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Theory of radiation from charged particles channeled in a crystal

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A new physical effect is theoretically predicted, namely intense spontaneous emission of γ photons by relativistic channeled particles. The theory of this phenomenon is developed by the method of quantum electrodynamics and classical mechanics. Both approaches give practically the same results. In the quantum approach, the radiation is due to the transitions between different levels formed in the potential of the atomic planes and chains. At particle energies ~0.1–10 GeV, the radiation is most intense in the band from 0.1 to several dozen MeV. Although the total radiation intensity is less than the bremsstrahlung intensity, the spectral density in the region of the maximum of the radiation can greatly exceed the bremsstrahlung density. The effective cross section for the photoabsorption on the nuclear transitions in this region is larger by several orders of magnitude for the radiation of the channeled particles than for the bremsstrahlung, thus uncovering, in principle, new possibilities in nuclear physics. It is shown than the radiation has several fundamental attributes that distinguish it from the ordinary types. The feasibility of observing this effect in experiment is discussed.

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INTRODUCTION

The channeling effect has been the subject of a large number of investigations (see, e.g., Gemmel's review).[1] In this article we consider mainly channeling of light particles, positrons and electrons. According to Lindhard's predictions[2] it has turned out that with increasing energy the treatment of channeling by classical mechanics becomes more and more correct.[3–5] The angular distributions and the different possible states of the particles were thoroughly measured and calculated.[6–11]

The behavior of relativistic positrons turned out to be close to the behavior of protons, and to explain electron channeling along an atomic chain, the "reseton" model has been proposed.[12] The electron turns out in this case in a bound state in the potential well of the chain and moves around the chain. It was shown experimentally[13] and theoretically[14] that when positrons are channeled the bremsstrahlung is substantially suppressed because the contribution of the short-range collisions decreases. At the same time, the bremsstrahlung increases in the case of electron channeling.[13]

Kalashnikov *et al.*[15] have shown that for nonrelativistic particles there is produced in the channel one level. In this case, naturally, radiation is then possible. A situation is possible, however,[15] in which the wave function of the particle entering the crystal differs from the ground state, so that radiation becomes possible by a transition to the ground state. Even earlier, Thompson[16] had suggested the possibility of stimulated emission by protons in transitions to different states that are formed in the channel. This stimulated emission, in Thompson's opinion, lies in the infrared region and is due to particle scattering by phonons.

In this paper we consider a new type of radiation, which is possible when particles are channeled. It is known[17] that a large number of levels is produced when the energy of the particles in a channel is increased. Spontaneous emission becomes possible, in principle, as a result of transitions between these levels. The relativistic electrons and positrons emit in this case hard γ rays. The possible existence of this effect was proposed in[18]. This effect can also be interpreted on the basis of classical mechanics. The radiation power in the frequency region in which it takes place, turns out to be unusually large, exceeding by approximately two orders of magnitude the bremsstrahlung and by 6–9 orders of magnitude the radiation power of modern synchrotrons.

In the construction of the theory, no account was taken of certain secondary processes, such as dechanneling (see, e.g.,[19–21] on this subject), the effect of the radi-

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ation itself on the trajectory, etc,. These processes can be additionally taken into account in the interpretation of the experiments.

I. RADIATION IN PLANAR CHANNELING OF ELECTRONS. CLASSICAL CALCULATIONS

At high electron velocities, the concepts of classical mechanics are applicable to channeling. The reason is that the number of produced levels becomes in this case much larger than unity. According to Chaderton[17] and Andersen *et al.*[3] the number of levels in a planar channel is equal to

$$n_p^- \approx 0.4\,\gamma^{1/2}, \tag{1.1}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic factor. Formula (1.1) was obtained under the assumption that the distance between the planes is approximately equal to 5 Bohr radii. It is seen from (1.1) that $n_p^- > 1$ in the relativistic case. For the calculations that follow we shall use the potential $Y(y)$ introduced by Lindhard[2] for the atomic plane:

$$Y(y) = 2\pi Z_1 Z_2 e^2 N d_p [(y^2 + C^2 a^2)^{1/2} - y], \tag{1.2}$$

where y is measured from the plane, d_p is the width of the channel, N is the density of the atoms, Z_1e and Z_2e are respectively the charges of the particle and of the atoms of the target, $C = \sqrt{3}$, and a is the screening parameter in the Thomas–Fermi model of the atom. For the electrons, the potential (1.2) is attracting (i.e., the right-hand side of (1.2) should be taken with a minus sign). The electrons are captured in the potential well and move towards the atomic plane, cross it, are then again returned by attraction, etc., until they become dechanneled. Thus, the electron oscillates near the plane and its trajectory recalls a sinusoid in first-order approximation.

For simple estimates, we expand the potential (1.2) in a series about the point $y = 0$, confining ourselves to the second term of the expansion, i.e., we assume that the electron is captured in a parabolic well. Of course, this approximation is not very exact and is used here only to simplify the calculations.[1)]

We direct the z axis along the longitudinal velocity of the electron, i.e., along the atomic plane. The equation of motion in a parabolic potential is given by

$$\frac{d}{dt}\frac{mv_y}{[1-(v_y^2+v_z^2)/c^2]^{1/2}} = -2V_0 y, \tag{1.3}$$

where $V_0 = 2\pi Z_1 Z_2 e^2 N d_p / Ca$; $v_y = dy/dt$ and $v_z = dz/dt$ are the velocities in the directions of y and z. In the case of channeling $v_y \ll v_z$. The solution of (1.3) is

$$y(t) = y_m \sin\bar{\omega} t, \tag{1.4}$$

$$\bar{\omega}^2 = \frac{2V_0}{m}\left(1 - \frac{v_z^2}{c^2}\right)^{1/2}; \tag{1.5}$$

y_m is the initial amplitude.

Since the particle trajectory is bent by the action of the potential of the plane, radiation should be produced. The power of the radiation of a relativistic particle ($v_z \approx c$) moving along a circle with radius R is

$$I = \frac{2}{3}\frac{e^2 c}{R^2}\left(\frac{E}{mc^2}\right)^4, \tag{1.6}$$

where R^2 is the square of the curvature radius, and E is the particle energy. Let us find R^2 for our case. When the particle moves with acceleration, the instantaneous curvature radius is

$$R = v^2/\dot{v}_\perp \approx c^2/\dot{v}_\perp, \tag{1.7}$$

where $\dot{v}_\perp$ is the change of the transverse velocity. In our case $\dot{v}_\perp = -\bar{\omega}^2 y_m^2 \sin\bar{\omega}t$, i.e., on the average $R^2 = 2c^4/\bar{\omega}^4 y_m^2$. Therefore the power is on the average

$$I = y_m^2 e^2 \bar{\omega}^4 \gamma^4 / 3c^3. \tag{1.8}$$

Since the gradients of the fields of the atomic chains and planes are ~ 10^{11-12} eV/cm, the average curvature radius in the case of channeling is very small. Therefore the power turns out to be very large, higher by approximately 6–9 orders of magnitude than in modern synchrotrons.

Table I shows a comparison of some of the radiation parameters in channeling and in bremsstrahlung. The bremsstrahlung was calculated under the assumption that there is no channeling. The number of photons was calculated in the radiation-frequency interval $0.1\omega_m \leqslant \omega \leqslant \omega_m$, where $\omega_m = (1 + v_z/c)\bar{\omega}\gamma^2$ is the maximum radiation frequency in channeling. It is seen that although the total intensity of the bremsstrahlung is much larger than the channeling radiation, however, since the spectral density of the radiation in channeling at $\omega \approx \omega_m$ is much larger than that of the bremsstrahlung, the number of photons is greater by more than one order of magnitude in the case of channeling than the number of bremsstrahlung γ photons. Thus, for example, at $E = 1$ GeV, in the {110} channel in the Si crystal, the value of the spectral density at $\omega = \omega_m$ is approximately 40 times larger than the corresponding value for the bremsstrahlung photons (for more details see Sec. 6 below and Fig. 1).

2. RADIATION IN THE CASE OF PLANAR CHANNELING OF ELECTRONS. QUANTUM CALCULATION

In our case the electron moves with relativistic velocity along the plane and executes at the same time non-

TABLE I. Comparison of the energy dependence of the radiation parameters in the case of channeling (I) of positrons in silicon in the {110} channel and bremsstrahlung* (II).

Energy, GeV	Radiation power, W		Maximum radiation frequency, 10^{22} sec^{-1}		Number of quanta, cm^{-1}	
	II	I	II	I	II	I
0,2	0,121	$1{,}03\cdot10^{-3}$	30	0,036	0,418	1,54
1	0,605	$2{,}57\cdot10^{-2}$	150	0,358	0,418	3,88
5	3,02	0,64	750	4	0,418	8,45

*The bremsstrahlung was calculated for a non-oriented target.

relativistic transverse motion. Therefore the longitudinal motion can be described by the Dirac equation, and the transverse motion by the Schrödinger equation.

We start with the Dirac equation in the two-component form

$$(\sigma\nabla)\psi_b - i\frac{E-Y(y)-mc^2}{\hbar c}\psi_a=0, \quad (2.1a)$$

$$(\sigma\nabla)\psi_a - i\frac{E-Y(y)+mc^2}{\hbar c}\psi_b=0, \quad (2.1b)$$

where $\sigma=(\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices; from (2.1b) and (2.1a) we obtain for ψ_a

$$(\sigma\nabla)\frac{\hbar c}{i(E-Y(y)+mc^2)}(\sigma\nabla)\psi_a - i\frac{E-Y(y)-mc^2}{\hbar c}\psi_a=0. \quad (2.2)$$

Recognizing that

$$E \gg (Y(y); Y'(y)), \quad \psi=\begin{pmatrix}\psi_a\\ \psi_b\end{pmatrix}, \quad \psi=\exp[i(k_x x+k_z z)]\psi(y)$$

it follows from (2.2) that

$$\frac{d^2\psi_a}{dy^2}+\frac{E^2-m^2c^4-\hbar^2c^2(k_x^2+k_z^2)-2EY(y)}{\hbar^2c^2}\psi_a=0. \quad (2.3)$$

Formula (2.3) is the Schrödinger equation for the transverse motion:

$$-\frac{\hbar^2}{2M}\frac{d^2\varphi(y)}{dy^2}+Y(y)\varphi(y)=\varepsilon\varphi(y), \quad \varepsilon=\frac{E^2-m^2c^4-\hbar^2c^2(k_x^2+k_z^2)}{2E}, \quad M=\frac{E}{c^2}. \quad (2.4)$$

It must be borne in mind that for the transverse motion the potential $Y(y)$ is a periodic function with period d_p. In the general case, therefore, to solve the wave equation it is necessary to introduce the Bloch condition

$$\psi_k(y+d_p)=e^{ikd_p}\psi_k(y),$$

where k is the quasimomentum. We, however, are interested primarily in well-channeled particles with low-lying levels. In this case, as shown by estimates, we can confine ourselves to the solution of the equation in one potential well. For particles with transverse energy close to the potential barrier allowance for the Bloch condition leads to the formation of a band structure. We propose to deal with this interesting problem in a different paper.

From (2.4) we obtain the eigenvalues for the transverse motion:

$$\varepsilon_n=\hbar c(2V_0/E)^{1/2}(n+1/2), \quad (2.5)$$

where n runs through integer values. The solution of (2.4) for a harmonic potential is expressed in terms of Hermite polynomials. The solution of (2.3) now takes the form

$$\psi_{na}=\begin{pmatrix}C_1\\ C_2\end{pmatrix}\exp[i(k_x x+k_z z)]\varphi_n(y). \quad (2.6)$$

This expression takes into account the fact that the solution must describe an electron with spin both up and down.

From (2.1b) we can obtain an expression for ψ_b, neglecting $Y(y)$ in comparison with E:

$$\psi_b\approx\frac{\hbar c}{i(E+mc^2)}(\sigma\nabla)\psi_a=\frac{c}{(E+mc^2)}(\sigma\mathbf{p})\psi_a, \quad (2.7)$$

where $\mathbf{p}=-i\hbar\nabla$ is the momentum operator.

To calculate the radiation power I we use the quantum theory of radiation[22,23]

$$I=\frac{ce^2}{2\pi}\int d^3\varkappa\,\delta(\varkappa-\varkappa_{\mathrm{I\,II}})\Phi; \quad (2.8)$$

Here

$$\Phi=\bar{\alpha}^*\bar{\alpha}-(\varkappa^0\bar{\alpha}^*)(\varkappa^0\bar{\alpha}),$$

$\varkappa^0=\varkappa/|\varkappa|$ is a unit vector and $\varkappa$ is the wave vector of the emitted photon.

The matrix element $\bar{\alpha}$ is equal to

$$\bar{\alpha}=\int\psi_{\mathrm{I}}\alpha e^{-i\varkappa r}\psi_{\mathrm{II}}\,d^3x, \quad \alpha=\begin{pmatrix}0&\sigma\\ \sigma&0\end{pmatrix}; \quad (2.9)$$

σ are Pauli matrices and $\varkappa_{\mathrm{I\,II}}=(E_{\mathrm{II}}-E_{\mathrm{I}})/\hbar c$, i.e., we are considering a spontaneous transition from the state E_{II} to the state E_{I}. It is seen from (2.7) that

$$\bar{\alpha}=\frac{c}{E+mc^2}\int[2\psi_{\mathrm{I}a}^*\mathbf{p}\psi_{\mathrm{II}a}-\hbar\psi_{\mathrm{I}a}\sigma(\sigma\varkappa)\psi_{\mathrm{II}a}]e^{-i\varkappa r}d^3x. \quad (2.10)$$

If the calculation is carried out for the potential $Y=V_0y^2$, then we obtain in the dipole approximation (i.e., $\exp(-i\varkappa_y y)\approx1$) an intensity value

$$I=\frac{e^2\bar{\omega}^2\hbar\bar{\omega}n}{m\gamma c^3}\frac{1}{3(1-\beta_z^2)^2}\left[1+\frac{1}{\beta_z^2}-\frac{2}{\beta_z^2}(1-\beta_z^2)+\frac{1-\beta_z^2}{2\beta_z^3}\ln\left|\frac{1+\beta_z}{1-\beta_z}\right|\right], \quad \beta_z=\frac{v_z}{c}, \quad (2.11)$$

where n is the principal quantum number; in this case the selection rule is $\Delta n=\pm1$ (dipole transition in a harmonic well). As $\beta_z\to1$ we obtain from (2.11) with the aid of the relation $\hbar\bar{\omega}n\approx\frac{1}{2}m_{\mathrm{rel}}\bar{\omega}^2y_m^2$

$$I\approx\bar{\omega}^4x_m^2\gamma^4e^2/3c^3=I_{\mathrm{cl}}, \quad (2.12)$$

i.e., a calculation based on the use of quantum electrodynamics confirms the simples classical estimate.

3. RADIATION IN AXIAL CHANNELING OF POSITRONS

The potential U_t in the axial channel is the sum of the potentials of the nearest atomic chains:

$$U_t=\sum_{i=1}^{n}U_i, \quad (3.1)$$

where U_i is the potential of the i-th atomic chain; for the standard Lindhard potential[12] we have

$$U_i=\frac{Z_1Z_2e^2}{d}\ln\left|\frac{(Ca)^2}{r^2+b_i^2-2b_ir\cos\theta_i}+1\right|, \qquad (3.2)$$

where d is the distance between the atoms in the chain, b_i is the distance from the i-th chain to the center of the channel, and r is measured from the center of the channel. The potential (3.2) can be expanded in a Taylor series about $r=0$, assuming azimuthal symmetry. In this case

$$U_i(r)=\frac{Z_1Z_2e^2(Ca)^2}{d}\frac{n}{[b^2+(Ca)^2]^2}r^2. \qquad (3.3)$$

In the calculation of (3.3) it assumed that the number of the nearest chains is equal to n and that all are located at a distance b from the center.

For the potential (3.3) the formula obtained for the intensity is similar to (1.8) with y_m replaced by the initial amplitude r_m of the particle in the channel; in addition we have in this case

$$\bar{\omega}^2=\frac{2Z_1Z_2e^2(Ca)^2n}{md}\frac{(1-\beta_z^2)^{1/2}}{[b^2+(Ca)^2]^2}. \qquad (3.4)$$

We see therefore that in axial channeling the radiation is harder than in planar channeling.

The quantum analysis shows that

$$I\approx I_{cl}. \qquad (3.5)$$

The energy levels in a two-dimensional harmonic well are given by[24]

$$E_n=\hbar\omega(1+n), \qquad (3.6)$$

where $n=|M|+2n_r$. Here $m=0, \pm1, \pm2$; $n_r=0, 1, 2$. The selection rules for the radiation are in our case

$$\Delta n_r=0, \pm1; \quad \Delta M=\pm1. \qquad (3.7)$$

In the case of planar channeling of positrons, the assumption of a harmonic well near the midpoint of the channel is a fair approximation. The formula for the power is then similar to (1.8), except that

$$\bar{\omega}^2=\frac{0.35}{m}\frac{8\pi NZ_1Z_2e^2l}{be^{bl}}(1-\beta_z^2)^{1/2}. \qquad (3.8)$$

where $l=d_p/2$ is half the distance between planes, and $b=0.3/a$. In the derivation of (3.8) we used a Thomas-Fermi potential in the Moliere approximation. The selection rules are $\Delta n=\pm1$, where n is the principal quantum number.

4. RADIATION PRODUCED BY AXIALLY CHANNELED ELECTRONS

Axially channeled electrons can execute motion of the rosette type.[12] To simplify the calculations we shall assume that the trajectory is a helix that winds around the atomic chain.

From the requirement that the angular momentum L and the transverse energy $E_\perp$ be conserved we can obtain in the ultrarelativistic approximation $\beta_z\to1$ the curvature radius

$$R\approx m\gamma c^2\Big/\left|\frac{\partial U}{\partial r}\right|, \qquad (4.1)$$

where U is the Lindard potential of the continuous atomic chain. This potential can be chosen in the form[9]

$$U=-Z_2e^2/r+\bar{C}, \qquad (4.2)$$

where parameter $\bar{C}$ is chosen such that U agrees best with the standard Linhard potential. The average curvature radius is then

$$\bar{R}\approx\frac{m\gamma c^2}{Z_2e^2}\bar{r}^2, \qquad (4.3)$$

where $\bar{r}^2$ is the mean squared radius of the helix around the atomic chain. Substituting (4.3) in (1.6) we obtain the value of the intensity.

In the quantum approach, the radiation is due to the change of the quantum number L. The selection rules are

$$\Delta L=\pm1. \qquad (4.4)$$

5. SPECTRAL AND THE POLARIZATION PROPERTIES OF THE RADIATION

As seen from the foregoing, in the ultrarelativistic limits the classical and quantum approaches yield close results. Classical electrodyanmics can therefore be used to cast light on certain properties of the radiation. Moreover, many results of the theory of ondulatory radiation[23–28] can be transferred to our case, the only difference being that the oscillation frequency in the channel depends on the relativistic factor γ, i.e., on the energy.

The energy dE radiated by a particle into a solid angle element d in the interval $d\Omega$ is given in the theory of ondulatory radiation[25–27] by the formula

$$\frac{dE}{d\Omega\,d\omega}=\frac{e^2}{4\pi^2c}|\mathbf{A}_\omega|^2, \qquad (5.1)$$

where $\mathbf{A}_\omega$ is a vector proportional to the Fourier components of the electric field intensity:

$$|\mathbf{A}_\omega|=\left|\int_{-\infty}^{+\infty}\frac{[[\mathbf{n}\times(\mathbf{n}-\boldsymbol{\beta})]\times\dot{\boldsymbol{\beta}}]}{(1-\boldsymbol{\beta}\mathbf{n})^2}\exp\left\{i\omega\left[t-\frac{\mathbf{n}\mathbf{r}(t)}{c}\right]\right\}dt\right|, \qquad (5.2)$$

where $\mathbf{n}$ is a unit vector in the direction of the emitted γ photon, $\boldsymbol{\beta}=\mathbf{v}/c$, $\mathbf{v}$ is the total velocity, and $\mathbf{r}(t)$ is specified by the particle trajectory.

As seen from (5.2), the radiation spectrum is formed on the entire particle trajectory. It is therefore convenient to use the averaged spectral radiation intensity, defined as the ratio of the energy radiated by the particle over the entire length λ_1 of the crystal into a unit solid angle and a unit frequency interval to the travel time:

$$\frac{dI}{d\omega\, d\Omega}=\frac{v_z}{l_1}\frac{dE}{d\omega\, d\Omega}. \tag{5.3}$$

We assume here that the longitudinal particle velocity v_z remains unchanged. To calculate the spectral distribution we now consider by way of example planar channeling of positrons. We direct the z axis along the plane and the x axis across the channel. In aharmonic well $x=x_m \sin\bar{\omega}t$, $z=v_z t$, $y=0$ we then have

$$\bar{\omega}^2=\frac{2V_0}{m}\left(1-\frac{v_z^2}{c^2}\right)^{1/2}. \tag{5.4}$$

Formula (5.4) specifies the particle trajectory $r(t)$. Substituting (5.4) in (5.2) and (5.3) we can calculate the emission spectrum in the same manner as for the ondulatory radiation.[27,28]

The result can be obtained in simpler fashion by using the formula for the radiation intensity:

$$I=-c\int \mathbf{j}^*\mathbf{E}\, d^3x, \tag{5.5}$$

which follows from the Maxwell-Lorentz equation; here $\mathbf{j}$ is the current-density vector ($\mathbf{j}^*=\mathbf{j}$) and $\mathbf{E}$ is the electric field vector. In our case, obviously, the charge and current densities are respectively equal to

$$\rho=-e\delta(x-x_m\sin\bar{\omega}t)\delta(z-v_z t)\delta(y),$$
$$\mathbf{j}=\mathbf{v}\rho/c. \tag{5.6}$$

To solve the wave equation for the radiation-field potential we use a series expansion of the functions that enter in the equation and a Fourier integral of the following type (for details see, e.g., [23]):

$$F(\mathbf{r},t)=\frac{1}{(2\pi)^3}\sum_{\nu=1}^{\infty}\int d^3k\exp[i\mathbf{kr}-i\nu\bar{\omega}t-iv_z k_z t)F_\nu(\mathbf{k}). \tag{5.7}$$

It turns out that

$$\mathbf{E}_\nu=i4\pi G(\bar{\omega}_\nu\mathbf{j}_\nu/c-\mathbf{k}\rho_\nu), \tag{5.8}$$

where G is the Green's function:

$$G=\frac{1}{k^2-(\bar{\omega}_\nu/c)^2}+i\pi\frac{\bar{\omega}_\nu}{|\bar{\omega}_\nu|}\delta\left(k^2-\frac{\bar{\omega}_\nu^2}{c^2}\right),$$
$$\bar{\omega}_\nu=\nu\bar{\omega}+v_z k_z. \tag{5.9}$$

The radiation can then be expressed in the form

$$I=-\frac{ice^2}{2\pi}\sum_{\nu=-\infty}^{\infty}\int d^3k(G_1+G_0)\left[\frac{\bar{\omega}_\nu}{c}\mathbf{j}_\nu^*\mathbf{j}_\nu-\mathbf{k}\mathbf{j}_\nu\rho_\nu\right]. \tag{5.10}$$

where G_1 and G_0 are the singular and nonsingular parts of the Green's function

$$G_1=\frac{1}{k^2-(\bar{\omega}_\nu/c)^2},\quad G_0=i\pi\frac{\bar{\omega}_\nu}{|\bar{\omega}_\nu|}\delta\left(k^2-\left(\frac{\bar{\omega}_\nu}{c}\right)^2\right).$$

In the ultrarelativistic limit, when $v_z/c\to 1$, we obtain from (5.10) the following angular distribution in the dipole approximation:

$$\frac{dI}{\sin\theta\, d\theta}=\frac{2x_m^2e^2\bar{\omega}^4}{c^3}\left[\frac{\gamma^{-4}+\theta^4}{(\gamma^{-2}+\theta^2)^5}\right] \tag{5.11}$$

where θ is the angle between the radiation direction and the z axis. This formula differs from the ondulator formula because in our case $\bar{\omega}$ depends on the relativistic factor γ, whereas in the case of the ondulator there is no such dependence.

In the dipole approximation, when radiation on the first harmonic is considered (the intensities of the remaining harmonics are negligibly small), it is customary to use[27] a spectral distribution normalized to unity:

$$i_\omega=\frac{dI}{d\omega}\frac{1}{I},\quad \int_0^\infty i_\omega\, d\omega=1. \tag{5.12}$$

An analysis of (5.10) with allowance for the fact that for the first harmonic we have

$$\omega=\bar{\omega}\Big/\left(1-\frac{v_z}{c}\cos\theta\right),$$

shows that in the ultrarelativistic limit

$$\frac{dI}{d\omega}=\frac{I}{\omega_m}\frac{3}{2}\frac{\omega}{\omega_m}\left[1+\frac{c}{v_z}\left(1+4\left(\frac{\omega}{\omega_m}\right)^2-4\frac{\omega}{\omega_m}\right)\right]$$
$$\approx\frac{3I}{\omega_m}\frac{\omega}{\omega_m}\left[1-2\frac{\omega}{\omega_m}+2\left(\frac{\omega}{\omega_m}\right)^2\right], \tag{5.13}$$

where I is given by (1.8). The value of $\bar{\omega}$ in I is given by (3.9) in the case of planar channeling of positrons.

In the case of planar channeling, just as for an ondulator with sinusoidal trajectory, [27] we have linear polarization. Let the projection of the radiation vector k on the xy plane make an angle φ with the z axis (the vector itself makes an angle θ with the z axis). We introduce a unit polarization vector $\mathbf{i}$ parallel to the electric field (see (5.8)). In the ultrarelativistic limit we then have

$$\frac{i_z}{i_y}\approx\frac{1}{2}\frac{\gamma^{-2}-\theta^2}{\sin\theta\sin\varphi}. \tag{5.14}$$

It is obvious therefore that at $\varphi=0$ and $\varphi=\pi/2$ the vector $\mathbf{i}$ lies in the plane of the orbit, i.e., in the xz plane.

6. EFFECTIVE ABSORPTION CROSS SECTION. COMPARISON WITH OTHER TYPES OF RADIATION

We consider a certain state of the nucleus with a lifetime $\tau=\hbar/\Gamma$, where Γ is the width of the level under consideration. The cross section for resonant scattering or absorption of γ photons is given in this case by the Breit–Wigner formula. Of practical interest is the effective cross section, which is obtained by averaging the resonant cross section over the spectral distribution of the radiation in question.

The effective bremsstrahlung cross section σ^b_{eff} is[29]

$$\sigma^b_{\text{eff}}=\pi\Gamma\sigma_0/2E_{\text{lim}} \tag{6.1}$$

where E_{lim} is the maximum value of the γ-photon energy, $E_{\text{lim}}=E$, and σ_0 is the resonant cross section.

In our case, when the distribution (5.13) is used, we obtain

$$\sigma^{\text{chan}}_{\text{eff}}\approx 2.5\sigma_0\Gamma/\hbar\omega_m. \tag{6.2}$$

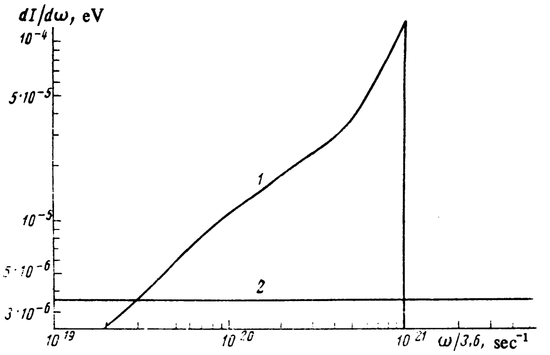

FIG. 1. Spectral distribution of radiation of channeled positrons with energy 1 GeV in the planar channel {110} of silicon (curve 1) and the bremsstrahlung spectrum (curve 2).

From (6.1) and (6.2) we get

$$\frac{\sigma_{\text{eff}}^{\text{chan}}}{\sigma_{\text{eff}}^{b}}=\frac{2.5}{\pi}\left(\frac{mc^2}{E}\right)^{1/2}\frac{mc^2}{\hbar\Omega_0}\qquad(6.3)$$

where $\Omega_0^2=2V_0/m$ is the natural frequency of the particle oscillation in the channel (without allowance for the relativistic factor). It is seen from (6.3) that at $E\sim 100$ MeV and $\Omega_0\sim 10^{16}$ sec^{-1} (a typical case) we have

$$\sigma_{\text{eff}}^{\text{chan}}/\sigma_{\text{eff}}^{b}>10^3.$$

Thus, the effectiveness of the action of the radiation produced by channeling on the nuclear transition is higher by several orders of magnitude than the effectiveness of the bremsstrahlung. The reason is that the bremsstrahlung spectrum is close to a constant quantity that extends from 0 to $E_{1\text{im}}$, and therefore the fraction of the photons used for resonance is small (of the order of $\sim\Gamma/E_{1\text{im}}$). In the case of channeling, the spectrum has a good degree of monochromaticity and therefore $\sigma_{\text{eff}}^{\text{chan}}$ is quite large.

As an absolute measure of the effectiveness of the action of the radiation on a certain nuclear transition we can use the spectral density of the radiation in the corresponding frequency band. Figure 1 shows the results of the calculation of the spectral density for positrons at $E=1$ GeV in silicon for the {110} channel, and also the results of the calculation for the bremsstrahlung. The spectral distribution for the case of channeling was calculated from the formulas

$$\frac{dI}{d\omega}=\frac{3I}{\omega_m}\frac{\omega}{\omega_m}\left[1-2\frac{\omega}{\omega_m}+2\left(\frac{\omega}{\omega_m}\right)^2\right],\qquad(6.4)$$

$$I=x_m^2e^2\varpi^4\gamma^4/3c^3.\qquad(6.5)$$

As seen from the figure, when $\omega=\omega_m$ ($\omega_m=3.58\cdot 10^{21}$ sec) the spectral density of the radiation is approximately 40 times larger in the case of channeling than the bremsstrahlung density. We note that with further increase of the particle energy, this ratio becomes even larger.

In the calculation of the bremsstrahlung spectrum it was assumed that the beam is so oriented that no channeling appears. From the experimental point of view, however, it is important to know also the background of the bremsstrahlung radiation in the case of channeling. It is known[13] that usually the positron bremsstrahlung is substantially suppressed (by a factor of 5–10) in the case of channeling. This means that the spectral density of the radiation of the channeled particles at $\omega=\omega_m$ exceeds by more than two orders of magnitude the density of the accompanying bremsstrahlung at $E\sim 1$ GeV, i.e., the effect should be easily observed.

Figure 2 shows the results of the calculation of the number of γ photons emitted on 1 cm of path in a unit frequency interval. This number, for the case of channeling, was calculated from the formula

$$\frac{d^2N}{dx\,d\omega}=\frac{1}{\hbar c}\left(\frac{dI}{d\omega}\right)\frac{1}{\omega}.\qquad(6.6)$$

The same number for bremsstrahlung was calculated from the approximate formula

$$\frac{d^2N}{dx\,d\omega}\approx\frac{4}{3}\frac{1}{\omega}\frac{1}{R}\qquad(6.7)$$

where R [cm] is the radiation length.

Comparison with synchrotron radiation also shows that the monochromaticity of the radiation is substantially better in channeling. In addition, it must be borne in mind that the minimum wavelengths obtained with modern synchrotrons are close to 0.1 Å (i.e., $\hbar\omega\sim 100$ keV); the beam energies are in this case of the order of 1–10 GeV.[30]

In the case of channeling one can obtain at these energies values of $\hbar\omega$ on the order of several dozen MeV, i.e., it is possible in practice to investigate all the nuclear transitions, and since the effective cross sections are large in this case, it is clear that the radiation of the channeled particles uncovers in principle new possibilities in nuclear physics. In the case when the angular divergence of the beam is less than $1/\gamma$ (a readily attainable value), one harmonic is emitted at a given angle, with a line width

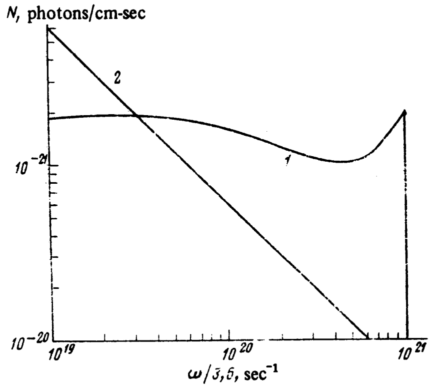

FIG. 2. Number of γ photons emitted per cm of length in a unit frequency interval by channeled electrons with $E=1$ GeV in silicon in the channel {110} (curve 1). Curve 2—the same for bremsstrahlung.

TABLE II. Comparison of certain characteristics of various types of radiation.

	Cerenkov radiation	Brems-strahlung	Synchrotron radiation	Ondulator	Radiation in channeling
Mass dependence of the intensity	No dependence	M_1^{-2}	M_1^{-2}	No dependence	M_1^{-2}
Energy dependence of the intensity	No dependence for relativistic particles	E	E^2	E^2	E^2
Energy dependence of the maximum frequency		E	E	E^2	$E^{3/2}$
Mass dependence of the maximum frequency			M_1^{-1}	No dependence	$M_1^{-1/2}$
Presence of radiation threshold	Present	Absent	Absent	Absent	Present

$$\Delta\omega=\omega/\mathcal{N}, \tag{6.8}$$

where $\mathcal{N}$ is the number of waves spanned by the entire trajectory of the channeled particles; the wavelength is $\lesssim 1$ μm for a particle with $E\sim 0.1-1$ GeV. The wavelength is defined here as the spatial periodicity of the particle trajectory. The attainable value of $\mathcal{N}$ is 10^3. Therefore at $\hbar\omega\sim 100$ keV we have $\Delta\omega\sim 100$ eV, i.e., when radiation at a given angle is considered, a high degree of monochromaticity can be attained. It turns out here that

$$\sigma_{\mathrm{eff}}^{\mathrm{chan}}\approx\frac{\pi}{4}\sigma_0\frac{\Gamma}{\overline{\Delta E}}\frac{1}{\sqrt{2\pi}},\quad \overline{\Delta E}=\frac{\hbar\omega}{\mathcal{N}}. \tag{6.9}$$

It is clear that in this case the intensity decreases, since we "cut out" a small angle when radiation is detected at a given angle. At $\mathcal{N}\gg 1$ the radiation frequency is uniquely connected with the emission direction by the relation (at $n=1$)

$$\omega=\frac{\Omega_0}{1-\beta\cos\theta}\approx\frac{2\Omega_0}{\theta^2+\gamma^{-2}}, \tag{6.10}$$

where $\Omega_0^2=2V_0/m$. It is seen from (6.10) that, generally speaking, the emission line width is determined by the cut-out angle at which the radiation is observed

$$|\Delta\omega|\approx 4\Omega_0\theta\Delta\theta/[\theta^2+\gamma^{-2}]^2. \tag{6.11}$$

Therefore, in order for relation (6.8), meaning also (6.9), to be valid if

$$2\gamma^2\theta\Delta\theta\approx 2\gamma^2\theta^2<1/\mathcal{N}. \tag{6.12}$$

The maximum radiation frequency and the intensity in one and the same crystal depend on the orientation and on whether the channeling is axial or planar. Therefore, in one and the same crystal (at a fixed beam energy) the radiation frequency can be varied by a factor 3-4, and the intensity by approximately two orders of magnitude. If the target is changed, then the frequency can be varied by one order of magnitude and the intensity by approximately four orders.

The maximum frequency ω_m depends on the potential of the channel, so that by measuring this potential it is possible to obtain information on the lattice potential. In addition, it is possible here to obtain also information on the initial energy of the beam. By exciting a fully defined nuclear transition with a known line width, it is possible to measure ω_m quite accurately, meaning also the energy of the primary particle beam. Incidentally, this method of exciting nuclei can be used to observe the radiation effect itself.

Table II shows a comparison of this radiation with other types of radiation. It is seen that the radiation produced by channeling differs in its properties from the known types of radiation. It is closest to ondulatory radiation. We therefore consider in greater detail the difference between the two.

7. RELATION BETWEEN THE RADIATION OF A CHANNELED PARTICLE AND RADIATION IN AN ONDULATOR

To demonstrate more clearly the difference between the radiation in a lattice channel and the ondulator radiation, we recall the derivation of the maximum radiation frequency ω_m. From the derivation (see (1.8)) it is seen that the equation of the particle oscillation in the channel is of the form

$$\ddot{x}+\bar{\omega}^2x=0, \tag{7.1}$$

where $\bar{\omega}^2=2V_0/m\gamma$, and $\bar{\omega}$ is the particle oscillation frequency in the laboratory frame. Equation (7.1) describes the natural oscillations of the particle.

The ondulator idea is due to V. L. Ginzburg.[31] The Ginzburg ondulator equation is[28]

$$\frac{d}{dt}\left(\frac{mv}{(1-v^2/c^2)^{1/2}}\right)=eE_0\cos\omega_0t, \tag{7.2}$$

where $\mathbf{E}=\mathbf{E}_0\cos\omega_0t$ is the external field. Equation (7.2) describes forced oscillations.

The difference between the solution (7.2) and the solu-

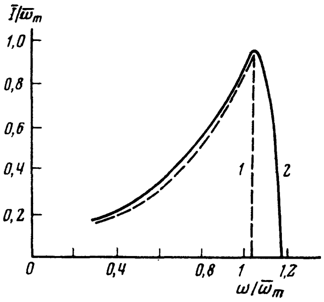

FIG. 3. Positron emission spectrum in silicon: 1—harmonic case; 2—anharmonic case.

tion (7.1) lies in the fact that in our case $\bar{\omega}$ depends on γ (i.e., on the energy), while in the ondulator the oscillation frequency is specified beforehand and is equal to ω_0. In the ondulator, the external field drags the particles, and in our case, owing to the relativistic increase of the particle mass with increasing energy of the particle, $\bar{\omega}$ decreases. This physical difference leads in final analysis to $\omega_m \propto \gamma^{3/2}$ in our case and $\omega_m \propto \gamma^2$ in the ondulator, i.e., the ondulator is a more effective frequency converter (and this is its principal function).

However, the minimal wavelengths in modern ondulators are ~10 Å, whereas in our case ~10^{-3} Å is attainable, i.e., the radiation produced by channeling is much harder than in the ondulator. The difference between these two types of radiation manifests itself most strongly in the quantum analysis of the radiation due to channeling. The channeling radiation, generally speaking, is a pure quantum effect (a spontaneous transition between levels). Indeed, according to formula (1.1), when the particle energy decreases and its velocity is nonrelativistic, the number of levels drops to one. In this case, naturally, the radiation considered by us vanishes.

To observe in experiment the channeling-radiation effect, the γ-photon spectrometer must be placed strictly behind an oriented target and tuned to the emission frequency $\omega \sim \omega_m$. It is also possible to use the polarization properties, as well as install a special monchromator that separates the required line.

8. EFFECT OF ANHARMONICITY OF THE POTENTIAL ON THE FREQUENCY SPECTRUM OF THE RADIATION

Let us estimate the influence of the anharmonicity in planar channeling of positrons. In the case of an anharmonic potential, the equation of motion of the particles in the channel is

$$\frac{d^2x}{dt^2}+\frac{2V_0}{m}\left(1-\frac{v_z^2}{c^2}\right)^{1/2}x(1+\alpha x^2)=0. \qquad (8.1)$$

Here α is the anharmonicity parameter; usually $\alpha x^2 \ll 1$.[21] The first-order approximation of the solution of this equation is

$$x(t)=x_m\sin[\bar{\omega}(1+{}^3/_8\alpha x_m^2)t],$$
$$\bar{\omega}^2=\frac{2V_0}{m}(1-\beta_z^2)^{1/2}. \qquad (8.2)$$

With the aid of (8.2) we easily estimate the influence of the anharmonicity on the spectrum–see Fig. 3. This figure shows the spectrum for positrons with $E=1$ GeV in silicon in the {110} channel. The quantities I and $\bar{\omega}_m$ are given by

$$\bar{I}=\frac{e^2}{3c^3}\left[\frac{2V_0}{m}\right]^2\gamma^2l^2,\quad \bar{\omega}_m=\left(\frac{2V_0}{m}\right)^{1/2}\left(1+\frac{v_z}{c}\right)\gamma^{3/2},$$

where l is the half-width of the channel. In this case $\bar{I}=0.03$ W and $\bar{\omega}_m=1.425\times10^{21}$ sec^{-1}. It is seen that the anharmonicity leads to a small broadening of the spectrum.

CONCLUSION

It is seen from the foregoing analysis that an experimental observation of the effect entails no substantial difficulties. At the same time, if the calculated emission parameters are experimentally confirmed, then this uncovers in principle new possibilities in nuclear physics, for in this case the experimenters are provided with a tool that is capable of acting much more effectively on the nuclear transitions than the presently known types of radiation.

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1) At the same time, in the case of planar channeling of positrons, the approximation of a parabolic well near the midpoint of two planes is quite correct. The intensity is calculated here in the same manner as for electrons. The influence of the anharmonicity can be taken into account separately (see below).

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Phase diagram of a ferromagnetic plate in an external magnetic field

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The phase diagram, in coordinates H and l, is investigated for a ferromagnetic plate located in an external magnetic field parallel to the surface of the plate. It is shown that in the high-anisotropy case, $\beta>4\pi$, the phase transition from the uniform state to a domain state on change of the plate thickness occurs as a second-order phase transition when $H>(\beta-4\pi)M_0$, and as a first-order transition when $H<(\beta-4\pi)M_0$. The phase diagram of a plate with large anisotropy, $\beta>4\pi$, is substantially more complicated than in the low-anisotropy case investigated earlier, and it has a number of peculiarities. In particular, an equilibrium domain structure exists for an arbitrarily small plate thickness.

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INTRODUCTION

A quantitative theory of the domain structure of ferromagnets was first developed by Landau and Lifshitz[1] (see also[2]). The domain structure of a ferromagnetic plate was investigated in the case in which the thickness of the plate greatly exceeds the thickness of the domain boundary. A criterion was also formulated for the single-domain state of a ferromagnet: a specimen of ferromagnetic material should be in the uniform state if $L\ll\alpha^{1/2}$, where L is a characteristic dimension of the specimen and α is an exchange constant. Later, domain structures of ferromagnets were extensively studied experimentally; this included intensive study of the properties of thin ferromagnetic films (those with parameters close to the single-domain criterion). For films with the axis of easiest magnetization perpendicular to the surface, the so-called stripe domain structure was discovered experimentally (see, for example, [3]), and also a transition from the uniform state of a film to a state with a stripe domain structure upon change of the plate thickness l (or of the external magnetic field H applied in the plane of the film). The experimental discovery of the transition from the uniform state of a film to a domain state led to a number of theoretical studies.[4,5] It was shown[4,5] that in ferromagnetic films with small anisotropy ($\beta<4\pi$, where β is an anisotropy constant), the state with a stripe domain structure occurs in consequence of an instability of the uniform state of the film, in which the magnetization lies in the plane of the film, with respect to small inhomogenous perturbations. A phase diagram, in coordinates H and l, was given for a ferromagnetic film with small anisotropy, $\beta<4\pi$.[5] The stability regions of the uniform phase $\Phi_\parallel$ and the nonuniform (domain) phase Φ_D were separated by a line of second-order phase transitions $H=H_c(l)$.

The form of the domain structure in the vicinity of the curve $H_c(l)$ has been investigated.[6] The variation of the parameters of the magnetization distribution (amplitude, period, etc.) was found, and also an analytical expression for $H_c(l)$ was obtained for large plate thicknesses:

$$\frac{H_c(l)}{M_0}=\beta-\frac{4\pi^{3/2}}{l}\left(\frac{\alpha\beta}{\beta+4\pi}\right)^{1/2}. \quad (1)$$

