Absorption of electromagnetic waves in a plasma in the presence of a quantizing magnetic field

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An expression for the effective electron-ion collision rate in the region of weak spatial dispersion is found for an electron-ion plasma located in a strong magnetic field. It is shown that the effective collision rate should decrease in a strong quantizing magnetic field.

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1. In investigating the influence of the effects of quantization of the orbital motion of the current carriers on the propagation of electromagnetic waves in a magnetized plasma, researchers are more often than not interested in the region of sufficiently high oscillation frequencies ω, when the collisions play a minor role ($\omega\tau_{eff}\gg 1$) and the wave dissipation is due mainly to Čerenkov and cyclotron absorptions. However, the contribution of the indicated effects is usually exponentially small, and, as a result, it becomes necessary to take into account the collision-induced dissipation (which is small, but not exponentially small).

On account of the difficulties encountered in the solution of the quantum-kinetic equation, the collision integral in it is usually approximated by a relaxation time τ_{eff} (the τ-approximation).[1] But in such an approach there is complete absence of information about the temperature, field, frequency, etc., dependences of the relaxation time τ_{eff} for the various scattering mechanisms. At the same time, it is well known[2] that strong magnetic fields, which modify the energy spectrum of the current carriers, can lead to the dependence on magnetic field of the rate of the relaxation processes, which, together with temperature, allows us to control the characteristic relaxation rates in the system with the aid of a magnetic field. In this connection, it is of particular interest to find an explicit expression for the collision rate τ_{eff}^{-1} for the various scattering mechanisms, for which purpose we need to solve the kinetic equation with the appropriate collision integral.

In the present paper we shall restrict ourselves to the consideration of a particle system with Coulomb interaction (an electron-ion plasma), located in a strong (quantizing) magnetic field in the presence of a high-frequency, alternating electromagnetic field. The quantum-kinetic equation for such a system with a collision integral that consistently takes the polarization effects into account was obtained by Silin.[3] In the limit $\omega\tau_{eff}\gg 1$, the collision integral in the kinetic equation is a small term, and the solution to this equation can be sought by the method of successive approximations, expanding it in "powers of the collision integral." In addition, we shall restrict ourselves to the region of weak spatial dispersion, which also allows the simplification of our problem.

As usual, the solution of the kinetic equation requires the determination of the correction, $\delta\mathbf{j}$, due to the collisions, to the induced-current density. In determining this correction, we shall take only the electron current into account, since the ion current makes a very small contribution, in view of the large ion mass. Further, as is well known,[4] in the frequency and wavelength regions under consideration, we can neglect the electron-electron collisions and assume the ion distribution to be close to the equilibrium distribution. Under all the indicated assumptions, for the steady-state process corresponding to a monochromatic oscillation ($\sim e^{-i\omega t}$) in a spatially homogeneous plasma, we find the collision correction to the induced electron current:

$$\delta\mathbf{j}_e^\omega=\mathrm{Sp}(\delta\rho_e^\omega\mathbf{j}_e)=e\sum_{\nu_e,\nu_e'}\langle\nu_e|\delta\rho_e^\omega|\nu_e'\rangle\langle\nu_e'|\mathbf{v}_e|\nu_e\rangle, \tag{1}$$

where ν_e is the set of quantum numbers characterizing the electron state in the absence of an electromagnetic disturbance, while $\mathbf{v}_e$ is the electron-velocity operator.

The expression for the collision correction $\delta\rho_e^\omega$, to the single-particle electron density matrix can be found under our assumptions by using Silin's results,[3] to wit:

$$\begin{aligned}\langle\nu_e|\delta\rho_e^\omega|\nu_e'\rangle=&\frac{-i\pi}{E(\nu_e')-E(\nu_e)+\hbar\omega}\int\frac{d\mathbf{k}}{(2\pi)^3}\sum_{\beta,\nu_e'',\nu_\beta,\nu_\beta'}\left(\frac{4\pi ee_\beta}{k^2}\right)^2\\&\times\Big\{f_{\nu_e'}f_{\nu_\beta}(1-e^{-\hbar\omega/T})|\langle\nu_\beta|e^{i\mathbf{kr}_\beta}|\nu_\beta'\rangle|^2\langle\nu_e|e^{-i\mathbf{kr}_e}|\nu_e''\rangle.\\&\times\sum_{\nu_e'''}\left(\frac{\langle\nu_e''|e^{i\mathbf{kr}_e}|\nu_e'''\rangle}{E(\nu_e')-E(\nu_e''')+\hbar\omega}\langle\nu_e'''|H_e^\omega|\nu_e'\rangle+\frac{\langle\nu_e''|H_e^\omega|\nu_e'''\rangle\langle\nu_e'''|e^{i\mathbf{kr}_e}|\nu_e'\rangle}{E(\nu_e'')-E(\nu_e''')-\hbar\omega}\right)\\&\times\delta(E(\nu_e')+E(\nu_\beta)-E(\nu_e'')-E(\nu_\beta')+\hbar\omega)+f_{\nu_e}f_{\nu_\beta}(1-e^{\hbar\omega/T})\\&\times|\langle\nu_\beta'|e^{i\mathbf{kr}_\beta}|\nu_\beta\rangle|^2\sum_{\nu_e'''}\left(\frac{\langle\nu_e|H_e^\omega|\nu_e'''\rangle}{E(\nu_e)-E(\nu_e''')-\hbar\omega}\langle\nu_e'''|e^{i\mathbf{kr}_e}|\nu_e''\rangle\right.\\&\left.+\frac{\langle\nu_e|e^{i\mathbf{kr}_e}|\nu_e'''\rangle\langle\nu_e'''|H_e^\omega|\nu_e''\rangle}{E(\nu_e'')-E(\nu_e''')+\hbar\omega}\right)\langle\nu_e''|e^{-i\mathbf{kr}_e}|\nu_e'\rangle\\&\times\delta(E(\nu_e)+E(\nu_\beta)-E(\nu_e'')-E(\nu_\beta')-\hbar\omega)\Big\},\end{aligned} \tag{2}$$

where the operator H_e^ω describes the resultant alternating self-consistent field in the medium; the remaining symbols have the same meanings as in[3]. It should be noted that, in deriving the last expression, we assumed the equality of the temperatures of the various kinds of carriers, which obey nondegenerate statistics, and also neglected the polarization effects.

In the case of a plasma located in a constant magnetic field $\mathbf{B}_0 = (0; 0; B_0)$ with the gauge $\mathbf{A}_0 = (0; B_0 x; 0)$, as the complete set ν_α, we can use the particle-momentum components p^z_α and p^y_α and the magnetic quantum number, n_α, characterizing the energy of the transverse motion ($\Omega_\alpha = |e_\alpha| B_0/m_\alpha c$):

$$E(\nu_\alpha) = E^\perp(\nu_\alpha) + E^z(\nu_\alpha) = \hbar\Omega_\alpha(n_\alpha + 1/2) + (p_\alpha^z)^2/2m_\alpha. \quad (3)$$

After choosing the basis, we can write down explicit expressions for the matrix elements entering into (1) and (2):

$$\langle\nu_\alpha|\exp(i\mathbf{kr}_\alpha)|\nu_\alpha'\rangle = \delta(p_\alpha'^z - p_\alpha^z + \hbar k_z)\delta(p_\alpha'^y - p_\alpha^y + \hbar k_y) \times \exp\left[\frac{i}{2}k_x(X_{0\alpha} + X_{0\alpha}')\right] I_{n_\alpha n_\alpha'},$$

$$I_{n_\alpha n'_\alpha} = \frac{\bar{n}_\alpha!}{(n_\alpha! n_\alpha'!)^{1/2}} \exp[-i\varphi(n_\alpha - n_\alpha')]\left(\frac{i\lambda_\alpha k_\perp}{2^{1/2}}\right)^{|n_\alpha - n_\alpha'|} \times \exp\left(-\frac{\lambda_\alpha^2 k_\perp^2}{4}\right) L_{\bar{n}_\alpha}^{|n_\alpha - n'_\alpha|}\left(\frac{\lambda_\alpha^2 k_\perp^2}{2}\right), \quad (4)$$

$$\langle\nu_\alpha|H_\alpha^\omega|\nu_\alpha'\rangle = \frac{ie_\alpha}{\omega}\mathbf{E}^\omega\langle n_\alpha|\mathbf{v}_\alpha|n_\alpha'\rangle\delta(p_\alpha'^z - p_\alpha^z)\delta(p_\alpha'^y - p_\alpha^y),$$

$$\langle n_\alpha|v_\alpha^x|n_\alpha'\rangle = i(\hbar\Omega_\alpha/2m_\alpha)^{1/2}[(n_\alpha+1)^{1/2}\delta_{n'_\alpha; n_\alpha+1} - n_\alpha^{1/2}\delta_{n'_\alpha; n_\alpha-1}],$$

$$\langle n_\alpha|v_\alpha^y|n_\alpha'\rangle = (\hbar\Omega_\alpha/2m_\alpha)^{1/2}[(n_\alpha+1)^{1/2}\delta_{n'_\alpha; n_\alpha+1} + n_\alpha^{1/2}\delta_{n'_\alpha; n_\alpha-1}],$$

$$\langle n_\alpha|v_\alpha^z|n_\alpha'\rangle = p_\alpha^z\delta_{n'_\alpha; n_\alpha}/m_\alpha,$$

where $X_{0\alpha} = \mathrm{sign}(e_\alpha)\lambda^2_\alpha k^\alpha_y$, $\lambda_\alpha = (c\hbar/|e_\alpha|B_0)^{1/2}$ is the magnetic length for the particle α, $\bar{n}_\alpha \equiv \min(n_\alpha, n'_\alpha)$, $\varphi = \mathrm{arctg}(k_y/k_x)$, $k^2_\perp = k^2_x + k^2_y$, and $L^s_n(x)$ is the Laguerre polynomial.

In writing down the expressions (4), we neglected the spatial dispersion and took for the self-consistent alternating field the following potential gauge:

$$\varphi^\omega = 0, \quad \mathbf{E}^\omega = i\omega\mathbf{A}^\omega/c. \quad (5)$$

2. Let us now proceed to the computation of the collision correction, $\delta\mathbf{j}^\omega$, to the induced-current density. For this purpose, it is necessary to substitute (2)–(4) into the formula (1). In view of the tediousness of the calculations, we shall consider only the correction, δj^ω_z, to the longitudinal current. The calculations for δj^ω_x and δj^ω_y are entirely similar.

Thus, after standard transformations, the expression for δj^ω_z can be written in the form

$$\delta j_z^\omega = \frac{4e^4 N_e E_z^\omega}{m_e^2\omega^3}(1 - e^{-\hbar\omega/T})\sum_\beta N_\beta e_\beta^2 \int_0^\infty dk_z k_z^2 \times \int_0^\infty dk_\perp k_\perp \int_{-\infty}^{+\infty} dt \sum_{s=-\infty}^{+\infty} A_s(k_\perp)\frac{1}{k^4}\exp\left\{-\frac{\hbar\Omega_e}{2T}s + it\left(\hbar\Omega_e s + \hbar\omega - \frac{\hbar^2 k_z^2}{2m_e}\right) - t^2\frac{\hbar^2 k_z^2}{2m_e}T\right\}; \quad (6)$$

$$A_s(k_\perp) = \exp\left(-\frac{\lambda_e^2 k_\perp^2}{2}\mathrm{cth}\frac{\hbar\Omega_e}{2T}\right) I_s\left(\frac{\lambda_e^2 k_\perp^2}{2}\mathrm{csch}\frac{\hbar\Omega_e}{2T}\right),$$

$I_s(x)$ is the Bessel function of imaginary argument. In deriving (6), we neglected the terms $\sim m_e/m_\beta$.

The last relation can be rewritten as follows:

$$\delta j_z^\omega = \frac{8e^4 N_e E_z^\omega}{\hbar\Omega_e m^2\omega^3}(1 - e^{-\hbar\omega/T})\sum_\beta N_\beta e_\beta^2 \int_0^\infty dk_z k_z^2 \times \int_0^\infty \frac{dk_\perp k_\perp}{(k_\perp^2 + k_z^2)^2}\int_0^\infty dt \cos\left[t\left(\frac{\omega}{\Omega_e} - \frac{\lambda_e^2}{2}k_z^2\right) - \frac{\lambda_e^2}{2}k_\perp^2 \sin t\right] \times \exp\left\{-\frac{t^2 r_L^2}{2}\left[k_z^2 + 2k_\perp^2\frac{\hbar\Omega_e}{T}\mathrm{cth}\frac{\hbar\Omega_e}{2T}\frac{\sin^2(t/2)}{t^2}\right]\right\} \quad (7)$$

where $r_L = v_T/\Omega_e$ is the Larmor electron radius.

It is easy to see that, in weak magnetic fields (where the role of maximum impact parameter is played by the distance traversed by an electron during a period of the oscillation), as well as in the region of strong magnetic fields (when the role of maximum impact parameter is played the Larmor radius),[5] the dominant contribution to the integration over t is made by small values of t ($t < 1$). This fact enables us to immediately write down the expression for the collision correction to the electrical-conductivity tensor component:

$$\delta\sigma_{zz}(\omega) = \frac{N_e e^2}{m_e\omega^2}\frac{1}{\tau_{eff}}. \quad (8)$$

Here

$$\tau_{eff}^{-1} = 4\left(\frac{2\pi}{m_e T}\right)^{1/2}\frac{e^2}{\hbar\omega}(1 - e^{-\hbar\omega/T})\sum_\beta N_\beta e_\beta^2 \int_0^1 dx\, x^2 \times \left(\frac{1-a}{1-ax^2}\right)^{1/2}\exp\left\{\frac{\hbar\omega}{2T}\frac{(1-a)}{(1-ax^2)}\right\}K_0\left(\frac{\hbar\omega}{2T}\frac{(1-a)}{(1-ax^2)}\right), \quad (9)$$

$$a = \left[1 - \frac{2T}{\hbar\Omega_e}\mathrm{th}\left(\frac{\hbar\Omega_e}{2T}\right)\right],$$

and $K_0(x)$ is the cylindrical function of imaginary argument.

We can similarly compute the collision corrections, δj^ω_x and δj^ω_y, to the currents, corrections which, together with (8), allow us to find the collision correction to the permittivity tensor, $\delta\varepsilon_{ij}(\omega)$, due to the electron-ion interaction:

$$\delta\varepsilon_{ij}(\omega) = \frac{4\pi i}{\omega}\delta\sigma_{ij}(\omega) = \begin{pmatrix} \delta\varepsilon_1 & i\delta g & 0 \\ -i\delta g, & \delta\varepsilon_1, & 0 \\ 0 & 0 & \delta\varepsilon_2 \end{pmatrix} \quad (10)$$

where

$$\delta\varepsilon_1 = i\frac{\omega_p^2(\omega^2 + \Omega_e^2)}{\omega(\omega^2 - \Omega_e^2)^2}\frac{1}{\tau_{eff}}, \quad \delta\varepsilon_2 = i\frac{\omega_p^2}{\omega^3}\frac{1}{\tau_{eff}}, \quad \delta g = i\frac{2\omega_p^2\Omega_e}{(\omega^2 - \Omega_e^2)^2}\frac{1}{\tau_{eff}}. \quad (11)$$

3. Let us discuss in greater detail the expression for the effective electron-ion collision rate τ^{-1}_{eff}. In the absence of a magnetic field, or in a sufficiently weak magnetic field ($\hbar\Omega_e \ll T$), we have from (9)

$$\frac{1}{\tau_{eff}} = \frac{8}{3}\left(\frac{2\pi}{m_e T}\right)^{1/2}\frac{e^2}{\hbar\omega}\mathrm{sh}\left(\frac{\hbar\omega}{2T}\right)\sum_\beta N_\beta e_\beta^2 K_0\left(\frac{\hbar\omega}{2T}\right), \quad (12)$$

which exactly coincides with the result obtained by Perel' and Éliashberg[6] in the region of high oscillation frequencies.

At the same time, in the presence of a strong, quantizing magnetic field ($\hbar\Omega_e > T$), the formula (9) goes over into the formula

$$\frac{1}{\tau_{eff}}=4\left(\frac{\pi}{m_e\hbar\Omega_e}\right)^{1/2}\frac{e^2}{\hbar\omega}(1-e^{-\hbar\omega/T})\sum_\beta N_\beta e_\beta^2\int_1^{\hbar\Omega_e/2T}dy$$
$$\times y^{-2}(y-1)^{1/2}\exp\left(\frac{\omega}{\Omega_e}y\right)K_0\left(\frac{\omega}{\Omega_e}y\right). \quad (13)$$

At high frequencies ($\omega \gg \Omega_e \gg T/\hbar$), the last relation assumes the form

$$\frac{1}{\tau_{eff}}=\frac{8\pi e^2}{3(2m_e)^{1/2}}(\hbar\omega)^{-3/2}\sum_\beta N_\beta e_\beta^2. \quad (14)$$

Thus, as was to be expected, at high frequencies, neither the temperature, nor the magnetic field exerts an appreciable influence on the dissipation processes. Therefore, the coincidence of the expression (14) with the high-frequency approximation in the absence of a magnetic field[6] appears to be quite natural.

Let us further discuss the region of sufficiently low frequencies ($\omega \ll T/\hbar \ll \Omega_e$), so as to be in a position to compare the results with similar zero-magnetic-field results. In this case

$$\frac{1}{\tau_{eff}}=\left(\frac{\pi}{m_e}\right)^{1/2}\frac{2\pi e^2}{T(\hbar\Omega_e)^{1/2}}\sum_\beta N_\beta e_\beta^2\ln\left(\gamma\frac{\Omega_e}{\omega}\right), \quad (15)$$

where $2\gamma=\exp\{-(1+C)\}$; C is the Euler constant.

It follows from a comparison of (15) with the analogous zero-field result[6] that the temperature dependence of the effective collision rate gets modified in a quantizing magnetic field. In addition, there arises a dependence on the magnetic-field intensity, such a field dependence obtaining in both the argument of the logarithm and the coefficient standing in front of the logarithm.

It is interesting to carry out a comparison of the effective collision rates for the low-frequency limit ($\omega \ll T/\hbar$) in weak and strong magnetic fields. It follows from (12) and (15) that

$$\frac{1}{\tau_{eff}}(\hbar\Omega_e\gg T)\infty\left(\frac{T}{\hbar\Omega_e}\right)^{1/2}\frac{1}{\tau_{eff}}(\hbar\Omega_e\ll T). \quad (16)$$

Thus, we can conclude that the effective electron-ion collision rate has, in a quantizing magnetic field, a tendency to decrease at low field-oscillation frequencies, whereas at high frequencies this collision rate cannot be varied by varying either the magnetic field or the temperature. Notice that, besides assuming the spatial dispersion to be weak, our results presuppose at the same time that the oscillation frequency, ω, is sufficiently far removed from the resonance frequency Ω_e, i.e., presuppose the fulfillment, in addition, of the condition:

$$|\omega-\Omega_e|\gg\tau_{eff}^{-1}. \quad (17)$$

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