Processes with fermion exchange in nonabelian gauge theories

V. S. Fadin and V. E. Sherman

Nuclear Physics Institute, Siberian Branch, USSR Academy of Sciences
(Submitted September 16, 1976)
Zh. Eksp. Teor. Fiz. 72, 1640–1658 (May 1977)

The amplitudes of elastic and inelastic processes with fermion exchange in channels with small momentum transfers in multireggeon kinematics are calculated in the leading logarithmic approximation for nonabelian gauge theories in which the mass arises by the Higgs mechanism. Integral equations are derived for the partial amplitudes of the elastic processes.

PACS numbers: 11.10.Np, 11.60.+c

1. INTRODUCTION

Field-theoretical models based on Yang-Mills gauge vector fields are attracting great attention at the present time.[1] A large number of papers are devoted to the study of the high-energy behavior of the amplitudes in such models and, in particular, to the question of the reggeization of the vector meson. In Ref. 2, in the leading logarithmic approximation, the different small-angle scattering amplitudes were calculated to sixth order of perturbation theory in the simplest model, based on the isotopic-rotation group, and the reggeization of the vector meson in this order was demonstrated. This result has been generalized to other models.[3] The results of the sixth-order calculation have been confirmed in a number of papers.[4,5] The eighth order of perturbation theory has been calculated.[6] In the calculation, as in Refs. 2, 3, the dispersion method was used; therefore, the amplitudes of inelastic processes in the multireggeon kinematics were also found. It was discovered that these amplitudes have a simple multireggeon form; this gave the possibility of generalizing the results of the eight-order calculations to arbitrary order in perturbation theory and of obtaining for the partial amplitudes of the elastic processes an integral equation[6,7] whose solution in the channel with isotopic spin $T=1$ is a reggeon (i.e., the vector meson is reggeized), while in the vacuum channel a stationary square-root branch point appears to the right of $j=1$, apparently because of the contribution of many-particle states in the t-channel.[7,8] The eighth-order calculations have been confirmed.[5]

We consider the question of the high-energy behavior of the amplitudes of processes with fermion exchange in a channel with small momentum transfer. To be more specific, we are concerned with calculating in the leading logarithmic approximation ($g^2 \ln s \sim 1$, $g^2 \ll 1$) the amplitude for scattering of a vector meson by a fermion at scattering angles close to 180°, and the amplitude for the annihilation at small angles of a pair of vector mesons with creation of a fermion-antifermion pair. If we are interested only in the positive-signature part of the amplitude, which is greater than the negative-signature part in each order of perturbation theory in $\ln s$, we can formulate a simple prescription for its determination.[9] It then turns out that the positive-signature part is determined by the contribution of the reggeon whose trajectory passes through $j=\frac{1}{2}$ for $\hat{q}=M$, i.e., the fermion is reggeized in the same way as in the well-known case of quantum electrodynamics with a massive photon.[10]

In the present work we have used the dispersion method, which enables us to calculate both the positive- and negative-signature parts of the amplitude. This method has been used successfully previously[2,3,6] for the problem of small-angle scattering. The method requires knowledge of the inelastic amplitudes in order to establish the elastic amplitudes from unitarity and analyticity. At the same time, therefore, we have calculated the inelastic amplitudes in the multi-reggeon kinematics, which gives the principal contribution to the unitarity relation. The calculations, carried out to eighth order of perturbation theory, show that the inelastic amplitudes have a simple multireggeon form. As in the problem of small-angle scattering,[6,7] this form is easily generalized to arbitrary order of perturbation theory. Using this generalization we can set up the elastic amplitude. The result can be written in the form of integral equations for the partial amplitudes; for the group $SU(2)$ we have published these in a previous note.[11]

The plan of the discussion is as follows. In the next section we obtain the Born amplitudes of the elastic processes and processes of creation of three particles in multireggeon kinematics and formulate simple rules for obtaining the Born amplitudes of processes of creation of an arbitrary number of particles in this kinematics (rules I–III of Sec. 2). In Sec. 3 we derive the rules for calculating the contribution of the $(n+2)$-particle intermediate state in the unitarity condition to the partial waves of the elastic process for the case when the amplitudes of the inelastic processes are taken in the Born approximation (rules 1–4 of Sec. 3), and the elastic amplitudes are calculated to order g^6. In Sec. 4 the rules I–III of Sec. 2 for inelastic processes are generalized to any order of perturbation theory and integral equations are derived for the partial amplitudes of the elastic processes. In the Appendix the $\sim g^5$ corrections to the Born amplitude for creation of three particles are calculated.

2. THE BORN AMPLITUDES

We shall consider, principally, the very simple model of Ref. 12, based on an isotriplet of Yang–Mills vector

 0038-5646/77/4505-0861$02.40

fields $\mathbf{V}_\mu$, with mass m that arises by the Kibble–Higgs mechanism[13] as a result of the appearance of a nonzero vacuum average of an isodoublet complex scalar field. The interaction of the fields $\mathbf{V}_\mu$ with the isodoublet of fermions has the form $-\frac{1}{2}g\bar{\psi}\hat{\mathbf{V}}\boldsymbol{\tau}\psi$. A detailed description of the model can be found in Refs. 12, 2, and 6. Our results are simply generalized to the case of the gauge group $SU(N)$ with spontaneous symmetry breaking that conserves the global $SU(N)$. A description of this model can be found in Ref. 6. The generalization to this case will be discussed in the course of the paper. In this model, the interaction of the Yang–Mills fields V_μ^a with the fermions has the form $-\frac{1}{2}g\bar{\psi}\hat{V}^a\lambda^a\psi$. We shall use the notation adopted in Ref. 6, i.e., we shall denote the particles by the letters A, B, etc., and the corresponding momenta by p_A, p_B; we shall denote the isotopic indices of the vector (spinor) particles A, B by the letters a, b (α, β), and their polarizations by λ_A, λ_B (r_A, r_B). We choose the polarization vectors as in Ref. 2:

$$(e_A^{(\lambda)}p_A)=0,\quad (e_A^{(\lambda)})^2=-1,\quad (\mathbf{e}_A^{(\lambda)}\mathbf{p}_A)|_{\lambda=1,2}=0,$$
$$e_A^{(3)}=\left(\frac{|\mathbf{p}_A|}{m};\quad \frac{p_A^0}{m}\frac{\mathbf{p}_A}{|\mathbf{p}_A|}\right). \tag{1}$$

To study the amplitudes for creation of $n+2$ particles in the multireggeon kinematics $A+B\to D_0+D_1+\cdots+D_{n+1}$ we introduce the momentum transfers

$$q_i=q_{i-1}+p_{D_{i-1}}\quad (i=1,\ldots,n+1),$$
$$q_0=-p_A \tag{2}$$

and the Sudakov variables for them:

$$q_i=\beta_i p_B-\alpha_i p_A+q_{\perp i}. \tag{3}$$

The kinematics is determined by the conditions

$$s=(p_A+p_B)^2\gg m^2,\ -t_i=-q_i^2\approx-q_{\perp i}^2\sim m^2,$$
$$1\gg\alpha_1\gg\ldots\gg\alpha_{n+1}\sim m^2/s,\ 1\gg\beta_{n+1}\gg\ldots\gg\beta_1\sim m^2/s,$$
$$s\alpha_i\beta_{i+1}=p_{D_i}^2-(q_i-q_{i+1})_\perp^2,$$
$$s_i=(p_{D_{i-1}}+p_{D_i})^2=s\alpha_{i-1}\beta_{i+1}\gg m^2,$$
$$\prod_{i=1}^{n+1}s_i=s\prod_{i=1}^{n}(p_{D_i}^2-(q_i-q_{i+1})_\perp^2). \tag{4}$$

For $n=0$ we shall use the notation A', B' for the final particles; $q=p_{A'}-p_A$, $q\approx q_\perp$, $-t=-q^2\sim m^2$. We use the spinor normalization $\bar{u}u=2M$.

The amplitudes in the multireggeon kinematics can be classified by the quantum numbers in the channels with momenta q_i. The amplitudes with the largest value ($\sim s$) are those which have the quantum numbers of the vector meson in all these channels. The magnitude ($\sim s$) of these amplitudes is explained by the fact that a vector meson with momentum q_i gives a factor $\sim s_i$, and, according to (4), the product of all the s_i gives s. These amplitudes were found in Refs. 6 and 7 in all orders in the coupling constant in the leading logarithmic approximation. Since we shall need them in the following, we give them here:

$$A_{2\to 2+n}=s\Gamma_{AD_0}^{i_1}\frac{(s_1/m^2)^{\alpha(t_1)}}{t_1-m^2}\gamma_{i_1i_2}^{D_1}(q_1,q_2)\frac{(s_2/m^2)^{\alpha(t_2)}}{t_2-m^2}\cdots$$
$$\cdots\gamma_{i_ni_{n+1}}^{D_n}(q_n,q_{n+1})\frac{(s_{n+1}/m^2)^{\alpha(t_{n+1})}}{t_{n+1}-m^2}\Gamma_{BD_{n+1}}^{i_{n+1}}, \tag{5}$$

where

$$j=1+\alpha(t)=1+\frac{g^2}{(2\pi)^3}(t-m^2)\int\frac{d^2k_\perp}{(k_\perp^2-m^2)((q-k)_\perp^2-m^2)} \tag{6}$$

is the trajectory of the Regge pole (the vector meson is reggeized); the vertices γ_{ij}^D are equal to

$$\gamma_{ij}^D=m\delta_{ij} \tag{7}$$

for emission of scalar particles σ, and

$$\gamma_{ij}^D(q_1,q_2)=ig\varepsilon_{dij}\left[-(q_1+q_2)^\mu+p_A^\mu\left(\frac{2(p_Bq_1)}{(p_Ap_B)}-\frac{q_1^2-m^2}{(p_Aq_2)}\right)\right.$$
$$\left.+p_B^\mu\left(\frac{2(p_Aq_2)}{(p_Ap_B)}-\frac{q_2^2-m^2}{(p_Bq_1)}\right)\right]e_D^\mu\equiv ig\varepsilon_{dij}e_D^\mu\mathcal{P}_\mu(q_1,q_2) \tag{8}$$

for emission of vector particles. The vertices Γ_{AD}^i are equal to

$$\Gamma_{AD}^i=2^{-1/2}g(\tau_i)_{\delta\alpha}\delta_{r_Ar_D}, \tag{9}$$

for fermions,

$$\Gamma_{AD}^i=2^{1/2}ig\varepsilon_{ida}a_{\lambda_A}\delta_{\lambda_A\lambda_D},$$
$$a_{\lambda_A}=\begin{cases}1, & \lambda_A=1,2,\\ 1/2, & \lambda_A=3,\end{cases} \tag{10}$$

for vector particles, and

$$\Gamma_{AD}^i=2^{-1/2}g\delta_{\lambda_A3}\delta_{ai} \tag{11}$$

for the transformation of a vector particle to a scalar particle. We note that formulas (9)–(11) give the vertices Γ_{AD}^i for well-defined polarizations in the center-of-mass frame of the initial particles. A covariant expression for them is given in Ref. 6. The generalization to the case of the group $SU(N)$ is also given there.

Only the given amplitudes were needed to study the process of small-angle scattering. In our problem (backward scattering; in accordance with the fact that the momentum q is carried by a fermion, the amplitude is of order $s^{1/2}$), we need all those amplitudes for which, in the channels with momenta q_i, the quantum numbers are either those of the vector meson or those of the fermion (each vector meson with momentum q_i gives a factor $\sim s_i$ and each fermion with q_j gives $\sim s_j^{1/2}$). We shall proceed to the calculation of these amplitudes in the Born approximation.

There exists a simple way of finding the Born amplitudes in multireggeon kinematics.[6] It consists in determining the pole parts, with respect to q_i^2, of the amplitudes. It turns out that these pole parts give the complete asymptotic behavior of the amplitudes. We do not have a rigorous proof of this fact for production of an arbitrary number of particles; for production of three and four particles it is verified by direct calculation of all the Feynman diagrams. The determination of the pole parts with respect to q_i^2 in the case when the momentum q_i is carried by a vector meson is described in detail in Ref. 6. The case when the momentum q_i is carried by a fermion will be illustrated below using a very simple example.

We shall consider the Born amplitude for almost-backward scattering of a vector meson by a fermion. One of the Feynman diagrams for this process is given in Fig. 1. In the channel with momentum q there is a fermion pole:

$$\mathrm{Im}_t A(AB\to A'B') = g^2\pi\delta(q^2-M^2)\cdot \times\sum_r \bar{u}_{A'}\hat{e}_A\frac{\tau^a}{2}u^r(q)\bar{u}^r(q)\hat{e}_{B'}\frac{\tau^{b'}}{2}u_B. \tag{12}$$

We are interested in the amplitude to within the leading terms $\sim s^{1/2}$ only ($s^{1/2}$ is provided by the spinors $\bar{u}_{A'}$ and u_B). Since at high energies the longitudinal polarization vectors $e_A^{(3)}$ and $e_{B'}^{(3)}$ almost coincide with p_A/m and $p_{B'}/m$ (cf. (1)), to our accuracy the right-hand side of (12) is equal to zero if at least one of the polarization vectors is longitudinal (the replacement $e_A\to p_A$ ($e_{B'}\to p_{B'}$) makes (12) vanish). The longitudinal polarizations can be eliminated by making the replacement

$$e_A\to\varepsilon_A=e_A-p_A\frac{(e_Ap_B)}{(p_Ap_B)},\quad e_{B'}\to\varepsilon_{B'}=e_{B'}-p_{B'}\frac{(e_{B'}p_A)}{(p_Ap_B)}. \tag{13}$$

In fact, this replacement does not alter (to our accuracy) the contribution of the transverse polarizations, and makes the longitudinal ones vanish. This replacement is completely analogous to that used in quantum electrodynamics.[10]

The extraction of the pole part of the amplitude with respect to q^2 reduces to the replacement

$$\pi\delta(q^2-M^2)\sum_r u^r(q)\bar{u}^r(q)\to\frac{1}{M-\hat{q}}.$$

In our case this pole part gives the whole amplitude. We write it in the form

$$A(AB\to A'B')=\bar{\Gamma}_{1/2}(A\to A')\frac{1}{M-\hat{q}_\perp}\Gamma_{1/2}(B\to B'), \tag{14}$$

where, for our case, i.e., when A and B' are vector mesons and A' and B are fermions,

$$\Gamma_{1/2}(B\to B')=-{}^1/_2 g\hat{\varepsilon}_{B'}\tau^{b'}u_B,\quad \bar{\Gamma}_{1/2}(A\to A')=(\Gamma_{1/2}(A'\to A))^+\gamma^0. \tag{15}$$

The subscript $\frac{1}{2}$ indicates the isospin in the t-channel. It is not difficult to convince oneself of the correctness

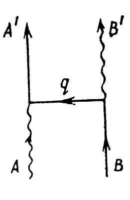

FIG. 1.

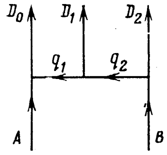

FIG. 2.

of expression (14) by direct analysis of the Feynman diagrams (there are three in all in the given case). If the particle B' is an antifermion and B is a vector meson, then, obviously,

$$\Gamma_{1/2}(B\to B')=-{}^1/_2 g\hat{\varepsilon}_B\tau^b v_{B'}, \tag{16}$$

where ε_B is given by formula (13) with the replacement $B'\to B$. We note that the vertices $\bar{\Gamma}_{1/2}$ and $\Gamma_{1/2}$ are gauge-invariant, as can be seen from (13).

In the model under consideration there is also the scalar particle σ; but since there is no direct $\sigma\bar{F}F$-interaction, in order g^2 there is no σF scattering and the backward scattering amplitude for $VF\to F\sigma$ is negligibly small ($\sim s^{-1/2}$), like the amplitude of the annihilation $V\sigma\to\bar{F}F$, in accordance with the fact that in a channel with small momentum transfer there is no fermion pole. This implies that the vertices $\bar{\Gamma}_{1/2}$ and $\Gamma_{1/2}$ are equal to zero for $F\rightleftharpoons\sigma$ transitions. The generalization of formulas (14)–(16) to the case of the group $SU(N)$ is trivial: it reduces to the replacement $\tau\to\lambda$. The method of setting up the amplitude from its pole parts with respect to q_i^2 makes it possible to write down simply all the three-particle production amplitudes, whereas an analysis of all the Feynman diagrams here would be rather cumbersome. We shall consider the process $A+B\to D_0+D_1+D_2$. We associate the diagram of Fig. 2 with it. This, of course, is not a Feynman diagram; it simply shows the kinematics of the process. We need to consider three cases: a) the case when there is a vector meson in the channel with momentum q_1 and a fermion in the channel with momentum q_2; b) the opposite case; c) the case when there is a fermion in each channel. In case a) the particle with momentum p_{D_1} is a fermion. Using the usual replacement of the spin matrix of a vector meson V with momentum q_1:

$$\left(\left(-g^{\mu\nu}+\frac{q_1^\mu q_1^\nu}{m^2}\right)\approx-\frac{2}{s}(p_B^\mu p_A^\nu+p_B^\nu p_A^\mu)\right),$$

we obtain

$$\mathscr{P}_1A^{(a)}_{2\to3}=s^{1/2}\Gamma^i_{AD_0}\frac{1}{q_1^2-m^2}A^i(VB\to D_1D_2), \tag{17}$$

where the sumbol $\mathscr{P}_1$ indicates that the pole part with respect to q_1^2 is taken; i is the isotopic index of the vector meson with momentum q_1, and in place of the polarization vector of this meson in A^i we must substitute $(2/s)^{1/2}p_A$. The amplitude A^i is given by formula (14); substituting the quantity $(2/s)^{1/2}p_A$ in place of e_V in ε_V and using the fact that

$$p_A-q_1\frac{(p_Ap_B)}{(q_1p_B)}\approx\frac{q_{1\perp}}{\alpha_1},$$

we obtain

$$\mathcal{P}_1 A_{2\to3}^{(a)} = s^{1/2}\Gamma_{AD_0}^{i}\left[-\left(\frac{2}{s}\right)^{1/2} g\right]\bar{u}_{D_1}\frac{\hat{q}_{1\perp}}{\alpha_1}\frac{\tau^i}{2}\frac{1}{M-\hat{q}_{2\perp}}\Gamma_{1/2}(B\to D_2). \quad (18)$$

Calculation of the pole part with respect to q_2^2 gives the same expression, so that, for case a), when we have a vector meson in the channel with momentum q_1 and a fermion in the channel with momentum q_2, the amplitude can be represented in the form

$$A_{2\to3}^{(a)} = s^{1/2}\Gamma_{AD_0}^{i}\frac{1}{q_1^2-m^2}\gamma_{iF}^{D_1}(q_1,q_2)\frac{1}{M-\hat{q}_{2\perp}}\Gamma_{1/2}(B\to D_2), \quad (19)$$

where we have introduced the vertex $\gamma_{iF}^{D_1}$, equal to

$$\gamma_{iF}^{D_1}(q_1,q_2) = \left(\frac{s}{2}\right)^{1/2} g\bar{u}_{D_1}\frac{\hat{q}_{1\perp}}{(q_1p_B)}\frac{\tau^i}{2}. \quad (20)$$

Case b) is treated completely analogously, and the result can be written down immediately:

$$A_{2\to3}^{(b)} = \Gamma_{1/2}(A\to D_0)\frac{1}{M-\hat{q}_{1\perp}}\gamma_{Fi}^{D_1}(q_1,q_2)\frac{1}{q_2^2-m^2}s^{1/2}\Gamma_{BD_2}^{i}, \quad (21)$$

where

$$\gamma_{Fi}^{D_1}(q_1,q_2) = \left(\frac{s}{2}\right)^{1/2} g\frac{\tau^i}{2}\frac{\hat{q}_{2\perp}}{(q_2p_A)}v_{D_1}. \quad (22)$$

In case c), an analysis of the pole parts with respect to q_1^2 and q_2^2 gives (in this case D_1 is a vector meson)

$$A_{2\to3}^{(c)} = \Gamma_{1/2}(A\to D_0)\frac{1}{M-\hat{q}_{1\perp}}\gamma^{D_1}(q_1,q_2)\frac{1}{M-\hat{q}_{2\perp}}\Gamma_{1/2}(B\to D_2), \quad (23)$$

where

$$\gamma^{D_1}(q_1,q_2) = -g\frac{\tau^{d_1}}{2}\left(\hat{e}_{D_1}-(M-\hat{q}_{1\perp})\frac{(e_{D_1}p_A)}{(q_2p_A)}-(M-\hat{q}_{2\perp})\frac{(e_{D_1}p_B)}{(\hat{q}_1p_B)}\right). \quad (24)$$

The formulas (19)–(24) give all the three-particle production amplitudes in multireggeon kinematics when exchange of a fermion occurs in at least one of the channels with small momentum transfer. The generalization of these formulas to the case of the group $SU(N)$ reduces to replacing $\tau\to\lambda$ and using the vertices $\Gamma_{AA'}^{i}$ found in Ref. 6 for the group $SU(N)$.

The method we have used to calculate the amplitudes also makes it possible to determine simply the amplitudes for production of a large number of particles. It leads to a prescription for determining the amplitude for production of an arbitrary number of particles that is already clear from the factorized form of formulas (19)–(24) and (5) and consists in the following. Suppose that we are considering a process $A+B\to D_0+D_1+\cdots+D_{n+1}$ in multireggeon kinematics, and the quantum numbers in the channels with small momentum transfers q_i are either those of the fermion or those of the vector meson. We associate the diagram of Fig. 3 with this process. The amplitude of the process in the Born approximation is calculated from this diagram in accordance with the following rules.

I. If there is a vector meson in the channel with momentum q_i, we associate the propagator $(q_i^2-m^2)^{-1}$ with the line q_i; if it is a fermion, the propagator is $(M-\hat{q}_{\perp i})^{-1}$.

II. If in the channel q_1 (q_{n+1}) there is a vector meson with isotopic index c, we associate $s^{1/2}\Gamma_{AD_0}^{c}$ ($s^{1/2}\Gamma_{BD_{n+1}}^{c}$) with the leftmost (rightmost) line (cf. (9)–(11)); if we have a fermion, the factors are $\bar{\Gamma}_{1/2}(A\to D_0)$ ($\Gamma_{1/2}(B\to D_{n+1})$) (cf. (15), (16)).

III. With each vertex formed by particles with momenta q_i, q_{i+1} and p_{D_i}, depending on the type of particles we associate

a) $\gamma_{cc'}^{D_i}(q_i,q_{i+1})$ (cf. (7), (8)) if q_i and q_{i+1} correspond to vector mesons with isotopic indices c and c';

b) $\gamma_{cF}^{D_i}(q_i,q_{i+1})$ (cf. (20)) if q_i corresponds to a vector meson with isotopic index c and q_{i+1} to a fermion;

c) $\gamma_{Fc}^{D_i}(q_i,q_{i+1})$ (cf. (22)) in the opposite case;

d) $\gamma^{D_i}(q_i,q_{i+1})$ (cf. (24)) if q_i and q_{i+1} correspond to fermions.

The rules I–III solve the problem of calculating the Born amplitudes. The generalization of these rules to the group $SU(N)$ reduces to replacing $\tau\to\lambda$ and using the vertices Γ_{AD}^{c} and $\Gamma_{cc'}^{D}$ found in Ref. 6 for the group $SU(N)$.

3. AMPLITUDES OF ELASTIC PROCESSES TO ORDER g_6

The determination of the scattering amplitudes in higher orders of perturbation theory reduces to finding their s-channel imaginary parts:

$$2\,\mathrm{Im}_s A(AB\to A'B') = \sum_N \int d\rho_N A(AB\to N)A^*(A'B'\to N). \quad (25)$$

Up to order g^6, only two- and three-particle intermediate states N give a contribution to the sum; it is more expedient, however, to deal straight away with an arbitrary number of particles. We shall consider a term in the sum in the right-hand side of (25) in the case when $A(AB\to N)$ and $A(A'B'\to N)$ are Born amplitudes for production of $2+n$ particles $D_0, D_1, \ldots, D_{n+1}$ in multireggeon kinematics. Then,

$$d\rho_N = d\rho_{2+n} = [(2\pi)^{3n+2}2^{n+1}s]^{-1}\prod_{i=1}^{n}\frac{d\alpha_i}{\alpha_i}\prod_{k=1}^{n+1}d^2q_{\perp k}, \quad (26)$$

where the range of variation of the α_i is determined by the conditions (4). In the Born approximation, $A^*(A'B'\to N)=A(N\to A'B')$; we therefore associate the diagram

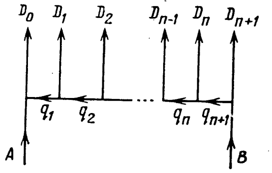

FIG. 3.

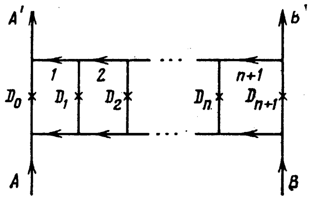

FIG. 4.

of Fig. 4 with the term under consideration. We are interested in the amplitude $A(AB\to A'B')$ with exchange of a fermion in the t-channel; therefore, in each section of the diagram of Fig. 4 (where the numbers from 1 to $n+1$ label the sections) one of the horizontal lines corresponds to a fermion and the other to a vector meson. The amplitude $A_{2\to 2+n}=A(AB\to D_0\cdots D_{n+1})$ is found using the rules I–III from the preceding section; the rules for determining $A(D_0\cdots D_{n+1}\to A'B')=A_{2+n\to 2}$ are easily obtained starting from the fact that $A(N\to A'B')=A^*(A'B'\to N)$: in II it is necessary to make the replacement $A\to D_0$, $D_0\to A'$, $B\to D_{n+1}$, $D_{n+1}\to B'$; in III(b) and III(c) we must replace $\gamma_{cF}^D\to\gamma_{c\bar F}^D$, $\gamma_{\bar F c}^D\to\gamma_{Fc}^D$; this change in the subscripts on γ signifies the change of spinors $\bar u\to\bar v$, $v\to u$; the other rules remain unchanged. But this is still not all. The point is that, because of the anticommutativity of the fermion operators, the sign of the amplitude depends on their order in the definition of the state vectors. We have defined the amplitudes without worrying about their sign or fixing this order, as if the operators were commutative. However, in (25) the relative signs of the amplitudes must be taken into account. In order to do this we must associate a factor -1 with each antifermion in the intermediate state.

Thus, the amplitudes $A_{2\to 2+n}$ and $A_{2+n\to 2}$ are determined. After multipolying them it is necessary to carry out the summation over the states of the intermediate particles. To perform the sum over the states of the "internal" particles $D_1,\ldots,D_n$, we stipulate that in the i-th section of the diagram the momentum q_i flows along the horizontal fermion line, so that $q-q_i$ flows along the vector-meson line. For the case when D_i is a fermion or antifermion we obtain, using (20), (22),

$$\sum_{D_i}\gamma_{Fc'}^{D_i}(q_i,q-q_{i+1})\gamma_{cF}^{D_i}(q-q_i,q_{i+1})$$
$$=-\sum_{D_i}\gamma_{\bar F c'}^{D_i}(q_i,q-q_{i+1})\gamma_{c\bar F}^{D_i}(q-q_i,q_{i+1})=2g^2K_F(q_i,q_{i+1})\sum_T d_T^F P_T^{cc'}, \tag{27}$$

where $\sum_{D_i}$ denotes summation over the spin and isospin states of the particle D_i,

$$K_F(q_1,q_2)=M-\hat q_\perp-(M-\hat q_{1\perp})\frac{1}{M-(q_{1\perp}+\hat q_{2\perp}-\hat q_\perp)}(M-\hat q_{2\perp}), \tag{28}$$

and $P_T^{cc'}$ are the projection operators on to the state with the isospin T in the t-channel:

$$P_{1/2}^{cc'}={}^1/_3\tau^c\tau^{c'},\quad P_{3/2}^{cc'}=(\delta^{cc'}-{}^1/_3\tau^c\tau^{c'}),\quad d_{1/2}^F={}^1/_4,\quad d_{3/2}^F=-{}^1/_2. \tag{29}$$

In K_F we have neglected quantities of the type $\beta_{i+1}\hat p_B$, $\alpha_i\hat p_A$, since p_B and p_A acting on the corresponding spinors give $\sim M$ and α_i, $\beta_{i+1}\ll 1$; in pulling, e.g., $\alpha_i\hat p_A$ through to its spinor it is necessary to commute it only with $\hat q_k$ for $k\le i$; this gives $\sim s\alpha_i\beta_k\ll m^2$.

The generalization of formula (27) to the case of the group $SU(N)$ is that $\sum_T$ in (27) for this case denotes a sum over three representations; the corresponding projection operators $P_T^{cc'}$ and coefficients d_T^F are equal to

$$\begin{aligned}
P_1^{cc'}&=\frac{2N}{N^2-1}\frac{\lambda^c}{2}\frac{\lambda^{c'}}{2}\\
P_2^{cc'}&=\frac{1}{2}\delta^{cc'}-\frac{1}{N+1}\frac{\lambda^c}{2}\frac{\lambda^{c'}}{2}+\frac{\lambda^{c'}}{2}\frac{\lambda^c}{2},\\
P_3^{cc'}&=\frac{1}{2}\delta^{cc'}-\frac{1}{N-1}\frac{\lambda^c}{2}\frac{\lambda^{c'}}{2}-\frac{\lambda^{c'}}{2}\frac{\lambda^c}{2},\\
d_1^F&=\frac{1}{2N},\qquad d_2^F=-d_3^F=-\frac{1}{2};
\end{aligned} \tag{30}$$

$P_1^{cc'}$ is the projection operator on to the fundamental representation according to which the fermion transforms.

In the case when D_i is a vector meson we obtain from (8), (24)

$$\sum_{D_i}\gamma_{cc'}^{D_i}(q-q_i,q-q_{i+1})\gamma^{D_i}(q_i,q_{i+1})=2g^2K_V(q_i,q_{i+1})\sum_T d_T^V P_T^{cc'}, \tag{31}$$

where

$$K_V(q_1,q_2)=M-\hat q_\perp-(M-\hat q_{1\perp})\frac{(q-q_2)_\perp^2-m^2}{(q_1-q_2)_\perp^2-m^2}-(M-\hat q_{2\perp})\frac{(q-q_1)_\perp^2-m^2}{(q_1-q_2)_\perp^2-m^2},$$

$$d_{1/2}^V=1,\qquad d_{3/2}^V=-{}^1/_2. \tag{32}$$

In the case of the group $SU(N)$,

$$d_1^V=N/2,\quad d_2^V=-d_3^V=-{}^1/_2. \tag{33}$$

It remains to carry out the summation for the external lines in Fig. 4. We shall consider the line BB'. Two variants are possible; either the lower or the upper horizontal line in the section $(n+1)$ is the fermion line. In the first variant the summation gives

$$s^{1/2}\sum_{D_{n+1}}\Gamma_{1/2}(B\to D_{n+1})\Gamma_{D_{n+1}B'}^c=(2s)^{1/2}g\sum_T d_T(B\to B')\Gamma_T^c(B\to B'), \tag{34}$$

where, for the case when B' is a vector meson and B a fermion,

$$\Gamma_T^c(B\to B')=-gP_T^{cb'}\hat e_{B'}u_B,\quad d_T(B\to B')=d_T^V \tag{35}$$

and for the opposite case, i.e., when B is a vector meson and B' a fermion (more precisely, an antifermion),

$$\Gamma_T^c(B\to B')=-gP_T^{cb}\hat e_B v_{B'},\quad d_T(B\to B')=-d_T^F. \tag{36}$$

Here the minus sign has arisen because there is an antifermion in the intermediate state. In the second variant we obtain

$$s^{1/2}\sum_{D_{n+1}}\Gamma_{BD_{n+1}}^c\Gamma_{1/2}(D_{n+1}\to B')=-(2s)^{1/2}g\sum_T d_T(B'\to B)\Gamma_T^c(B\to B'). \tag{37}$$

The formulas (34)–(37) are obtained using the explicit form of the vertices (9), (10) and (15), (16). We note that, inasmuch as there is no direct interaction of σ with fermions, the particle D_{n+1} can only be either a fermion or a vector meson. Since the vertices $\Gamma_{1/2}$ are nonzero only for transverse polarizations of the vector meson, and $\Gamma^c_{V\sigma}$ is nonzero only for transformation of σ into an arbitrarily polarized vector meson, in our approximation and in higher orders of perturbation theory backward scattering (or annihilation) processes with participation of σ do not appear.

The summation formulas for the line AA' are obtained from (34), (37) by taking the conjugate and changing the indices:

$$s^{1/2}\sum_{D_0}\Gamma_{1/2}(A\to D_0)\Gamma^c_{D_0A'}=-g(2s)^{1/2}\sum_T d_T(A\to A')\Gamma_T{}^c(A\to A'),$$
$$s^{1/2}\sum_{D_0}\Gamma^c_{AD_0}\Gamma_{1/2}(D_0\to A')=g(2s)^{1/2}\sum_T d_T(A'\to A)\Gamma_T{}^c(A\to A'), \quad (38)$$
$$\bar{\Gamma}_T{}^c(A\to A')=(\Gamma_T{}^c(A'\to A))^+\gamma^0.$$

The formulas (34)–(38) also remain valid for the case of $SU(N)$, with P_T and d_T defined in (30), (33). The relations (26), (27), (31), and (34)–(38) make it possible to calculate the contribution to the imaginary part from any diagram of the type of Fig. 4. As can be seen from these relations, the matrix element associated with a diagram depends only on $q_{i\perp}$ and does not depend on α_i; the dependence on $q_{i\perp}$ is such that all the integrals over $q_{i\perp}$ converge in the region $q_{i\perp}\sim m$, and the integration over α_i (cf. (26)) gives powers of logarithms; i.e., the multireggeon kinematics makes the principal contribution to the integral.

It is convenient to transform from the amplitude $A(AB\to A'B')$ to partial waves with well-defined isospin in the t-channel and signature. We represent the amplitude in the form

$$A(AB\to A'B')=\sum_T\Gamma_T{}^c(A\to A')M_T(AB\to A'B')\Gamma_T{}^c(B\to B'). \quad (39)$$

The possibility of such a representation is clear from (34), (37) and (38). Next we separate the parts of M_T with positive and negative signature:

$$M_T(AB\to A'B')=M_T{}^++M_T{}^-,$$
$$M_T{}^\pm={}^1/{}_2[M_T(AB\to A'B')\pm M_T(A\bar{B}'\to A'\bar{B})] \quad (40)$$

and transform to the j-representation ($\omega=j-\frac{1}{2}$)

$$M_T{}^\pm=\frac{1}{4i}\int_{\delta-i\infty}^{\delta+i\infty}d\omega\left(\frac{s}{m^2}\right)^\omega\frac{(e^{-i\pi\omega}\pm1)}{\sin\omega\pi}F_T{}^\pm(\omega,q). \quad (41)$$

In the Born approximation only $M^+_{1/2}=\frac{3}{4}(M-\hat{q})^{-1}$ is nonzero; (for the group $SU(N)$ we have $M^+_1=(N^2-1)/2N(M-\hat{q})$); according to (41), in this approximation we obtain

$$F_T^{(0)\pm}=\frac{C_T{}^\pm}{M-\hat{q}},\quad C_{1/2}{}^+=\frac{3}{4},\quad C_{3/2}{}^+=0,\quad C_{1/2}{}^-=C_{3/2}{}^-=0. \quad (42)$$

For the group $SU(N)$ we have $C^+_1=(N^2-1)/2N$, $C^+_{2,3}=C^-_{1,2,3}=0$. We note that the C^-_T can be defined as arbitrary quantities independent of ω, since, because of the cancellation of the pole at $\omega=0$ in the negative signature (41), their contribution is negligibly small.

In higher orders $F^\pm_T(\omega,q)$ is determined from the s-channel imaginary part:

$$F_T{}^\pm(\omega,q)=-\frac{2}{\pi}\int_1^\infty d\left(\frac{s}{m^2}\right)\left(\frac{s}{m^2}\right)^{-\omega-1}\operatorname{Im}_s M_T{}^\pm. \quad (43)$$

We can now formulate simple rules for determining the contribution to $F^\pm_T$ from the $(n+2)$-particle intermediate state in the unitarity relation (25) when the amplitudes $A_{2\to2+n}$ and $A_{2+n\to2}$ are taken in the Born approximation.

First we shall see what the integration over α_i and over s/m^2 gives us. As we have already said, the whole "inner part" of the diagram of Fig. 4 depends only on q_i; s^{-1} in the phase volume cancels with the product $\sqrt{s}\sqrt{s}$ from the lines AA' and BB'; using (4), we obtain

$$\int_1^\infty d\left(\frac{s}{m^2}\right)\int_{m^2/s}^1\frac{d\alpha_1}{\alpha_1}\int_{m^2/s}^{\alpha_1}\frac{d\alpha_2}{\alpha_2}\cdots\int_{m^2/s}^{\alpha_{n-1}}\frac{d\alpha_n}{\alpha_n}\left(\frac{s}{m^2}\right)^{-\omega-1}$$
$$=\prod_{i=1}^n\int_1^\infty d\left(\frac{s_i}{m^2}\right)\left(\frac{s_i}{m^2}\right)^{-\omega-1} \quad (44)$$

i.e., with each section of the diagram we associate $1/\omega$.

Since the factors associated with the lines AA' and BB' do not depend on the $q_{i\perp}$, over which it remains to integrate, the lines D_0 and D_{n+1} can be contracted to a point; therefore, with the contribution of an $(n+2)$-particle intermediate state to $F^\pm_T$ we shall associate the set of diagrams of the type depicted in Fig. 5, with all possible paths of propagation of the fermion from the extreme left to the extreme right of the vertex. In the case when the amplitudes $A_{2\to2+n}$ and $A_{2+n\to2}$ of the inelastic processes are taken in the Born approximation the contribution to $F^\pm_T$ corresponding to each diagram of this set is calculated by the following rules.

1) With the horizontal fermion line in the i-th section we associate $(M-\hat{q}_{i\perp})^{-1}$, and with the vector meson line, $[(q-q_i)^2_\perp-m^2]^{-1}$.

2) With the transverse lines between the i-th and $(i+1)$-th sections we associate $d^F_T K_F(q_i,q_{i+1})$ (for a fermion line) and $d^V_T K_V(q_i,q_{i+1})$ (for a vector-meson line) (cf. (27)–(33)).

3) With each section we associate a factor $g^2(2\pi)^{-3}\omega^{-1}\times d^2q_i$. With each diagram we associate a factor equal to $[d_T(B\to B')\pm d_T(B'\to B)]d_T(A\to A')$ ($[d_T(B'\to B)\pm d_T(B\to B')]d_T(A'\to A)$) in the case when the fermion line is the lower (upper) line in both the $(n+1)$-th and the first section, a factor $-[d_T(B\to B')\pm d_T(B'\to B)]d_T(A'\to A)$ in the case when the fermion line in the $(n+1)$-th section is the lower line and that in the first section is the upper line, and $-[d_T(B'\to B)\pm d_T(B\to B')]d_T(A\to A')$ in the opposite

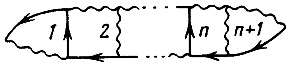

FIG. 5.

case ((34)–(38)). Since, as follows from rules 1)–3), the whole "inner part" of the diagram is unchanged under interchange of "upper" and "lower," this rule can be formulated in a simpler form:

4) We consider only diagrams in which the fermion line in the $(n+1)$-th section is the lower line. With each such diagram, in the case when the fermion line in the first section is the lower line, we associate a factor $d_T^\pm$, or, if it is the upper line, $\mp d_T^\pm$, where

$$d_T^\pm=[d_T(B\to B')\pm d_T(B'\to B)][d_T(A\to A')\pm d_T(A'\to A)],$$

(the upper signs are for the positive signature and the lower for the negative). Here and below all momenta are two-dimensional: $q_i=q_{i\perp}$. The γ-matrix factors are arranged in the usual order—from the first section, against the fermion line. The rules are valid for the group $SU(N)$ for any N, with d_T defined in (30), (33).

Here we must note an important fact. The positive-signature part of the amplitude is nonzero, as follows from rule 4) and the definition of d_T, only in a channel with the quantum numbers of the fermion; this, generally speaking, is a consequence of the gauge-invariance of the original theory, or, more concretely, of the fact that the generators $\frac{1}{2}\lambda^i$ of the group appear in the vertex of the interaction of the vector meson with fermions while the structure constants f_{ijk} appear in the three-point Yang–Mills vertex. We note also that, since the positive-signature part of the amplitude is greater than the negative-signature part in each order of perturbation theory in $\ln s$, only it is used in the unitarity relation (25).

The rules 1)–4) that we have formulated make it possible to obtain the partial amplitudes $F_T^\pm$ to order g^4 (i.e., $A(AB\to A'B')$ to order g^6; cf. (39) and the definition of Γ_T^c). Next, for definiteness, we shall consider the scattering of a vector meson by a fermion, i.e., the particles A and B' are vector mesons and B and A' are fermions (changing to the other channel can only change the sign of F_T^-). Then,

$$d_1^\pm=\pm\left(\frac{N^2\mp1}{2N}\right)^2,\quad d_{2,3}^+=0,\quad d_{2,3}^-=-1. \tag{45}$$

The relations (45) are written for the group $SU(N)$. For $SU(2)$,

$$d_{1/2}^+=\left(\frac{3}{4}\right)^2,\quad d_{3/2}^+=0,\quad d_{1/2}^-=-\left(\frac{5}{4}\right)^2,\quad d_{3/2}^-=-1.$$

In order g^2 there is only one diagram for $F_T^\pm$ according to our rules;

$$F_T^{(2)\pm}=\frac{d_T^\pm}{\omega}\eta(q),$$
$$\eta(q)=\frac{g^2}{(2\pi)^3}\int\frac{d^2q_1}{(M-\hat{q}_1)((q-q_1)^2-m^2)}. \tag{46}$$

In order g^4 the contribution from the three-particle intermediate state is given by two diagrams; we obtain

$$F_T^{(4)\pm}(2\to3\to2)=\frac{d_T^\pm}{\omega^2}[(M-q)\eta^2(q)(d_T^V\mp d_T^F)-(2d_T^V\mp d_T^F)\rho(q)],$$
$$\rho(q)=\left(\frac{g^2}{(2\pi)^3}\right)^2\int\frac{d^2q_1d^2q_2}{(M-\hat{q}_1)(q_2^2-m^2)((q-q_1-q_2)^2-m^2)}. \tag{47}$$

However, in order g^4 there is also a contribution to $F_T^\pm$ from the two-particle intermediate state, when one of the amplitudes in (25) is taken in the Born approximation and corrections $\sim g^2$ are taken into account in the other. According to formula (5), allowance for this correction in the amplitude with exchange of a vector meson in the channel with momentum q leads to the replacement $(q^2-m^2)^{-1}\to(q^2-m^2)^{-1}(1+\alpha(q^2)\ln s)$ (for the group $SU(N)$ the replacement $\alpha(q^2)\to\alpha_N(q^2)=\frac{1}{2}N\alpha(q^2)$ is necessary[6]), and in the amplitude with exchange of a fermion, according to (46), (42), (41), to the replacement $(M-\hat{q})^{-1}\to(M-\hat{q})^{-1}(1+\delta_N(q)\ln s)$, where for $SU(N)$

$$\delta_N(q)=\frac{N^2-1}{2N}(M-\hat{q})\eta(q). \tag{48}$$

Using (44) we find that, to take into account the correction $\sim g^2$ to the amplitude with fermion exchange, in rule 1) it is necessary to make the replacement $(M-\hat{q})^{-1}\to(M-\hat{q})^{-1}(1+\omega^{-1}\delta_N(q))$, and, for the correction to the amplitude with exchange of a vector meson, to make the replacement $(q^2-m^2)^{-1}\to(q^2-m^2)^{-1}(1+\omega^{-1}\alpha_N(q^2))$; as a result we find the contribution from the two-particle intermediate state:

$$F_T^{(4)\pm}(2\to2\to2)=\frac{d_T^\pm}{\omega^2}\frac{g^2}{(2\pi)^3}\int\frac{d^2q_1[\alpha_N((q-q_1)^2)+\delta_N(q)]}{(M-\hat{q}_1)((q-q_1)^2-m^2)}$$
$$=\frac{d_T^\pm}{\omega^2}\left(\frac{N}{2}+\frac{N^2-1}{2N}\right)\rho(q). \tag{49}$$

Combining (49) with (47) we see that in the channel with positive signature (with the quantum numbers of the fermion) the term proportional to $\rho(q)$ has been cancelled; to order g^4,

$$F_1^+=\frac{C_1^+}{M-\hat{q}}\left(1+\frac{\delta_N(q)}{\omega}+\left(\frac{\delta_N(q)}{\omega}\right)^2\right), \tag{50}$$

i.e., the fermion is reggeized. In the negative-signature parts the cancellation of $\rho(q)$ does not occur, i.e., they are not an expansion of simple Regge poles

4. INTEGRAL EQUATIONS FOR THE PARTIAL AMPLITUDES

For the calculation of $F_T^\pm$ in the next orders of perturbation theory, corrections to the Born amplitudes of the inelastic processes are necessary. In the Appendix, corrections to the amplitude for creation of three particles in multireggeon kinematics are calculated. The calculation is rather cumbersome, but the result is perfectly simple: taking the $\sim g^2$ correction into account reduces to the result that if the particle in the channel with momentum q_i is a fermion we must make the replacement $(M-\hat{q}_i)^{-1}\to(M-\hat{q}_i)^{-1}(1+\delta_N(q_i)\ln s_i)$ in the Born amplitude, while if it is a vector meson we must make the replacement $(q_i^2-m^2)^{-1}\to(q_i^2-m^2)^{-1}(1+\alpha_N(q_i^2)\times\ln s_i)$, i.e., the usual propagators are replaced by reggeized propagators. We generalize the result obtained to arbitrary order of perturbation theory; i.e., we assume that in the leading logarithmic approximation the amplitude of the process $A+B\to D_0+D_1+\cdots+D_{n+1}$ can be obtained from the rules I–III of Sec. 2 if in rule I we make the replacement

$$(q_i^2-m^2)^{-1}\to(q_i^2-m^2)^{-1}\left(\frac{s_i}{m^2}\right)^{\alpha_N(q_i^2)},$$
$$(M-\hat{q}_i)^{-1}\to(M-\hat{q}_i)^{-1}\left(\frac{s_i}{m^2}\right)^{\delta_N(q_i)}. \quad (51)$$

This generalization enables us to obtain integral equations for the partial amplitudes of the elastic processes. In Sec. 3 we derived rules for determining the contribution to $F_T^{\pm}$ in an $(n+2)$-particle intermediate state in the case when the amplitudes $A_{2\to 2+n}$ and $A_{2+n\to 2}$ are taken in the Born approximation; in order to obtain this contribution when these amplitudes are taken in all orders of perturbation theory, it is necessary only, according to (51) and (44), to make the replacement

$$\omega^{-1}\to[\omega-\alpha_N((q-q_i)^2)-\delta_N(q_i)]^{-1} \quad (52)$$

in rule 3). We represent $F_T^{\pm}(\omega,q)$ in the form

$$F_T^{\pm}(\omega,q)=\frac{C_T^{\pm}}{M-\hat{q}}+d_T^{\pm}\frac{g^2}{(2\pi)^3}\int\frac{d^2q_1}{((q_1-q)^2-m^2)(M-\hat{q}_1)}f_T^{\pm}(\omega;q_1,q). \quad (53)$$

We denote the contribution to $f_T^{\pm}(\omega;q_1,q)$ from an $(n+2)$-particle intermediate state by $f_T^{(n)\pm}(\omega;q_1,q)$. According to the rules 1)–4) (with the replacement (52)), we obtain

$$f_T^{(n+1)\pm}(\omega;q_1,q)=[\omega-\delta_N(q_1)-\alpha_N((q-q_1)^2)]^{-1}\frac{g^2}{(2\pi)^3}\int\frac{d^2q_2}{(q-q_2)^2-m^2}$$
$$\times[d_T{}^VK_V(q_1,q_2)\mp d_T{}^FK_F(q_1,q_2)](M-\hat{q}_2)^{-1}f_T\quad(\omega;q_2,q). \quad (54)$$

Taking into account that $f_T^{(0)\pm}(\omega;q_1,q)=[\omega-\delta_N(q_1)-\alpha_N((q-q_1)^2)]^{-1}$, for $f_T^{\pm}(\omega;q_1,q)$ we obtain the equation

$$[\omega-\delta_N(q_1)-\alpha_N((q-q_1)^2)]f_T^{\pm}(\omega;q_1,q)=1+\frac{g^2}{(2\pi)^3}\int d^2q_2$$
$$\times d_T{}^VK_V(q_1,q_2)\mp d_T{}^FK_F(q_1,q_2)][(q-q_2)^2-m^2]^{-1}(M-\hat{q}_2)^{-1}f_T^{\pm}(\omega;q_2,q), \quad (55)$$

where d_T and $K(q_1,q_2)$ are defined in (28)–(33) and $\delta_N(q)$ in (48); $\alpha_N(q^2)=\frac{1}{2}N\alpha(q^2)$ (cf. Ref. 6). It is easy to verify that in the case of positive signature (the channel with the quantum numbers of the fermion) the solution of (55) is

$$f_1^{+}(\omega;q_1,q)=(\omega-\delta_N(q))^{-1}. \quad (56)$$

In this case,

$$F_1^{+}(\omega,q)=\frac{C_1^{+}}{(M-\hat{q})}\frac{\omega}{(\omega-\delta_N(q))} \quad (57)$$

i.e., the fermion is reggeized, which demonstrates the self-consistency of our assumption. For the negative signature it has not been possible to solve Eq. (55).

For the group $SU(2)$, Eq. (55) was published by us earlier.[11] There the case of quantum electrodynamics was considered, for which the results that we obtained coincide with those of McCoy and Wu.[14]

5. CONCLUSIONS

The Born amplitudes that we found in Sec. 2, for inelastic processes in multireggeon kinematics for cases when in some (or all) channels with small momentum transfers q_i the quantum numbers correspond to exchange of a fermion, have a simple factorized form, as for the case treated earlier[6] in which the quantum numbers in all channels are those of the vector meson. The claculation of the $\sim g^5$ corrections to the Born amplitudes for creation of three particles gave the possibility of generalizing this form to arbitrary order of perturbation theory. The generalization consists in replacing the propagators of the fermion and vector meson by reggeized propagators. Using this generalization we obtain integral equations for the partial amplitudes of elastic processes with fermion exchange in a channel with small momentum transfer. The solution of the equation for the positive signature shows that the fermion is reggeized; thus, our generalization is self-consistent. For the negative signature we can state that, besides poles, the partial amplitudes have branch points arising from the exchange of a reggeized fermion and a reggeized vector meson. Since there operates here a mechanism that leads to the appearance of a stationary branch point in the vacuum channel[7] (the existence of arbitrarily high thresholds with respect to t), we may also expect the appearance of stationary branch points.

APPENDIX

We shall seek the amplitude $A_{2\to 3}$ in order g^5 from its s-, s_1-, and s_2-channel discontinuities, represented schematically in Figs. 6a, b, and c, respectively. We must consider three cases: 1) there is a vector meson in the q_1 channel and a fermion in the q_2 channel; 2) the opposite case; 3) there is a fermion in both charnels. The case when we have vector mesons in both cnannels was considered in detail in Ref. 6.

We begin with case 1). We have (see Fig. 6a)

$$2\,\mathrm{Im}_s A_{2\to 3}=\sum_{A',B'}(2s)^{-1}(2\pi)^{-2}\int d^2q_\perp[A(AB\to A'B')A(A'B'\to D_0D_1D_2)$$
$$+A(AB\to A'D_1B')A(A'B'\to D_0D_2)], \quad (A.1)$$

where the summation symbol denotes summation over the types of particle and over the spin and isospin (unitary) states of the particles. Substituting the amplitudes (there are vector mesons in the q- and (q_1-q)-channels and a fermion in the (q_2-q)-channel), we obtain

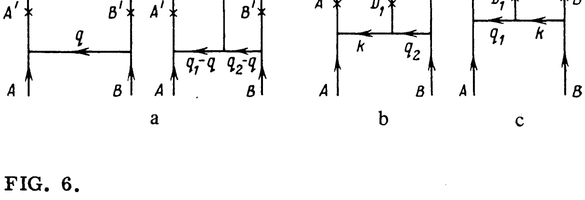

FIG. 6.

$$2\,\mathrm{Im}_s A_{2\to3}=\Big(\sum_{A'}\Gamma^i_{AA'}\Gamma^j_{A'D_0}\Big)s^{1/2}$$
$$\times\Big[R_j\sum_{B'}\Gamma^i_{BB'}\Gamma_{1/2}(B'\to D_2)+R_i\sum_{B'}\Gamma_{1/2}(B\to B')\Gamma^j_{B'D_2}\Big],$$
$$R_i=\frac{1}{2(2\pi)^2}\int\frac{d^2q_\perp\gamma_{iF}{}^{D_1}(q_1-q_\perp,q_2-q_\perp)}{(q_\perp{}^2-m^2)((q_1-q)_\perp{}^2-m^2)}\frac{1}{M-(\hat{q}_2-\hat{q})_\perp}. \tag{A.2}$$

The summation over B' is performed in accordance with (34), (37) and that over A' in accordance with formulas (13), (A.6) of Ref. 6. We need the amplitudes in the leading logarithmic approximation, i.e., we do not distinguish between $\ln s$ and $\ln u$, and, therefore, after the s- and u-channel contributions are combined, only the part that is antisymmetric in i,j survives in the formulas for summation over A':

$$\sum_{A'}\Gamma^i_{AA'}\Gamma^j_{A'D_0}\to2^{-1/2}ig f_{kij}\Gamma^k_{AD_0}, \tag{A.3}$$

so that, when the u-channel contribution is taken into account (as will be understood to be the case in the following), the diagram of Fig. 6a give

$$A^{(a)}_{2\to3}=-\frac{2(\ln s)}{2\pi}ig^2f_{kij}s^{1/2}\Gamma^k_{AD_0}R_i\sum_T(d_T(B\to D_2)+d_T(D_2\to B))\Gamma_T{}^j(B\to D_2). \tag{A.4}$$

Since $d_T(B\to D_2)+d_T(D_2\to B)=d_T^V-d_T^F$ is nonzero only for the fundamental representation (then, $d_1^V-d_1^F=(N^2-1)/2N$) and $\frac{1}{4}if_{kij}\lambda^j\lambda^i=\frac{1}{4}N\lambda^k$, we have

$$A^{(a)}_{2\to3}=(\ln s)s^{1/2}\Gamma^i_{AD_0}\frac{N}{2}\frac{g^2}{\pi}R_i\Gamma_{1/2}(B\to D_2). \tag{A.5}$$

We shall consider the s_1-channel discontinuity (Fig. 6b; there are vector mesons in the k- and (q_1-k)-channels)

$$2\,\mathrm{Im}_{s_1}A_{2\to3}=\frac{1}{(2\pi)^2 2s_1}\sum_{A',D_1'}\int d^2k_\perp A(AB\to A'D_1'D_2)A(A'D_1'\to D_0D_1) \tag{A.6}$$

Substituting the expressions for the amplitudes and performing the summation over A' with allowance for the u_1-channel contribution in accordance with (A.3), we obtain

$$A^{(b)}_{2\to3}=-(\ln s_1)\frac{g}{(2\pi)^3}\Big(\frac{s}{2}\Big)^{1/2}if_{kij}\Gamma^k_{AD_0}\int\frac{d^2k_\perp}{(k_\perp{}^2-m^2)((q_1-k)_\perp{}^2-m^2)}$$
$$\times\sum_{D_1'}\tilde{\Gamma}^j_{D_1'D_1}\gamma_{iF}^{D_1'}(k,q_2)\frac{1}{M-\hat{q}_{2\perp}}\Gamma_{1/2}(B\to D_2). \tag{A.7}$$

Here we have written $\tilde{\Gamma}^j_{D_1'D_1}$ in order to emphasize that it is necessary to take the covariant expression for the vertex $\Gamma^j_{D_1'D_1}$ (cf. Ref. 6). Performing the summation over D_1', we obtain, after straightforward transformations,

$$A^{(b)}_{2\to3}=(\ln s_1)s^{1/2}\Gamma^i_{AD_0}\frac{N}{2}\frac{g^2}{(2\pi)^3}\int\frac{d^2k_\perp}{(k_\perp{}^2-m^2)((q_1-k)_\perp{}^2-m^2)} \tag{A.8}$$
$$\times\Big[\gamma_{iF}{}^{D_1}(q_1,q_2)\frac{1}{M-\hat{q}_{2\perp}}-\gamma_{iF}{}^{D_1}(q_1-k_\perp,q_2-k_\perp)\frac{1}{M-(\hat{q}_2-\hat{k})_\perp}\Big]\Gamma_{1/2}(B\to D_2).$$

We turn to the s_2-channel discontinuity. In the diagram of Fig. 6c the fermion line can be either the upper or the lower line. The summation over B' is performed in accordance with (34), (37); after taking the u_2-channel into account, using the fact that $d_T(B\to D_2)+d_T(D_2\to B)=d_T^V-d_T^F$ is nonzero only for the fundamental representation, we have

$$A^{(c)}_{2\to3}=-(\ln s_2)s^{1/2}\Gamma^i_{AD_0}\frac{1}{(q_1{}^2-m^2)}\frac{1}{(2\pi)^3}\int\frac{d^2k_\perp}{2s_2}\Big\{\frac{s_2}{(q_2-k)_\perp{}^2-m^2}$$
$$\times\sum_{D_1'}\tilde{\Gamma}^j_{D_1'D_1}\gamma_{iF}^{D_1'}(q_1',k)\frac{1}{M-\hat{k}_\perp}-\sum_{D_1'}\bar{\Gamma}_{1/2}(D_1'\to D_1)\gamma_{ij}^{D_1'}(q_1,k)$$
$$\times\frac{s^{1/2}}{(M-(\hat{q}_2-\hat{k})_\perp)}\frac{1}{(k_\perp{}^2-m^2)}\Big\}2^{1/2}g\frac{\lambda^j}{2}\Gamma_{1/2}(B\to D_2). \tag{A.9}$$

Performing the summation over D_1' and using the relations

$$\frac{\lambda^j}{2}\frac{\lambda^i}{2}\frac{\lambda^j}{2}=-\frac{1}{2N}\frac{\lambda^i}{2},\quad if_{dij}\frac{\lambda^d}{2}\frac{\lambda^j}{2}=\frac{N}{2}\frac{\lambda^i}{2},$$

we obtain

$$(A)^{(c)}_{2\to3}=(\ln s_2)s^{1/2}\Gamma^i_{AD_0}\frac{1}{(q_1{}^2-m^2)}\frac{g^2}{(2\pi)^3}\int d^2k_\perp$$
$$\times\Big\{-\frac{1}{2N}\frac{\gamma_{iF}{}^{D_1}(q_1,q_2)}{((q_2-k)_\perp{}^2-m^2)}\frac{1}{M-\hat{k}_\perp}+\frac{N}{2}\Big[\gamma_{iF}{}^{D_1}(q_1,q_2)$$
$$-\gamma_{iF}{}^{D_1}(q_1-k_\perp,q_2-k_\perp)\frac{(q_1{}^2-m^2)}{(q_1-k)_\perp{}^2-m^2}\Big] \tag{A.10}$$
$$\times\frac{1}{(M-(\hat{q}_2-\hat{k})_\perp)}\frac{1}{(k_\perp{}^2-m^2)}\Big\}\Gamma_{1/2}(B\to D_2).$$

Summing the individual contributions and taking into account that $\ln s=\ln s_1+\ln s_2$, we find

$$A_{2\to3}=A^{(a)}_{2\to3}+A^{(b)}_{2\to3}+A^{(c)}_{2\to3}=s^{1/2}\Gamma^i_{AD_0}\frac{1}{(q_1{}^2-m^2)}\gamma_{iF}{}^{D_1}(q_1,q_2)$$
$$\times\frac{1}{M-\hat{q}_{2\perp}}(\alpha_N(q_1{}^2)\ln s_1+\delta_N(q_2)\ln s_2)\Gamma_{1/2}(B\to D_2). \tag{A.11}$$

Comparing (A.11) with the Born approximation, we see that the corrections reduce to the reggeization of the vector meson and fermion.

The case 2) is treated entirely analogously and, obviously, leads to the same result. We turn to the case 3). In the diagrams of Fig. 6a the fermion can now pass along either the line with momentum q or the lines with q_1-q, q_2-q. After performing the summation over A', B' using formulas (33), (37) and (38), we obtain for the contribution of Fig. 6a

$$A^{(a)}_{2\to3}=(\ln s)(d_1{}^V-d_1{}^F)^2\Gamma_1{}^i(A\to D_0)\frac{g^2}{(2\pi)^3}\int d^2q_\perp\Big\{\frac{1}{q_\perp{}^2-m^2}$$
$$\times(M-(\hat{q}_1-\hat{q})_\perp)^{-1}\delta_{ij}\gamma^{D_1}(q_1-q_\perp,q_2-q_\perp)(M-(\hat{q}_2-\hat{q})_\perp)^{-1}$$
$$+((q_1-q)_\perp{}^2-m^2)^{-1}\gamma_{ij}{}^{D_1}(q_1-q_\perp,q_2-q_\perp)((q_2-q)_\perp{}^2$$
$$-m^2)^{-1}(M-\hat{q}_\perp)^{-1}\Big\}\Gamma_1{}^j(B\to D_2). \tag{A.12}$$

We turn to the diagram of Fig. 6b. Here the fermion can pass either along the line with momentum k or along that with q_1-k. After the summation over A' we obtain

$$A^{(b)}_{2\to3}=(\ln s_1)2^{1/2}(d_1{}^V-d_1{}^F)\Gamma_1{}^i(A\to D_0)\frac{g}{(2\pi)^3}\int\frac{d^2k_\perp}{2s_1}$$
$$\times\Big\{\frac{s_1}{(q_1-k)_\perp{}^2-m^2}\frac{1}{M-\hat{k}_\perp}\sum_{D_1'}\Gamma^i_{D_1'D_1}\gamma^{D_1'}(k,q_2)$$
$$-\frac{s^{1/2}}{(k_\perp{}^2-m^2)}\frac{1}{(M-(q_1-\hat{k})_\perp)}$$
$$\times\sum_{D_1'}\Gamma_{1/2}(D_1'\to D_1)\gamma_{iF}{}^{D_1'}(k,q_2)\Big\}\frac{1}{(M-\hat{q}_{2\perp})}\Gamma_{1/2}(B\to D_2). \qquad (A.13)$$

Again it is necessary to take the covariant expression for $\Gamma^i_{D_1'D_1}$. The summation over D_1' leads, after simple but rather unwieldy transformations, to the form

$$A^{(c)}_{2\to3}=(\ln s_1)\Gamma_{1/2}(A\to D_0)\frac{g^2}{(2\pi)^3}\int d^2k_\perp\Big\{\frac{N}{2}\frac{1}{((q_1-k)_\perp{}^2-m^2)}$$
$$\times\frac{1}{(M-\hat{k}_\perp)}\Big[\gamma^{D_1}(q_1,q_2)+g\frac{\lambda^{d_1}}{2}\frac{(M-\hat{q}_{2\perp})}{(q_2-k)_\perp{}^2-m^2}e_{D_1}{}^{\mu}\mathscr{P}_\mu(q_1-k_\perp,q_2-k_\perp)\Big]$$
$$-\frac{1}{2N}\frac{1}{(k_\perp{}^2-m^2)}\frac{1}{(M-(\hat{q}_1-k)_\perp)}[\gamma^{D_1}(q_1,q_2)-\gamma^{D_1}(q_1-k_\perp,q_2-k_\perp)$$
$$\times(M-(\hat{q}_2-\hat{k})_\perp)^{-1}(M-\hat{q}_{2\perp})]\Big\}(M-\hat{q}_{2\perp})^{-1}\Gamma_{1/2}(B\to D_2), \qquad (A.14)$$

where $\mathscr{P}_\mu(q_1, q_2)$ is defined in (8).

The diagram of Fig. 6c is treated entirely analogously; the result can be written out immediately:

$$A^{(c)}_{2\to3}=(\ln s_2)\Gamma_{1/2}(A\to D_0)\frac{1}{(M-\hat{q}_{1\perp})}\frac{g^2}{(2\pi)^3}$$
$$\times\int d^2k_\perp\Big\{\frac{N}{2}\frac{1}{((q_2-k)_\perp{}^2-m^2)}\Big[\gamma^{D_1}(q_1,q_2)$$
$$+g\frac{\lambda^{d_1}}{2}\frac{(M-\hat{q}_{1\perp})}{((q_1-k)_\perp{}^2-m^2)}e_{D_1}{}^{\mu}\mathscr{P}_\mu(q_1-k_\perp,q_2-k_\perp)\Big]\frac{1}{(M-\hat{k}_\perp)}$$
$$-\frac{1}{2N}\frac{1}{(k_\perp{}^2-m^2)}\Big[\gamma^{D_1}(q_1,q_2)-(M-\hat{q}_{1\perp})\frac{1}{(M-(\hat{q}_1-\hat{k})_\perp)}$$
$$\times\gamma^{D_1}(q_1-k_\perp,q_2-k_\perp)\Big]\frac{1}{((M-(\hat{q}_2-\hat{k})_\perp)}\Big\}\Gamma_{1/2}(B\to D_2). \qquad (A.15)$$

Summing the individual contributions for the case 3) under consideration gives

$$A_{2\to3}=A^{(a)}_{2\to3}+A^{(b)}_{2\to3}+A^{(c)}_{2\to3}=\overline{\Gamma}_{1/2}(A\to D_0)\frac{1}{(M-\hat{q}_{1\perp})}\{\delta_N(q_1)\gamma^{D_1}(q_1,q_2)\ln s$$
$$+\gamma^{D_1}(q_1,q_2)\delta_N(q_2)\ln s_2\}\frac{1}{(M-\hat{q}_{2\perp})}\Gamma_{1/2}(B\to D_2). \qquad (A.16)$$

Comparing with the Born approximation, we see that the corrections reduce to the reggeization of the fermions in the q_1- and q_2-channels.

[1]C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).
[2]L. N. Lipatov, Yad. Fiz. 23, 642 (1976) [Sov. J. Nucl. Phys. 23, 338 (1976)].
[3]L. L. Frankfurt and V. E. Sherman, Yad. Fiz. 23, 1099 (1976) [Sov. J. Nucl. Phys. 23, 581 (1976)].
[4]B. M. McCoy and T. T. Wu, Phys. Rev. D13, 1076 (1976); L. Tyburski, Phys. Rev. D13, 1107 (1976).
[5]C. Y. Lo and H. Cheng, Phys. Rev. D13, 1131 (1976); P. S. Yeung, Phys. Rev. D13, 2306 (1976).
[6]É. A. Kuraev, L. N. Lipatov and V. S. Fadin, Zh. Eksp. Teor. Fiz. 71, 840 (1976) [Sov. Phys. JETP **44**, 443 (1976)].
[7]V. S. Fadin, É. A. Kuraev and L. N. Lipatov, Phys. Lett. 60B, 50 (1975).
[8]É. A. Kuraev, L. N. Lipatov and V. S. Fadin, Zh. Eksp. Teor. Fiz. 72, 377 (1977) [Sov. Phys. JETP **45**, 199 (1977)].
[9]V. S. Fadin and V. E. Sherman, Preprint, I. Ya. F. (Nuclear Physics Institute), 1976.
[10]M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx and F. Zachariasen, Phys. Rev. 133, B145 (1964).
[11]V. S. Fadin and V. E. Sherman, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 599 (1976) [JETP Lett. 23, 548 (1976)].
[12]G. 't Hooft, Nucl. Phys. B35, 167 (1971).
[13]P. W. Higgs, Phys. Rev. **145**, 1156 (1966); T. W. B. Kibble, Phys. Rev. **155**, 1554 (1967).
[14]B. M. McCoy and T. T. Wu, Phys. Rev. D13, 369 (1976).

Translated by P. J. Shepherd.