–4] for a hydrogen plasma. The results, however, can be used also for other two-component systems where hydrogen-like ions and electrons are present.

An attempt to go outside the framework of formula (1) was undertaken in[6]. An expansion of the grand partition function in group integrals yielded here for $\Delta\Omega$ a formal expression containing arbitrary powers of the quantity ζ. Numerical calculations for a hydrogen plasma were performed on the basis of an expression for the second virial coefficient with an effective chain potential. No expansion in powers of ζ in explicit form was obtained, so that it is difficult to speak of the limits of the applicability of the results.

When the enthalpy calculated with the aid of (1) for a hydrogenlike cesium plasma is compared with its experimental values[7,8] obtained with a cesium shock tube,[9] a discrepancy is observed between the theoretical and experimental data even at relatively low values of the interaction parameter $\gamma=\beta e^2[4\pi\beta e^2\times(\zeta_i+\zeta_e)]^{1/2}$.

This suggests the natural assumption that to describe satisfactorily the experimental results it is necessary to add to (1) the next higher terms of the expansion in ζ. These next higher terms are $D\zeta^{5/2}\ln\zeta$, $E\zeta^{5/2}$, and $F\zeta^3$. A rigorous theoretical calculation of the coefficient F encounters great mathematical difficulties, since the problem reduces here to the quantum-mechanical three-body problem. The terms $D\zeta^{5/2}\ln\zeta$ and $E\zeta^{5/2}$ are due, as is the Debye term $A\zeta^{3/2}$ in (1), to polarization effects in the plasma, and the calculation of the coefficients D and E can be reduced to the two-body quantum-mechanical problem. We present below the results of the calculation of these coefficients.

2. ANALYSIS OF THE DIAGRAMS

To calculate the thermodynamic potential it is convenient to use a diagram technique. The required diagrams are chosen on the basis of an analysis of the

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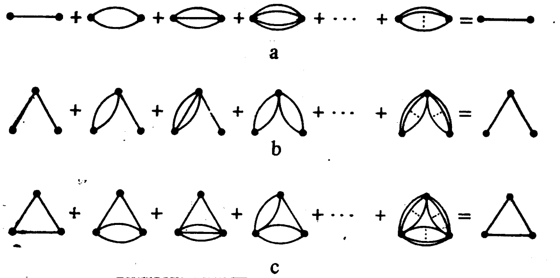

FIG. 1. Classical diagrams corresponding to the second and third virial coefficients.

perturbation-theory series for the Bloch equation[3,10] and is a cumbersome and tiresome procedure. We choose therefore a simple path towards the necessary explanations, which is made possible by the existing analogy between the construction of the virial series for ordinary gases and the construction of an expansion of the type (1) for a plasma. The necessary principles of the indicated analogy can be explained by using classical diagrams as an example.

It is known that in the classical theory of ordinary nonideal gases the contribution to the second virial coefficients comes from a sum of diagrams of type a, while the contribution to the third virial coefficient comes from a sum of diagrams of type b and c in Fig. 1. The points mark here the coordinates of the particles, the thin lines correspond to the potentials of the interaction between these particles. The sums of the diagrams with thin lines describe terms of the perturbation-theory series for the canonical Gibbs distribution.[11] The diagrams with the thick lines correspond to the usual Mayer diagrams[12] and are the results of summation of perturbation-theory diagrams.

The analogy between the diagrams for the virial series, shown in Fig. 1, and the diagrams for the plasma expansion of type (1) arises if the problem of finding this expansion is formulated as a problem of the interaction of two, three, etc. particles situated in the field of the remaining particles. The problem consists then of finding the effective potential that characterizes the interaction of the chosen particles with one another in the presence of the field particles. If the effective potential is impaired in this case, then under certain conditions the conversion of the diagrams of Fig. 1 into plasma diagrams reduces to a formal replacement of the interaction lines by effective interaction lines.

In fact, it was shown in[3,10] that at $\lambda_e\varkappa \ll 1$ (λ_e is the de Broglie wavelength of the electron and $\varkappa = [4\pi\beta e^2(\zeta_i+\zeta_e)]^{1/2}$ is the reciprocal Debye radius) the many-particle diagrams that describe collective and individual interactions in a plasma within the framework of the applicability of the expansion (1) actually reduce to a sum of ladder diagrams with a Debye effective potential whose classical analog is the sum of diagrams a in Fig. 1. A similar situation obtains also in the case when account is taken in the expansion (1) of the next higher terms: $D\zeta^{5/2}\ln\zeta$, $E\zeta^{5/2}$, and $F\zeta^3$. These terms are described by quantum analogs of the diagrams b and c of Fig. 1, and the calculation of the contributions made by them to the thermodynamic potential is connected with the solution of the three-particle Schrödinger equation with a Debye effective potential. However, if we confine ourselves to the terms $D\zeta^{5/2}\ln\zeta$ and $E\zeta^{5/2}$, then the problem becomes much simpler and reduces to an approximate solution of the two-particle Schrödinger equation. To this end it is necessary to select from the indicated diagrams all the diagrams that make contributions $\sim\zeta^{5/2}\ln\zeta$ and $\zeta^{5/2}$. The selection procedure is greatly simplified if account is taken of two circumstances that take place for both classical and quantum diagrams. First, all the diagrams having a point connected by only one interaction line each with other points make a summary zero contribution to the thermodynamic potential, since the plasma is electrically neutral.[10] Second, diagrams in which all points are interconnected by not less than three interaction lines yield the lowest order ζ^3, with the exception of the fourth diagram of Fig. 1c, the contribution of which to the thermodynamic potential is $\sim\zeta^{5/2}$. It is then necessary to consider from among the diagrams of Fig. 1b only those in which one of the points is connected to two interaction lines (the first of them is the fourth diagram in the indicated firm). The remaining diagrams should be excluded from consideration because all the diagrams, starting with the fifth, make contributions $\sim\zeta^3$ and $\zeta^3\ln\zeta$, while the first three diagrams $\sim\zeta^{3/2}$, $\zeta^{5/2}\ln\zeta$, $\zeta^{5/2}$ are contained in the diagrams of Fig. 1a. The latter situation is the result of the use of the Debye effective potential which corresponds, as is well known, to a sum of chain diagrams where each point is connected with the remaining ones only by two interaction lines and integration is carried out over the coordinates of all the points. Therefore if, for example, integration is carried out in the first three diagrams of Fig. 1c with respect to the coordinate of one of the points connected to two interaction lines, then the obvious result are the diagrams contained in the second, third, and fourth diagrams of Fig. 1a, respectively.

The sums of the classical diagrams and their quantum analogs that must be considered are shown in Fig. 2. In the quantum diagrams the loops correspond, as usual to particle propagation and the lines denote the interaction potentials. Diagrams of similar type were used earlier in the calculation of the effect of the shift of the ground level of an atom on the thermodynamic functions of a plasma.[13] It must be noted that consideration of quantum analogs of classical diagrams is equivalent to the use of Boltzmann statistics in quantum calculations. This leads to an incorrect allowance for the exchange effects; in particular, if the results are obtained in the form of expansion in the interaction parameter γ and the quantum parameter $\lambda_e\varkappa$, then the coefficients of the powers of the last parameters have incorrect values. To obtain the correct values in terms of the parameters $\lambda_e\varkappa$ it is necessary, naturally, to use quantum statistics.

We confine ourselves below to consideration of quantum diagrams that have classical analogs and give the correct dependence on the interaction parameter γ. In

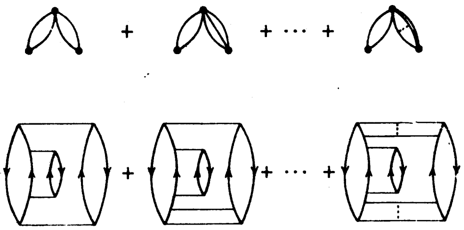

FIG. 2. Three-particle classical and quantum diagrams that make contributions $\sim\zeta^{5/2}\ln\zeta$ and $\zeta^{5/2}$ to the thermodynamic potential of the plasma.

the calculation of the contributions from these diagrams we shall neglect the exchange terms, i.e., assume that the quantum parameter $\lambda_e\varkappa$ is small, but we shall take into account in the final results the exchange effect in first order in the indicated parameter, using the results of[1,14].

Thus the expressions obtained below for the thermodynamic potential contain all the terms $\sim\zeta^{5/2}\ln\zeta$ and $\zeta^{5/2}$ proportional to the parameter $\gamma(\zeta^{5/2}\ln\zeta \sim \zeta\gamma^3\ln\gamma$, $\zeta^{5/2}\sim\zeta\gamma^3)$, and the analogous terms proportional to $(\lambda_e\varkappa)^2(\zeta^{5/2}\ln\zeta \sim \zeta\gamma(\lambda_e\varkappa)^2\ln\gamma$; $\zeta^{5/2}\sim\zeta\gamma(\lambda_e\varkappa)^2)$ have been left out. This approach is justified for a low-temperature plasma, where the conditions $(\lambda_e\varkappa)^2 \ll 1$ and $\gamma \lesssim 1$ hold in the temperature and pressure range of practical significance.[15]

To complete the construction of the theory, accurate to terms $\zeta^{5/2}\ln\zeta$ and $\zeta^{5/2}$ inclusive, it is necessary also to take into account a diagram shown in Fig. 3 in the classical and quantum variants.

3. CALCULATION OF THE PLASMA THERMODYNAMIC POTENTIAL

By summing ladder diagrams with a Debye effective potential, an expression was obtained for $\Delta\Omega$ in a preceding paper[3] and is an analog of the second virial coefficient for ordinary gases

$$-\frac{\beta\Delta\Omega_{ab}^{(1)}}{V} = -\zeta_a\zeta_b\lambda_{ab}^3\beta\int_0^{e^2}\frac{de^2}{e^2}\int\sum_{\{n\}}[\psi_n^*(\mathbf{r})e^{-\beta E_n}\psi_n(\mathbf{r}) - \psi_n^{*0}(\mathbf{r})e^{-\beta E_n^0}\psi_n^0(\mathbf{r})]\Phi_{ab}(\mathbf{r})\,d\mathbf{r}. \tag{2}$$

Here $\psi_n(\mathbf{r})$ and E_n are respectively the eigenfunction and eigenvalues of the three-dimensional Schrödinger equation with a Debye effective potential

$$\Phi_{ab}(\mathbf{r}) = \frac{Z_aZ_b}{|\mathbf{r}|}e^2\exp(-\varkappa|\mathbf{r}|);$$

$\psi_n^0(\mathbf{r})$ and E_n^0 are the eigenfunctions and eigenvalues of the Schrödinger equation for free motion; Z_a is a number characterizing the value of the charge. Calculation of the contribution made to the Coulomb part of the thermodynamic potential by the sum of the quantum diagrams of Fig. 2 is carried by the same method as the derivation of (2).

As a result we obtain the expression

$$-\frac{\beta\Delta\Omega_{ab}^{(2)}}{V} = -\frac{3}{8}\zeta_a\zeta_b\lambda_{ab}^3\beta^2(Z_a^2+Z_b^2)\int_0^{e^2}\varkappa\,de^2\int\sum_{\{n\}}[\psi_n^*(\mathbf{r})e^{-\beta E_n}\psi_n(\mathbf{r}) - \psi_n^{*0}(\mathbf{r})e^{-\beta E_n^0}\psi_n^0(\mathbf{r})]\Phi_{ab}(\mathbf{r})\,d\mathbf{r}. \tag{3}$$

In view of the dependence of the quantities $\psi_n(\mathbf{r})$, E_n, and $\Phi_{ab}(\mathbf{r})$ on the interaction parameter γ, the expansions of (2) and (3) contain arbitrary powers of this parameter, and, in addition, terms of the type $\gamma^n\ \ln\gamma$ appear. The problem is to separate from (2) and (3) all those terms of the expansions in the parameter γ, whose contributions to $\Delta\Omega$ are of the order $\zeta^{5/2}\ln\zeta$ and $\zeta^{5/2}$, inclusive. Let us indicate the main procedures used for this purpose. For like charged particles that have only a positive energy spectrum, it is possible to use in (2) and (3) the quasiclassical values of $\psi_n(\mathbf{r})$ and to replace the sum over $\{n\}$ by an integral over the number of states. For particles of unlike sign, the energy-level spectrum consists of a discrete spectrum of negative values and a continuous spectrum of positive values. In this case it is possible to go over in (2) and (3) from sums to integrals by the indicated method not only for positive energies, but also for the quasiclassical part of the negative energies. It is then necessary to choose in the sum over $\{n\}$ a principal quantum number n_0 such that $\beta|E_{n0}|\ll 1$. Then, in view of the quasiclassical character of the states with large quantum numbers, the spectrum of the negative energies $E_{n_0} \leq E_n \leq 0$ will be dense enough and the corresponding part of the sum over $\{n\}$ can be replaced by an integral.[2,3] This integral is written jointly with the integral of the positive-energy spectrum.

Separating in (2) the contribution made to the thermodynamic potential by the continuous spectrum, we have

$$-\frac{\beta(\Delta\Omega_{ab}^{(1)})_H}{V} = \frac{\varkappa_a^2\varkappa_b^2}{12\pi\varkappa}I_{ab}(\gamma), \tag{4}$$

where for particles with the same charge

$$I_{ab}(\gamma) = \frac{3}{Z_aZ_b\gamma}\int_0^1 dg\int_0^\infty[1-\exp(-\beta\Phi_{ab})]\exp(-g^{1/2}r)\,r\,dr, \tag{5}$$

and for oppositely charged particles

$$I_{ae}(\gamma) = \frac{3}{Z_a\gamma_0}\int_0^1 dg\left\{\int_0^{r_0}\left[\frac{2}{\sqrt{\pi}}\exp(-\beta\Phi_{ae})\Gamma\left(\frac{3}{2}; -\beta\Phi_{ae}+\beta E_{n_0}\right)-1\right] \times (-g^{1/2}r)\,r\,dr + \int_{r_0}^\infty[\exp(-\beta\Phi_{ae})-1]\exp(-g^{1/2}r)\,r\,dr\right\}. \tag{6}$$

The integrals are written in dimensionless variables in the coordinate $r\varkappa\to r$ and the charge $e^2\to ge^2$; $\Gamma(3/2; x)$ is the incomplete gamma function; r_0 is the root of the equation $E_{n_0} - \Phi_{ae}(r_0) = 0$. If the interaction parameter γ is regarded as a small quantity, then we can find the approximate values of the integrals written out above. It is necessary for this purpose to expand the integrands of (5) and (6) in the perturbation-theory series, where the perturbation is the difference between the Debye and the nucleon potentials, the maximum value of this dif-

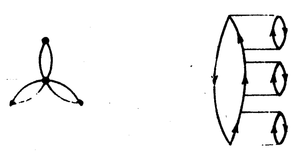

FIG. 3. Four-particle diagram (classical and quantum) that makes a contribution $\sim\zeta^{5/2}$ to the thermodynamic potential of the plasma.

ference not exceeding the small quantity γ.

As a result, accurate to terms $\sim\gamma^2\ln\gamma$ and γ^2 inclusive, we get

$$I_{ab}(\gamma)=1+\frac{Z_aZ_b}{2}\gamma(2C+\ln 3Z_aZ_b\gamma-2)+\frac{4}{9}Z_a^2Z_b^2\gamma^2\left(2C+\ln 4Z_aZ_b\gamma-\frac{35}{12}\right); \tag{7}$$

$$I_{ae}(\gamma)=1-\frac{3\sqrt{\pi}}{4}(2\gamma a\varkappa)^{1/2}n_0+\frac{9\sqrt{\pi}}{14}Z_a\gamma(2\gamma a\varkappa)^{1/2}n_0+\frac{\sqrt{\pi}}{6}\frac{2a\varkappa}{Z_a}(2\gamma a\varkappa)^{1/2}\left(n_0^3+\frac{3}{2}n_0^2+\frac{1}{2}n_0\right)+\frac{Z_a^2}{n_0}\gamma\left(\frac{\pi\gamma}{2a\varkappa}\right)^{1/2}-\frac{15}{44}\frac{Z_a^3\gamma^2}{n_0}\left(\frac{\pi\gamma}{2a\varkappa}\right)^{1/2}-\frac{Z_a\gamma}{2}(2C+\ln 3Z_a\gamma-2)+\frac{4}{9}Z_a^2\gamma^2\left(2C+\ln 4Z_a\gamma-\frac{35}{12}\right); \tag{8}$$

where $C=0.5772$ is the Euler constant; $a=\hbar^2/m_ee^2$ is the Bohr hydrogen radius. The n_0-dependent terms in (8) describe the contribution made by the quasiclassical part of the negative spectrum; the last two terms correspond to a positive spectrum and coincide with (7). As $n_0\to\infty$ expression (8) tends to infinity owing to the presence of terms proportional to n_0 and n_0^3, but it will be shown below that this divergence is canceled by corresponding terms in the expression for the discrete spectrum.

The contribution made to the thermodynamic potential by the discrete spectrum is determined by part of the sum (2) over the principal quantum number $1\leqslant n\leqslant n_0$. Here, too, it is necessary to expand $\psi_n(\mathbf{r})$ and E_n in the perturbation-theory series and retain those powers of the parameter γ which contribute to the thermodynamic potential, accurate to terms $\sim\zeta^{5/2}$ inclusive. To this end, we can regard the $\psi_n(\mathbf{r})$ in (2) as Coulomb wave functions, and we can retain for E_{n_0} the first-order correction to the Coulomb level $\beta E_{n_0}\approx\beta E_{n_0}^{\text{Coul}}+Z_n\gamma$. We then obtain for the discrete part of the spectrum the expression

$$-\frac{\beta(\Delta\Omega_{ae}^{(1)})_d}{V}=\zeta_a\zeta_e\lambda_e^3\sum_{n=1}^{n_0}n^2e^{\beta I_n}\left[1-e^{-\beta I_n}-\frac{2}{3}Z_a\gamma F\left(1;\frac{7}{4};-\beta I_n\right)-\frac{4}{7}Z_a\gamma\beta I_nF\left(1;\frac{11}{4};-\beta I_n\right)\right], \tag{9}$$

where

$$F(1;b;-z)=(b-1)\int_0^1 e^{-zx}(1-x)^{b-2}dx\quad(b>1)$$

is a confluent hypergeometric function; $I_n=-E_n^{\text{Coul}}$. It is easily noted that as $n\to\infty$ expression (9) tends to infinity. But if (9) is combined with the n_0-dependent terms of (8), then the result is finite as $n_0\to\infty$. To this end it is necessary to write the numbers n_0 and n_0^3 in (8) in the form of the sums

$$n_0=\sum_{n=1}^{n_0}1;\quad n_0^3=3\sum_{n=1}^{n_0}n^2-\frac{3}{2}n_0^2-\frac{1}{2}n_0.$$

The combined result for the entire negative spectrum is written in the form

$$-\frac{\beta(\Delta\Omega_{ae}^{(1)})^-}{V}=\zeta_a\zeta_e\lambda_e^3\left[\sum_{n=1}^{\infty}n^2e^{\beta I_n}[\omega_n^{(1)}-Z_a\gamma\omega_n']+\Delta(n_0)\right]. \tag{10}$$

Here $\omega_n^{(1)}$, ω_n', and $\Delta(n_0)$ stand for the expressions

$$\omega_n^{(1)}=1-e^{-\beta I_n}-\beta I_ne^{-\beta I_n}, \tag{11}$$

$$\omega_n'=\frac{2}{3}F\left(1;\frac{7}{4};-\beta I_n\right)+\frac{4}{7}\beta I_nF\left(1;\frac{11}{4};-\beta I_n\right)-\frac{2}{3}e^{-\beta I_n}-\frac{6}{7}\beta I_ne^{-\beta I_n}, \tag{12}$$

$$\Delta(n_0)=\frac{4}{3}\frac{1}{n_0}(\beta I_1)^2\left(1-\frac{15}{44}Z_a\gamma\right)-\sum_{n=n_0+1}^{\infty}n^2e^{\beta I_n}(\omega_n^{(1)}-Z_a\gamma\omega_n'). \tag{13}$$

The factor $\omega_n^{(1)}-Z_a\gamma\omega_n'$ has the measuring of a weighting function and ensures convergence of the sum (10). We note that the sum over n contained in (10) with the weighting factor (11) is the Planck-Larkin formula.

We discuss now the possibility of going to the limiting form of (1) as $n_0\to\infty$ and of eliminating by the same token the uncertainty in the choice of the number n_0. We investigate for this purpose the quantity (13). Since $(\beta I_n)^2\ll 1$ at $n>n_0$, the expression $e^{\beta I_n}(\omega_n^{(1)}-Z_a\gamma\omega_n')$ can be expanded in powers of βI_n with only terms $\sim(\beta I_n)^2$ retained. If we put by way of estimate also $\gamma=0$, then at values of n from 1 to 5 the expression $\Delta(n_0)/(\beta I_1)^2$ decreases rapidly from 1.21 to 0.155 and then tends slowly to zero with increasing n_0. If $n_0>5$, then $\Delta(n_0)\lesssim 0.155(\beta I_1)^2$. At $\beta I_1>1$ the quantity $\Delta(n_0>5)$ then becomes negligibly small in comparison with the first term of the sum (10); on the other hand, this quantity can be neglected at $\beta I_1>1$ because of the smallness of the numerical coefficient.

Let us write out the expression for the contribution made to the thermodynamic potential by the positive spectrum. It is obtained with the required accuracy if $I_{ab}(\gamma)$ in (4) is taken in the form (7):

$$-\frac{\beta(\Delta\Omega_{ab}^{(1)})^+}{V}=\frac{\varkappa_a^2\varkappa_b^2}{12\pi\varkappa}\left[1+\frac{Z_aZ_b}{2}\gamma(2C+\ln 3|Z_aZ_b|\gamma-2)+\frac{4}{9}Z_a^2Z_b^2\gamma^2\left(2C+\ln 4|Z_aZ_b|\gamma-\frac{35}{12}\right)\right]. \tag{14}$$

This expression is valid for particles with either like or unlike charges. The sum of (10) and (14) describes the total contribution made to the thermodynamic potential by the ladder diagrams, accurate to terms $\sim\zeta^{5/2}\ln\zeta$ and $\zeta^{5/2}$ inclusive.

Expression (3) differs from (2) only by a factor $\frac{3}{8}\beta\varkappa e^2(Z_a^2+Z_b^2)$, so that all the calculation methods used for (2) are fully applicable also to (3). As a result we obtain for the positive and negative spectra the expressions

$$-\frac{\beta(\Delta\Omega_{ab}^{(2)})^+}{V}=\frac{\varkappa_a^2\varkappa_b^2}{12\pi\varkappa}(Z_a^2+Z_b^2)\left[\frac{Z_aZ_b}{8}\gamma^2\left(2C+\ln 3|Z_aZ_b|\gamma-\frac{11}{6}\right)+\frac{3}{16}\gamma\right], \tag{15}$$

$$-\frac{\beta(\Delta\Omega_{ae}^{(2)})^-}{V}=\frac{3}{8}\zeta_a\zeta_e\lambda_e^3\gamma(Z_a^2+1)\sum_{n=1}^{\infty}n^2e^{\beta I_n}\omega_n'', \tag{16}$$

where

$$\omega_n'' = \frac{4}{7}\beta I_n\left[F\left(1;\frac{11}{4};-\beta I_n\right)-e^{-\beta I_n}\right].$$

Relation (16) has a limiting form ($n_0 \to \infty$) that can be obtained under the same conditions as (10). The function ω_n'' has, just as in (11) and (12), the meaning of a weighting function.

We consider now the fourth diagram of Fig. 1c and the diagram of Fig. 3. They are calculated in accordance with the usual principles, and in the classical limit the final results have the respective forms

$$-\frac{\beta(\Delta\Omega_{abc}^{(3)})^+}{V} = \frac{\varkappa_a^2\varkappa_b^2\varkappa_c^2}{12\pi\varkappa^3}\frac{9\cdot 1.0379}{88}\gamma^2 Z_a Z_b Z_c (Z_a+Z_b+Z_c), \quad (17)$$

$$-\frac{\beta(\Delta\Omega_{ab}^{(4)})^+}{V} = -\frac{\varkappa_a^2\varkappa_b^2}{12\pi\varkappa}\frac{9}{88}\gamma^2(Z_a^4+Z_b^4). \quad (18)$$

The sum of the expressions (10) and (14)–(18) over all particle species, together with the thermodynamic potential of an ideal gas

$$-\frac{\beta\Omega_0}{V} = \sum_a \zeta_a$$

and the exchange corrections from [14], yields the value of the total thermodynamic potential of the plasma in the employed approximation;

$$-\frac{\beta\Omega}{V} = \beta p = \sum_a \zeta_a + \sum_{a,b}\frac{\varkappa_a^2\varkappa_b^2}{12\pi\varkappa}[1+f_{ab}(\gamma)]$$
$$-\frac{\varkappa_e^4}{12\pi\varkappa}\left(\frac{1+\ln 2}{\sqrt{2}}-1\right)\frac{3}{16}\lambda_e\varkappa - \frac{\varkappa_e^2\varkappa}{12\pi}\frac{3}{16}\lambda_e\varkappa$$
$$+\frac{1}{2}\sum_a \zeta_a\zeta_e\lambda_e^3\sum_{\{n\}} g_n e^{\beta I_n}(\omega_n^{(1)}-\gamma\omega_n^{(2)}), \quad (19)$$

where

$$f_{ab}(\gamma) = \frac{Z_a Z_b}{2}\gamma(2C+\ln 3|Z_a Z_b|\gamma - 2) + \frac{3}{16}\gamma(Z_a^2+Z_b^2)$$
$$-\frac{9}{88}\gamma^2(Z_a^4+Z_b^4) + \frac{27\cdot 1.0379}{88}\gamma^2 Z_a Z_b\left(\sum_c \frac{\varkappa_c^2}{\varkappa^2}Z_c^2\right)$$
$$+\frac{4}{9}Z_a^2 Z_b^2\gamma^2\left(2C+\ln 4|Z_a Z_b|\gamma - \frac{35}{12}\right)$$
$$+\frac{1}{8}Z_a Z_b(Z_a^2+Z_b^2)\gamma^2\left(2C+\ln 3|Z_a Z_b|\gamma - \frac{11}{6}\right), \quad (20)$$

$$\omega_n^{(2)} = Z_a\omega_n' - \frac{3}{8}(Z_a^2+1)\omega_n'' = \frac{2}{3}Z_a(1-e^{-\beta I_n})$$
$$+\frac{4}{7}\beta I_n\left[\frac{Z_a}{3}-\frac{3}{8}(Z_a^2+1)\right]F\left(1;\frac{11}{4};-\beta I_n\right)$$
$$-\frac{3}{7}\beta I_n\left[2Z_a - \frac{1}{2}(Z_a^2+1)\right]e^{-\beta I_n}. \quad (21)$$

The contributions $\sim \zeta^{5/2}\ln\zeta$ and $\zeta^{5/2}$ to formula (19) are made by the last four terms of (20) and by the last term of (19). The remaining terms coincide with expression (1).[1–4] With an aim of using in the future formula (19) not only for a hydrogen plasma, we have introduced in the last expression, under the summation sign, the quantity g_n that serves as the statistical weight of an isolated particle ($g_n = 2n^2$ for hydrogen). In this form, formula (19) can be used in a certain approximation also to calculate a non-hydrogen plasma. If the ion is not a nucleus, then it is necessary to use for the level energies either experimental or approximately calculated values. If we add to (19) the equations for the charged-particle densities

$$n_a = -\frac{\beta}{V}\zeta_a\left(\frac{\partial\Omega}{\partial\zeta_a}\right)_{\beta,V} \qquad (a=1,2,\ldots),$$

then we obtain the equation of state of the plasma in parametric form.

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