

# Generalized susceptibilities of a one-dimensional system of electrons in a magnetic field

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The effect of a magnetic field on the low-temperature behavior of the generalized susceptibilities of a one-dimensional electron system which describe singlet ( $\chi_{SS}$ ) and triplet ( $\chi_{TS}$ ) superconducting fluctuations and also fluctuations of the dielectric ( $\chi_{CDW}$ ) and antiferromagnetic ( $\chi_{SDW}$ ) types is investigated. The limiting case  $\omega_0 \gg T$  ( $2\omega_0$  is the magnitude of the Zeeman splitting) is considered. It is shown that in this limit there is a displacement of the positions of the singularities in the functions  $\chi_{SS}(k)$  and  $\chi_{CDW}(q)$ : the latter may exhibit singular behavior as  $T \rightarrow 0$  at a total momentum  $|k| \sim 2\omega_0/v_F$  and at a momentum transfer  $|q| \approx 2p_F \pm 2\omega_0/v_F$  ( $p_F$  and  $v_F$  are the Fermi momentum and velocity). Therefore, a phase transition to an inhomogeneous superconducting state or to a Peierls state with two density waves may occur in a quasi-one-dimensional crystal located in a sufficiently strong magnetic field. The asymptotically exact form of the susceptibility components which are singular as  $T \rightarrow 0$  is found in the case when the coupling constant at small distances  $g_1 \geq 0$  or when the strong field condition  $g_1 \ln(\Lambda/\omega_0) \ll 1$  ( $\Lambda$  is the cutoff parameter) is satisfied for any sign of  $g_1$ . The effect of a magnetic field on the low temperature behavior of the impurity part of the electrical resistivity of a quasi-one-dimensional conductor is discussed.

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## 1. INTRODUCTION

The generalized susceptibilities of a one-dimensional system of interacting electrons in the absence of any magnetic field have been calculated in a whole series of articles.<sup>[1-4]</sup> The effect of a magnetic field on the low temperature dynamical instabilities of a one-dimensional system of weakly interacting electrons<sup>[1]</sup> is investigated in the present article. The formulation of such a problem makes sense because in real quasi-one-dimensional conductors, in view of the small probability of tunneling skips of the electrons from filament to filament over a wide range of field strengths, one can neglect the field's influence on the orbital motion and regard it as having an effect only on the electron spins. (The appropriate quantitative criteria are given by Dzyaloshinskii and Kats.<sup>[5]</sup>)

The generalized susceptibilities describing singlet ( $\chi_{SS}$ ) and triplet ( $\chi_{TS}$ ) superconducting fluctuations and also fluctuations of the dielectric ( $\chi_{CDW}$ ) and antiferromagnetic ( $\chi_{SDW}$ ) types are evaluated below by the method of parquet summation in the limiting case  $\omega_0 \gg T$  ( $\omega_0 = \mu H$ ,  $\mu$  denotes the Bohr magneton). The distinguishing features of this case is that logarithmic singularities with respect to the temperature arise in the electron-electron and electron-hole channels depending on the spin structure of the channel, these singularities appearing not only for total momentum  $k=0$  and momentum transfer  $q = \pm p_F$  as in the absence of a magnetic field,<sup>[1,6]</sup> but also for  $k = \pm 2\omega_0/v_F$  and  $|q| = 2p_F \pm 2\omega_0/v_F$  ( $p_F$  is the limiting momentum in zero field and  $v_F$  is the Fermi velocity). In this connection it is found that the response functions  $\chi_{TS}(k)$  and  $\chi_{SDW}(q)$  exhibit singular behavior as  $T \rightarrow 0$  if  $k=0$  and  $q = \pm 2p_F$ , that is, just as for the case  $H=0$ , but the functions  $\chi_{SS}(k)$  and  $\chi_{CDW}(k)$  only exhibit singular behavior for  $k = 2\omega_0/v_F$  and  $|q| = 2p_F \pm 2\omega_0/v_F$ .

One can easily establish the positions of the singulari-

ties with respect to the total momentum and momentum transfer in generalized susceptibilities of different symmetry if it is taken into consideration that in the limit  $\omega_0 \gg T$  of interest to us the separation of the electron sub-bands corresponding to the two projections of the spin along the direction of the magnetic field appreciably exceeds the temperature broadening of the Fermi distribution functions near the limiting momenta  $\pm p_F^\sigma$ , where  $p_F^\sigma = p_F - \sigma\omega_0/v_F$  and  $\sigma = \pm 1$  is an index labelling the sub-band. The function  $\chi_{TS}(k)$  characterizes the superconducting correlations of the electrons of a single sub-band having momenta near  $p_F^\sigma$  and  $-p_F^\sigma$  so that  $k \sim 0$ , and  $\chi_{SDW}(q)$  characterizes the correlations between an electron in one sub-band and a hole in another for which  $q \sim p_F^\sigma + p_F^{\sigma'} = 2p_F$ . For superconducting correlations of the electrons in different sub-bands ( $\chi_{SS}(k)$ )  $k \sim p_F^\sigma - p_F^{\sigma'} = -2\sigma\omega_0/v_F$  and the Peierls fluctuations of the density ( $\chi_{CDW}(q)$ ) in each sub-band are characterized by their own "diameter":  $q \sim 2p_F^\sigma = 2p_F - 2\sigma\omega_0/v_F$ .

The parquet vertex parts  $\Gamma_{\sigma_1\sigma_2\sigma_3\sigma_4}(p_1p_2|p_3p_4)$  were evaluated by Dzyaloshinskii and Kats<sup>[5]</sup> for a one-dimensional electron system in the region  $\omega_0 \gg T$  with the following ordering of the momenta:  $p_1 \sim -p_2 \sim -p_3 \sim p_4 \sim p_F$ , that is, for  $|k|v_F \sim |q - 2p_F|v_F \lesssim T$ . In this connection pole singularities appeared in the vertex functions  $\Gamma_{\sigma\sigma\sigma\sigma}$  and  $\Gamma_{\sigma\sigma\sigma\sigma'}$  at a finite temperature.<sup>[5]</sup> As is indicated below, for evaluation of the generalized susceptibilities it is necessary to know the vertex parts  $\Gamma_{\sigma\sigma'\sigma\sigma'}$  in the region  $\omega_0 \gg T$  for

$$p_1 \sim p_4 \sim p_F^{\sigma'}, \quad p_2 \sim p_3 \sim -p_F^{\sigma'}, \quad \sigma' = \pm\sigma,$$

which corresponds to the two possibilities:

$$|k|v_F \sim 2\omega_0, \quad |q - 2p_F|v_F \sim T,$$

or

$$|k|v_F \sim T, \quad |q - 2p_F|v_F \sim 2\omega_0.$$

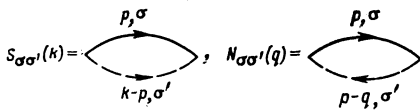


FIG. 1.

It is found that in these regions  $\Gamma$  only depends on the logarithmic field variable  $h = \ln(\Lambda/\omega_0)$  ( $\Lambda$  is the cutoff parameter) coinciding with its value for  $H=0$ <sup>[1,2]</sup> after the formal replacement of  $T$  by  $\omega_0$ .

If the interaction of the electrons at small distances is repulsive ( $g_1 \geq 0$ ) or if  $H$  is so large that the condition  $|g_1| h \ll 1$  is satisfied for arbitrary sign of the constant  $g_1$ , the behavior of  $\Gamma(h)$  turns out to be nonsingular. In this case the weak coupling regime is realized, when the parquet approximation used in the present work leads to asymptotically exact results for the generalized susceptibilities. In this connection the singular parts of the latter quantities take the form of power functions  $T^\gamma$ , where the exponent  $\gamma$  is proportional to a definite vertex part and is small. In the concluding sections of this article we discuss the phase diagram for the possible states of a one-dimensional system for  $T=0$ ,  $H \neq 0$  and also investigate the effect of a magnetic field on the low temperature behavior of the impurity part of the electrical resistivity of a quasi-one-dimensional conductor in regard to objects of the type (SN)<sub>x</sub><sup>[7,8]</sup> and HMTSF-TCNQ,<sup>[9]</sup> which possess metallic conductivity down to temperatures of the order of 1°K.

## 2. PARQUET ELEMENTS

In the presence of a magnetic field the electron spectrum near the Fermi energy has the form

$$\xi_\sigma(p) = \xi(p) + \sigma\omega_0 = v_F(|p| - p_F^\sigma), \quad \sigma = \pm 1.$$

In the following diagrams the electron Green's function will be depicted by a solid line if  $p \sim p_F^\sigma$  and by a dashed line if  $p \sim -p_F^\sigma$ .

The basic elements of the parquet—representing the graphs for the second-order vertex part—are shown in Fig. 1. They represent the diagrams of the zero-order approximation for the generalized susceptibilities, where

$$\chi_{rs}^{(0)} \sim S_{\sigma\sigma}, \quad \chi_{ss}^{(0)} \sim S_{\sigma,-\sigma}, \quad \chi_{CDW}^{(0)} \sim N_{\sigma\sigma}, \quad \chi_{SDW}^{(0)} \sim N_{\sigma,-\sigma}.$$

For  $|k|$  and  $|q - 2p_F| \ll p_F$  we have

$$\begin{aligned} \xi_\sigma(k-p) &= \xi_\sigma(p) - [kv_F + (\sigma - \sigma')\omega_0], \\ \xi_\sigma(p-q) &= -\xi_\sigma(p) + [(q - 2p_F)v_F + (\sigma + \sigma')\omega_0]. \end{aligned} \quad (1)$$

Changing from an integration over  $p$  to an integration with respect to  $\xi_\sigma(p)$  and taking Eq. (1) into consideration, we obtain

$$S_{\sigma\sigma}(k, \omega_m) \sim \ln \frac{\Lambda}{\max\{|kv_F + (\sigma - \sigma')\omega_0|, |\omega_m|, T\}}, \quad (2)$$

$$N_{\sigma\sigma}(q, \omega_m) \sim \ln \frac{\Lambda}{\max\{|(q - 2p_F)v_F + (\sigma + \sigma')\omega_0|, |\omega_m|, T\}}, \quad (3)$$

where  $\omega_m = 2m\pi T$ <sup>[10]</sup> and  $\Lambda \gg T$ ,  $|\omega_m|$ ,  $|k|v_F$ ,  $|q - 2p_F|v_F$ ,  $\omega_0$ .

In the parquet diagrams of higher order, the singularities of type (2) and (3) begin to interfere. For  $\omega_0 < T$  the magnetic field falls off everywhere with logarithmic accuracy, and we arrive at the results for the case  $H=0$ .<sup>[1-3]</sup> However, in the limit  $\omega_0 \gg T$  of interest to us, the difference between the positions of the singularities in the elements  $S_{\sigma\sigma}$  and  $N_{\sigma\sigma}$  for  $\sigma = \sigma'_\pm$  and  $\sigma = -\sigma'_\pm$  becomes substantial. In Sec. 3 below an analysis is made of the scheme of parquet summation for the function  $\chi_{SS}$  which one can then easily adapt to the evaluation of  $\chi_{CDW}$ ,  $\chi_{TS}$ , and  $\chi_{SDW}$  (see Sec. 4).

## 3. THE GENERALIZED SUSCEPTIBILITY $\chi_{SS}$

We shall regard  $\chi_{SS}$  as a function of  $T$  and  $\omega_0$  for  $\omega_m = 0$  and small values of  $k$  ( $|k| \ll p_F$ ) in the region  $\omega_0 \gg T$ . In the parquet approximation one can represent  $\chi_{SS}(k) - \chi_{SS}^{(0)}(k)$  in the form of an infinite sequence of diagrams of the form shown in Fig. 2, where the vertical "brick" is the vertex part which is irreducible in the superconducting channel. Let us introduce the notation

$$x = \ln \frac{\Lambda}{T}, \quad x_\sigma = \ln \frac{\Lambda}{\max\{|kv_F + 2\sigma\omega_0|, T\}}, \quad h = \ln \frac{\Lambda}{\omega_0}.$$

In Fig. 2 the  $i$ -th intersection with respect to a pair of parallel lines corresponds to integration with respect to the logarithmic variable  $y_i$  within limits from 0 to  $x_{\sigma_i}$ . For  $|k|v_F \sim T \ll \omega_0$  and  $x_\sigma \approx h$  all of the intersections are suppressed by the field and  $\chi_{SS}$  does not depend on  $T$ . The singularity in  $\chi_{SS}(k)$  as  $T \rightarrow 0$  may arise only for  $kv_F \sim \pm 2\omega_0$ . For the sake of definiteness we shall assume that  $|kv_F - 2\omega_0| \sim T \ll \omega_0$  (due to the obvious symmetry, the case  $|kv_F + 2\omega_0| \sim T$  leads to the same result for  $\chi_{SS}$ ). Then  $x_\pm \approx h$ ,  $x_\pm \approx x \gg h$ , and the  $i$ -th intersection in Fig. 2 will correspond to logarithmic integration over the region  $0 < y_i < x$  if  $\sigma_i = -1$  and over the region  $0 < y_i < h$  if  $\sigma_i = +1$ .

Thus, for  $kv_F \approx 2\omega_0$  the function  $\chi_{SS}$  depends on two logarithmic variables:  $x$  and  $h$ . It can be evaluated by Sudakov's method<sup>[11]</sup> which is based on isolation of the intersection having a maximal value of the logarithmic integration variable  $y$  and summing over all such intersections. In this connection the distinguishing feature of the problem under consideration consists in the fact that, as a consequence of the inequality  $x > h$ , two regions of integration arise at each Sudakov intersection, namely  $0 < y < h$  for  $\sigma = \pm 1$  and  $h < y < x$  for  $\sigma = -1$ . In accordance with this,  $\chi_{SS}(x, h)$  turns out to be the sum of two terms:

$$\chi_{SS}(x, h) = \chi_{SS}^{(1)}(x, h) + \chi_{SS}^{(2)}(h).$$

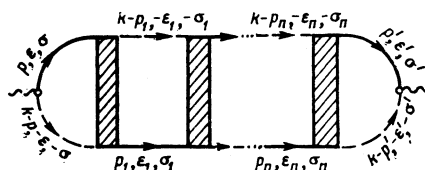


FIG. 2.

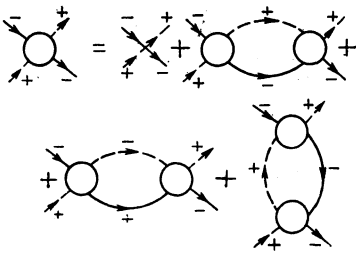


FIG. 3.

where

$$\chi_{SS}^{(1)}(x, h) = \int_h^x |\Delta_{-+}(y, h)|^2 dy. \quad (4)$$

$$\chi_{SS}^{(2)}(h) = \int_0^h [|\Delta_{-+}(y)|^2 + |\Delta_{+-}(y)|^2] dy. \quad (5)$$

Here  $\Delta_{\sigma, -\sigma}$  are the total parquet amplitudes of the interaction with a weak generalized field, producing a pair of electrons with momenta near  $p_F^{\sigma}$  and  $-p_F^{-\sigma}$ , [1] and moreover in Eq. (5) it is taken into consideration that the quantities  $\Delta_{\sigma, -\sigma}$  only depend on  $y$  in the region  $y < h$ . We are interested in the singular part of the susceptibility  $\chi_{SS}^{(1)}$  and to obtain this part it is sufficient to evaluate  $\Delta_{-+}(x, h)$  for  $x > h$ .

In accordance with the existence of two regions of variation of the variable  $y$ , we obtain the following system of parquet equations:

$$\Delta_{-+}(x, h) = \Delta_{-+}(h) + \frac{1}{2} \int_h^x \Gamma_2(y, h) \Delta_{-+}(y, h) dy, \quad (6)$$

$$\Delta_{-+}(h) = 1 + \frac{1}{2} \int_0^h [\Gamma_2(y) \Delta_{-+}(y) + \Gamma_3(y) \Delta_{+-}(y)] dy. \quad (7)$$

The vertex part  $\Gamma_2 \equiv \Gamma_{\rightarrow\rightarrow}(p_1 p_2 | p_3 p_4)$  appearing in Eq. (6) is taken with the following ordering of the momenta:  $p_1 \sim p_4 \sim p_F^+$ ,  $p_2 \sim p_3 \sim -p_F^+$ , that is, for

$$k = p_1 + p_2 \sim 2\omega_0/v_F, \quad |q - 2p_F| = |p_1 - p_3 - 2p_F| \sim T/v_F.$$

For such an ordering of the momenta, in the region  $x > h$  the vertex  $\Gamma_2$  generally depends on two variables:  $x$  and  $h$ . In Eq. (7) the vertices  $\Gamma_2$  and  $\Gamma_3 \equiv \Gamma_{\rightarrow\leftarrow}(p_1 p_2 | p_3 p_4)$  (here  $p_1 \sim -p_3 \sim p_F^+$ ,  $p_4 \sim -p_2 \sim p_F^+$ ) depend only on the single variable  $y$ , owing to the restriction  $y < h$ , and coincide with their values at  $H=0$ . [1] Therefore  $\Delta_{-+}(h)$  coincides with the corresponding expression for  $\Delta_{-+}(x)$  obtained in Refs. 1 and 2 after the formal substitution  $x \rightarrow h$ :

$$\Delta_{-+}(h) = \left(1 + g_1 \ln \frac{\Lambda}{\omega_0}\right)^{-2/4} \left(\frac{\Lambda}{\omega_0}\right)^{(g_1 - 2g_2)/4}, \quad (8)$$

where the small dimensionless coupling constants  $g_1$  and  $g_2$  correspond to scattering with large ( $\sim 2p_F$ ) and small momentum transfer.

Now it is necessary to determine the vertex part  $\Gamma_2(x, h)$  in the region  $x > h$ . The parquet equation for  $\Gamma_2$

is shown in Fig. 3. One can easily verify that on the right hand side of this equation the contributions of the second and fourth diagrams mutually cancel and as a result

$$\Gamma_2(x, h) = \Gamma_2^0 + \frac{1}{2} \int_0^h \Gamma_1^2(y) dy, \quad (9)$$

where  $\Gamma_2^0 = -g_2$ . From Eq. (9) it follows that  $\Gamma_2(x, h) = \Gamma_2(h)$  for  $x > h$ , that is, it only depends on the field. Noting that Eq. (9) for  $\Gamma_2(h)$  coincides with the equation for  $\Gamma_2(x)$  when  $H=0$ , let us utilize the result of Dzyaloshinski and Larkin [1]:

$$\Gamma_2(h) = \frac{1}{2} \left[ (g_1 - 2g_2) - \frac{g_1}{1 + g_1 h} \right]. \quad (10)$$

The solution of Eq. (6) has the form

$$\Delta_{-+}(x, h) = \Delta_{-+}(h) \exp \left\{ \frac{1}{2} \Gamma_2(h) (x - h) \right\} \quad x > h, \quad (11)$$

and after the substitution of (11) into Eq. (4) we obtain the final answer for the singular part of the generalized susceptibility  $\chi_{SS}$ :

$$\chi_{SS}^{(1)}(T, \omega_0) = \frac{|\Delta_{-+}(h)|^2}{\Gamma_2(h)} \left[ \left( \frac{\omega_0}{T} \right)^{\Gamma_2(h)} - 1 \right]. \quad (12)$$

#### 4. THE GENERALIZED SUSCEPTIBILITIES $\chi_{CDW}$ , $\chi_{TS}$ AND $\chi_{SDW}$

A situation completely analogous to the case considered in Sec. 3 arises in connection with the evaluation of the singular part of  $\chi_{CDW}(q)$ . In actual fact  $\chi_{CDW}(q) - \chi_{CDW}^{(0)}(q)$  takes the form of the collection of diagrams shown in Fig. 4 where the  $i$ -th electron-hole intersection corresponds to a logarithmic integration within the limits from zero to  $z_{\sigma_i}$ , where

$$z_{\sigma_i} = \ln \frac{\Lambda}{\max\{|(q - 2p_F)v_F + 2\sigma_i \omega_0|, T\}}.$$

The singularity in  $\chi_{CDW}(q)$  with respect to the temperature arises for  $q - 2p_F \sim \pm 2\omega_0/v_F$ . Choosing  $|(q - 2p_F)v_F - 2\omega_0| \sim T$  we have:  $z_+ \approx h$ ,  $z_- \approx x \gg h$ , and all subsequent calculations repeat the calculation of  $\chi_{SS}(k)$  in Sec. 3.

The singular part of the susceptibility is given by

$$\chi_{CDW}^{(1)}(x, h) = \int_h^x |n_{--}(x, h)|^2 dy, \quad (13)$$

where the amplitude

$$n_{--}(x, h) = n_{--}(h) \exp \left\{ -\frac{1}{2} \int_h^x \Gamma_1(y, h) dy \right\}, \quad (14)$$

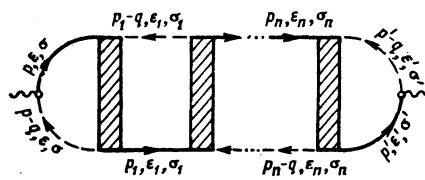


FIG. 4.

and moreover<sup>[2,3]</sup>

$$n_{--}(h) = \left(1 + g_1 \ln \frac{\Lambda}{\omega_0}\right)^{-2/4} \left(\frac{\Lambda}{\omega_0}\right)^{-(g_1 - 2g_2)/4} \quad (15)$$

The vertex part  $\Gamma_1 \equiv \Gamma_{--}(p_1 p_2 | p_3 p_4)$  is taken for  $p_1 \sim p_4 \sim p_F$ ,  $p_2 \sim p_3 \sim -p_F$  which corresponds to

$$|k| = |p_1 + p_2| \sim T/v_F, \quad q = p_1 - p_3 \sim 2p_F + 2\omega_0/v_F.$$

Constructing a parquet equation for  $\Gamma_1$  analogous to the one shown in Fig. 3 one can easily verify that for  $x > h$  the vertex  $\Gamma_1(x, h) = \Gamma_1(h)$  where<sup>[11]</sup>

$$\Gamma_1(h) = \frac{1}{2} \left[ (g_1 - 2g_2) + \frac{g_1}{1 + g_1 h} \right]. \quad (16)$$

From Eq. (14) we obtain the following result:

$$\chi_{CDW}^{(1)}(T, \omega_0) = \frac{|n_{--}(h)|^2}{\Gamma_1(h)} \left[ 1 - \left(\frac{\omega_0}{T}\right)^{-\Gamma_1(h)} \right]. \quad (17)$$

The response function  $\chi_{TS}$  is represented by a set of diagrams analogous to those shown in Fig. 2, the only difference being that all of the lines joining the "bricks" correspond to the same projection of the spin. Therefore in  $\chi_{TS}(k)$  the logarithmic singularities in the temperature arise for  $|k| v_F \sim T \ll \omega_0$  and all intersections with regard to a pair of lines are equivalent. The function  $\chi_{CDW}(q)$  is represented by graphs of the type shown in Fig. 4 in which all of the solid lines correspond to the index  $\sigma$ , and all of the dashed lines correspond to the index  $-\sigma$ . The singularity in  $\chi_{SDW}(q)$  exists for  $|q - 2p_F| - 2p_F | v_F \sim T$ .

Let us present the final results for the singular parts of  $\chi_{TS}$  and  $\chi_{SDW}$ :

$$\chi_{TS}(T, \omega_0) = \frac{|\Delta_{++}(h)|^2}{\Gamma_1(h)} \left[ \left(\frac{\omega_0}{T}\right)^{\Gamma_1(h)} - 1 \right], \quad (18)$$

$$\chi_{SDW}(T, \omega_0) = \frac{|\sigma_{+-}(h)|^2}{\Gamma_2(h)} \left[ 1 - \left(\frac{\omega_0}{T}\right)^{-\Gamma_2(h)} \right], \quad (19)$$

where<sup>[2-4]</sup>

$$\Delta_{++}(h) = \left(1 + g_1 \ln \frac{\Lambda}{\omega_0}\right)^{1/4} \left(\frac{\Lambda}{\omega_0}\right)^{-(g_1 - 2g_2)/4}, \quad (20)$$

$$\sigma_{+-}(h) = \left(1 + g_1 \ln \frac{\Lambda}{\omega_0}\right)^{1/4} \left(\frac{\Lambda}{\omega_0}\right)^{(g_1 - 2g_2)/4}. \quad (21)$$

## 5. THE PHASE DIAGRAM

Expressions (12) and (17) - (19) obtained in Secs. 3 and 4 for the generalized susceptibilities are valid over the entire low temperature region  $T \ll \omega_0$ ; however, they have a restricted range of applicability with regard to the magnitude of the field if  $g_1 < 0$ . In the latter case singularities appear in the vertex parts  $\Gamma_1$  and  $\Gamma_2$  and in the amplitudes  $\Delta$ ,  $n$ , and  $\sigma$  for

$$H \sim H_{c0} = (\Lambda/\mu) \exp(-1/|g_1|),$$

characterizing the onset of the strong coupling regime ( $H \lesssim H_{c0}$ ). Therefore, for  $g_1 < 0$  one can use formulas (12) and (17) - (19) in the region of fields  $H \gg H_{c0}$ .

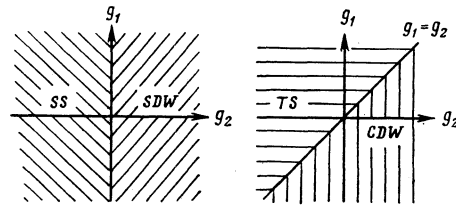


FIG. 5.

The weak coupling regime is realized for  $g_1 > 0$ ;  $\Gamma_1$  and  $\Gamma_2$  are small due to the smallness of the bare coupling constants  $g_1$  and  $g_2$  and the singular parts of all response functions  $\chi$  have power-law asymptotic forms  $T^\gamma$  with a small exponent  $\gamma$ . In the limit of a weak field ( $g_1 h \gg 1$ )

$$\Gamma_1(h) = \Gamma_2(h) = (g_1 - 2g_2)/2 = g/2,$$

and we arrive at the case  $H=0$  when the ground state of a one-dimensional system turns out to be of the type *CDW* or *SDW* if  $g_1 < 0$  and of the type *SS* or *TS* if  $g_1 > 0$ .<sup>[2,3]</sup>

As follows from Eqs. (10) and (16), for  $g_1 > 0$  the vertex function  $\Gamma_1(h)$  increases with increasing field, but  $\Gamma_2(h)$  decreases. In this connection the domain of the states *TS* and *SDW* enlarges but the domain of the states *SS* and *CDW* shrinks.

For  $|g_1| h \ll 1$  the expressions for all  $\chi$  are valid for any sign of  $g_1$ . In this limit of a very strong field

$$\chi_{SS} \propto T^{\delta_1}, \quad \chi_{SDW} \propto T^{-\delta_1}, \quad \chi_{TS} \propto T^{\delta_1 - \delta_2}, \quad \chi_{CDW} \propto T^{\delta_1 - \delta_2}. \quad (22)$$

For simplicity, the phase diagrams describing the possible symmetry of the ground state of the system in the case  $|g_1| h \ll 1$  is represented by two diagrams in Fig. 5.

Formulas (22) can also be obtained in the model of Luther and Emery<sup>[12]</sup> if the constant  $U_1$  describing the interaction between electrons with opposite spins at small distances is set equal to zero. Such processes are frozen out in strong fields. It is well known that for  $U_1 = 0$  the Hamiltonian of Luther and Emery<sup>[12]</sup> reduces to two Hamiltonians of the Tomonaga-Luttinger type,<sup>[13]</sup> describing collective excitations of density and spin with gapless spectra. The absence of a gap also indicates the weak coupling regime (22), which is realized in the limit of a strong field  $|g_1| h \ll 1$ .

## 6. DISCUSSION OF THE RESULTS. IMPURITY ELECTRICAL RESISTIVITY

As was shown above, in a magnetic field  $H \gg T/\mu$  there is a displacement of the positions of the singularities in the one-dimensional response functions  $\chi_{SS}(k)$  and  $\chi_{CDW}(q)$ . Owing to this property a phase transition into an inhomogeneous superconducting state<sup>[14,15]</sup> or into a Peierls dielectric state with two density waves is possible in principle in a quasi-one-dimensional metal placed in a sufficiently strong magnetic field. It is well known, however,<sup>[16]</sup> that the majority of quasi-one-dimensional metallic systems undergo dielectric transitions at rather high temperatures:  $T_p \sim 60$  to  $250^\circ \text{K}$ .

In such systems the realization of the condition  $H \gtrsim T_p/\mu$  presents substantial experimental difficulties.

Meanwhile substances exist whose metallic properties are retained down to  $T \sim 1^\circ\text{K}$  and in which the strong field condition  $\mu H \gg T$  may consequently be realized. The crystals  $(\text{SN})_x$  [7,8] and HMTSF-TCNQ [9] are such objects. Owing to the characteristics of their crystal-line structure, the transverse effects in these systems are markedly weakened. This allows one to use the results for a purely one-dimensional model in order to describe their low temperature properties.

In crystals of  $(\text{SN})_x$  and HMTSF-TCNQ a minimum of the resistivity is observed in the temperature range 30 to 50 °K with its subsequent increase upon a further lowering of the temperature, reminiscent of the Kondo effect in nonmagnetic metals containing magnetic impurities. Such a behavior of the resistivity in the indicated systems may be caused by the scattering of electrons on nonmagnetic impurity atoms, enhanced by one-dimensional correlations of the dielectric type. [17,18] As is shown in Refs. 17 and 18, in the Born approximation the electron's transport relaxation time, determined by impurity scattering processes involving a change of the momentum by  $2p_F$ , is given by the formula<sup>2)</sup>

$$\frac{1}{\tau} = \frac{1}{\tau_0} |n(x)|^2 = \frac{1}{\tau_0} \left(1 + g_1 \ln \frac{\Lambda}{T}\right)^{-g_1} \left(\frac{\Lambda}{T}\right)^{-g_1/2}, \quad (23)$$

where  $1/\tau_0 = 2c|u|^2/v_F$  is the scattering probability in the absence of any interaction between the electrons ( $c$  is the impurity concentration,  $u$  is the bare amplitude of the electron-impurity interaction), and  $n(x)$  is the electron-hole amplitude evaluated in the parquet approximation<sup>[1,2]</sup> (see Eq. (15) with the replacement of  $h$  by  $x$ ).

As follows from formula (23), in the absence of a magnetic field the impurity resistivity may increase with a lowering of the temperature in two cases: (a) for  $g_1 > 0$ ,  $g_2 > 0$  and (b) for  $g_1 < 0$ . Case (a) corresponds to the weak coupling regime in which the resistivity slowly increases due to the smallness of the coupling constants  $g_1$  and  $g_2$ . The strong coupling regime in which a more rapid increase of the resistivity occurs is realized in case (b). In the temperature range  $T > T_{c0}$  ( $T_{c0} = \mu H_{c0}$ ) we have

$$\rho_{imp} \sim (T - T_{c0})^{-g_1}. \quad (24)$$

The nature of the temperature dependence of the resistivity in  $(\text{SN})_x$  and HMTSF-TCNQ<sup>[7,9]</sup> apparently favors case (b). Then it is reasonable to assume that the temperature of the parquet instability  $T_{c0}$  is below the temperature of the resistivity minimum  $T_{\text{min}}$  (precisely such a situation is realized in Kondo systems where the Kondo temperature  $T_K \ll T_{\text{min}}$ ). In addition the smallness of  $T_{c0}$  may be one of the reasons for the absence of a dielectric transition in the indicated systems even in the region of low temperatures. The strong field condition  $\mu H \gg T_{c0}$  may be realized for values of  $T_{c0} \lesssim 10^\circ\text{K}$  and it is of interest to clarify the nature of the effect of such a field on the temperature dependence of the impurity resistivity.

In a field  $H \gg T/\mu$  the relaxation time of an electron

with spin projection  $\pm 1/2$  is determined by scattering processes involving a change of the momentum by  $2p_F^*$ . Constructing the parquet equation for the effective amplitude of impurity scattering in the region  $x > h$ , we obtain:

$$\begin{aligned} \frac{1}{\tau_{\pm}} &= \frac{1}{\tau_0} |n(h)|^2 \left(\frac{\omega_0}{T}\right)^{-r_1(h)} \\ &= \frac{1}{\tau_0} \left(1 + g_1 \ln \frac{\Lambda}{\omega_0}\right)^{-g_1} \left(\frac{\Lambda}{\omega_0}\right)^{-g_1/2} \left(\frac{\omega_0}{T}\right)^{-r_1(h)}. \end{aligned} \quad (25)$$

It follows from formula (25) that at  $g_1 < 0$  the application of a strong field  $H > H_{c0}$  will lead to a suppression of the temperature dependence (24). In such fields the resistivity will vary according to the law

$$\rho_{imp} \sim (\omega_0/T)^{-r_1(h)}$$

and depending on the sign of  $\Gamma_1(h)$ ,  $\rho_{imp}$  will either slowly increase ( $\Gamma_1(h) < 0$ ) or slowly decrease ( $\Gamma_1(h) > 0$ ) as the temperature is lowered. In the last case a weak maximum may be observed in the temperature dependence of the resistivity below  $T_{\text{min}}$ . The conclusion about the suppression of the rapid increase in the resistivity by a magnetic field, reflecting the shift from the weak coupling regime to the strong coupling regime in fields  $H > H_{c0}$ , is of interest from the point of view of its experimental verification.

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<sup>1)</sup>It is assumed everywhere below that in the absence of a magnetic field the band is not half filled and umklapp processes<sup>[1]</sup> do not play a role.

<sup>2)</sup>One can use formulas (23) and (25) for estimates of the impurity resistivity if  $W\tau \gg 1$  and  $T\tau \gg 1$  where  $W$  is the energy of the electrons' transverse motion ( $W \ll \epsilon_F$ ). The first of these conditions removes the question of Mott localization of the electrons in a single dimension and the second allows one to consider the single-impurity problem, neglecting the influence of the impurities on the nature of the one-dimensional correlations between the electrons.

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## Angular dependence of the coefficient of specular reflection of bismuth electron from the binary plane

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We report further development of the method of determining the angular dependence of the coefficient  $q$  of specular reflection of electrons (V. S. Tsoi, Sov. Phys. JETP **41**, 927, 1975) on the basis of focusing the electrons with a transverse homogeneous magnetic field (V. S. Tsoi, JETP Lett. **19**, 70, 1974). The angular dependence of  $q$  is determined for the reflection of the conduction electron of bismuth from a plane perpendicular to  $C_2$ .

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### INTRODUCTION

A method for determining the angular dependence of the specular reflection coefficient  $q$  was developed in<sup>[1]</sup> on the basis of electron focusing (EF) in metals by a transverse uniform magnetic field.<sup>[2]</sup> Also investigated was the dependence of  $q$  of bismuth electrons on the incidence angle  $\theta$  in the case of reflection from the trigonal plane (plane perpendicular to  $C_3$ ), and it was established that electron reflection from a perfect surface is practically specular ( $q \approx 0.8$  at normal incidence). Even though an insignificant deviation ( $\sim 5\%$ ) from the optimally chosen  $q(\theta)$  dependence leads to a noticeable discrepancy with the experimental data, the small range of variation of  $q(0.8-1.0)$  does not make it possible to establish with high accuracy the analytic form of the  $q(\theta)$  dependence. The absence of a theoretical calculation of  $q(\theta)$  for arbitrary  $\theta$  permits a large leeway in the choice of the analytic form of  $q(\theta)$ .

In this communication we develop further the method of<sup>[1]</sup> for the measurement of  $q(\theta)$  and measure the function  $q(\theta)$  for bismuth electrons reflected from the binary plane (plane perpendicular to  $C_2$ ), from which normally incident electrons are reflected practically diffusely ( $q \approx 0.13$ ).

### EXPERIMENT

The experimental setup for the observation of the EF is the following.<sup>[2]</sup> Two pinpoints are placed on a single-crystal metallic sample—an emitter and a collector.

Current is passed through the emitter and the voltage  $U$  on the collector is measured as a function of the applied magnetic field  $H$ .

The samples were single-crystal bismuth disks of 10 mm diam and 2 mm thickness, with a specified crystallographic orientation, grown in a polished dismountable quartz mold.<sup>[3]</sup> The sample was cut from the seed crystal in such a way that a stub 5-10 mm long was left on the disk. The measurements were made on samples with two crystallographic orientations: 1)  $C_3 \parallel n$  and 2)  $C_2 \parallel n$  ( $n$  is the normal to the sample surface).

The construction of the measuring head is shown in Fig. 1. Sample 1 was placed on a copper disk 2, which was mounted on a rotating copper table 3 placed on needle supports 4. The sample was glued to the disk

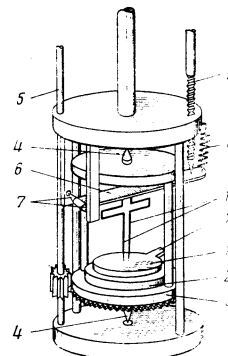


FIG. 1. Diagram of measurement head.