Interaction of J/ψ particles with nucleons and nuclei in the soliton model

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It is shown that, within the framework of the soliton model of J/ψ particles, the $(J/\psi)N$ interaction can be regarded as the scattering of the nucleon by an external classical field. The phase shifts are then real and the total and elastic cross sections for this interaction are equal (if diffractional dissociation processes are neglected). The model is in qualitative agreement with existing experimental data on the photoproduction of J/ψ particles on nucleons and nuclei. The coherent and noncoherent cross sections for the photoproduction of J/ψ particles on nuclei are calculated, and it is shown that measurement of these cross sections as functions of the number of nucleons in the nucleus, the momentum transfer to the nucleus, and the photon energy can be used to determine the real and imaginary parts of the amplitude for $(J/\psi)N$ scattering and the slope of the differential cross section.

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1. INTRODUCTION

The soliton model for particles such as the J/ψ bosons was proposed by one of us in[1,2]. The essence of this model is the proposal that particles of this kind can be described by a nonspreading wave packet of the classical boson field, i.e., it is the soliton solution of a certain relativistically invariant nonlinear equation. A one-dimensional equation, similar to the Ginzburg-Landau equation for a scalar field was discussed in[2] as a heuristic example. Kudryavtsev has shown[3] that this equation has localized soliton-like solutions of the quasistatic type (oscillating slowly and only slightly attenuated in time). Analogous solutions were found by Bogolyubskiĭ and Makhan'ko[4] for the three-dimensional equation (see the note in[5] in this connection).

In the case of weak nonlinearity, the field amplitude and energy for the solutions found in[3] turn out to be large. This leads us to expect the existence of relatively heavy particles, the gross properties of which can be described by the classical solution or a quasiclassical approximation to it.

An approximate (quasiclassical) quantum solution in the form of a Glauber coherent state was used in[2] to consider the decay of J/ψ particles on the basis of the wave packet (soliton) model of a classical field. This state is a boson condensate in which the mean number of light bosons ("pions") is $\bar{n}\gg 1$ and the pion number is described by the Poisson distribution around this mean. The example of a scalar field mentioned above was used in[2] to demonstrate that $\bar{n}\approx N/m_0$, where M is the mass of a particle such as the J/ψ boson and m_0 is the mass of a light boson which, in this example, plays the role of the pion. In this model, the decay of the J/ψ particle into a small ($n\ll\bar{n}$) number of pions is suppressed exponentially in $\bar{n}$ and, consequently, in the mass M of the J/ψ particle. The distinctive feature of ψ particles (J/ψ and ψ') is thus the fact that the purely pion decay channels are suppressed, and this restriction is organically related to their large mass. In other words, the stability of the ψ particles is explained in the above model precisely by the fact that the field corresponding to them is quasiclassical. In fact, when $\bar{n}\gg 1$, the state vector is an eigenvector of the field operator, and the result of this is that the amplitude-phase uncertainty principle is minimized. This immediately leads to the Poisson distribution for the number of pions, and hence to the suppression of the above factor which is exponential in $\bar{n}$ and suppresses the decay.

It is shown below that these properties also lead to the "conservation" of ψ particles when they collide with hadrons so that, in the first approximation, the ψN interaction can be looked upon simply as the scattering of a nucleon in a localized external classical field corresponding to the soliton-like solution. Hence, it follows that the phase shifts are real (when diffractional dissociation is neglected), or the total cross section for the ψN interaction is equal to the elastic cross section. This prediction will have to be verified by experiments on, say, the photoproduction of ψ particles on nuclei.

In accordance with the foregoing, we shall begin with a detailed analysis of the reasons for the suppression of inelastic interactions involving ψ particles in the soliton model (Sec. 2). In Sec. 3, we shall discuss the parameters of the ψ, N interaction and, in Secs. 4 and 5, we shall calculate the cross sections for the coherent and noncoherent photoproduction of ψ particles on nuclei and will discuss possible experiments that may verify the predictions of the theory.

2. SUPPRESSION OF INELASTIC INTERACTIONS OF ψ PARTICLES

By inelastic processes, we shall understand reactions involving the creation and annihilation of ψ particles, i.e., processes in which the ψ particle appears only in the ground state, initial or final. Reactions of the form

$$\psi+X\to\psi+Y$$

where $X\neq Y$, will be called diffractional inelastic pro-

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cesses. The latter will not be discussed in detail in the present paper (it will be assumed that their cross sections form a small fraction of the total interaction cross section, just as in the case of the interaction of ordinary hadrons (see, for example, [6]).

Following[2], we assume, for simplicity, that the state $|\psi\rangle$ of the ψ particle is a coherent state of a neutral spinless boson field. If the operator for this field in the Schroedinger representation is

$$\hat{\varphi}(\mathbf{r})=\hat{\varphi}^{(+)}(\mathbf{r})+\hat{\varphi}^{(-)}(\mathbf{r}),$$

where $\hat{\varphi}^{(+)}$ and $\hat{\varphi}^{(-)}$ are the positive and negative frequency parts, respectively, then $|\psi\rangle$ is the eigenvector of the operator $\hat{\varphi}^{(+)}$:

$$\hat{\varphi}^{(+)}(\hat{\mathbf{r}})|\varphi\rangle={}^1/_2 g(\mathbf{r})|\psi\rangle. \quad (1)$$

The eigenvalue $g(\mathbf{r})$ should, in this case, be the soliton or soliton-like solution of the nonlinear equation for the classical field. An explicit expression for the vector $|\psi\rangle$ in terms of the classical solution g, the field operators, and the vacuum state $|0\rangle$, which is reduced to zero by the operator $\hat{\varphi}^{(+)}$, is given in[2] [Eqs. (1)–(3)]. The amplitude of the classical field g is proportional to $(\bar{n})^{1/2}$ [see[2], Eq. (12)]. If we use the normalization $\langle\psi|\psi\rangle=1$, we have

$$\langle 0|\psi\rangle=e^{-\bar{n}/2}. \quad (2)$$

For the scalar products of the coherent states $|\psi_1\rangle$ and $|\psi_2\rangle$, on the other hand, which are described by the functions g_1 and g_2, we have

$$|\langle\psi_2|\psi_1\rangle|^2=\exp\left[-\frac{1}{2}\int\omega|\tilde{g}_1-\tilde{g}_2|^2\frac{d^3\mathbf{k}}{(2\pi)^3}\right]. \quad (3)$$

In this expression, $\omega=(\mathbf{k}^2+m_0^2)^{1/2}$ and $\tilde{g}(\mathbf{k})$ is the Fourier component of the function $g(\mathbf{r})$. The expansion of $|\psi\rangle$ into a series over the states $|\mathbf{k}_1, \ldots, \mathbf{k}_n\rangle$ with a given number n of particles with momenta $\mathbf{k}_1, \ldots, \mathbf{k}_n$, i.e.,

$$|\psi\rangle=\sum_{\mathbf{k},n}\prod_i(2\omega_i)^{-1/2}g_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)|\mathbf{k}_1,\ldots,\mathbf{k}_n\rangle, \quad (4)$$

is such that

$$\int|g_n|^2\prod_i\frac{d^3\mathbf{k}}{(2\pi)^3\cdot 2\omega_i}=N_n^2=\frac{\bar{n}^n}{n!}e^{-\bar{n}}, \quad (5)$$

i.e., we have the Poisson distribution for the number of particles.

It is convenient to write

$$g_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)=N_n\Phi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n), \quad (6)$$

where Φ_n are normalized by the condition[1)]

$$\int|\Phi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)|^2\prod_i\frac{d^3\mathbf{k}_i}{(2\pi)^3\cdot 2\omega_i}=1. \quad (7)$$

These functions can be expressed in terms of $\tilde{g}(\mathbf{k})$ as follows:

$$\Phi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)=\prod_i(2\pi)^{3/2}(2\omega_i)^{1/2}\Phi(\mathbf{k}_i)=\prod_i\omega_i(\bar{n})^{-1/2}\tilde{g}(\mathbf{k}_i), \quad (8)$$

where

$$\int|\Phi(\mathbf{k}_i)|^2d^3\mathbf{k}_i=\int\frac{\omega_i}{2\bar{n}}|\tilde{g}(\mathbf{k}_i)|^2\frac{d^3\mathbf{k}_i}{(2\pi)^3}=1. \quad (9)$$

For ψ particles moving with velocity v along the z axis, the normalized single-particle density has the form

$$|\Phi_v(\mathbf{k})|^2d^3\mathbf{k}=|\Phi(\mathbf{k}_\perp,(1-v^2)^{1/2}k_z)|^2d^2\mathbf{k}_\perp d((1-v^2)^{1/2}k_z). \quad (10)$$

We emphasize that the difference between the coherent state and the usual multiboson system lies not only in the undetermined number of bosons but also in the structure of the wave function Φ_n. The formula given by (8) indicates that all the particles are in the same state with wave function $\tilde{g}(\mathbf{k})$. The multiboson wave function, on the other hand (for the noncoherent state) should, in general, contain the symmetrized sum of products of different states

$$\sum_P\prod_{j=1}^n\omega_i\tilde{g}_j(\mathbf{k}_i).$$

If we have an n-boson state of the usual form

$$|\alpha_n\rangle=\sum_{\mathbf{k}_i}\alpha_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)|\mathbf{k}_1,\ldots,\mathbf{k}_n\rangle,$$

the probability of the $|\psi\rangle\to|\alpha_n\rangle$ transition is proportional to

$$\int|\langle\alpha_n|\psi\rangle|^2d\Gamma_n, \quad (11)$$

where d_n is the phase volume of the n-boson state $|\alpha\rangle$ and

$$\langle\alpha_n|\psi\rangle=\frac{\bar{n}^{n/2}e^{-\bar{n}/2}}{2^n(2\pi)^{3n/2}}\int\alpha_n^*\Phi_n\prod_{i=1}^n\frac{d^3\mathbf{k}_i}{\omega_i}. \quad (12)$$

The equations given by (1)–(3) and (12) express the "conservation" of the state $|\psi\rangle$, and this leads, in particular, to the fact that the elastic ψN interaction predominates over inelastic processes (which involve the disintegration of ψ). It is precisely this property that enables us to consider the ψN interaction as the scattering of the nucleon N by the "external" (i.e., classical) field. In fact, the relation given by (1) ensures that any operator containing the normal product of the field operators $\hat{\varphi}$, acting on the state $|\psi\rangle$, will generate this state. The element of the S matrix corresponding to the inelastic process $|\psi N\rangle\to|\alpha_n N'\rangle$ will always, therefore, contain the scalar product $\langle\alpha_n|\psi\rangle$, and the cross section will be proportional to the quantity given by (12). For $\bar{n}\gg 1$ and $n\ll\bar{n}$, the cross section for this inelastic process will therefore be suppressed by the presence of the Poisson factor N_n^2 in $|\langle\alpha_n|\psi\rangle|^2$.

It would seem at first sight that the suppression of inelastic processes at sufficiently high energies (ensuring the necessary multiplicity of final particles) will not occur in this model for $n\approx\bar{n}$. Closer analysis will readily show, however, that this is not so for the following reasons. If the invariant mass of the resulting system of pions is close to the mass of the ψ particle, $M \approx \bar{n} m_0$, the inelastic cross section for $n \approx \bar{n}$ is small because the phase volume of the final state is also small, i.e., for the same reason for which the pion decay $\psi \to \bar{n}\pi$ is suppressed (see[2]). Insofar as large effective masses are concerned, we have here two suppression factors: one is due to the unavoidable fact that the overlap integral for the wave functions Φ_n and α_n is small, and the other is the necessarily large longitudinal transfer of momentum. In point of fact, Eq. (1) ensures that the element of the S matrix corresponding to the inelastic process will, in addition to $\langle \alpha_n \mid \psi \rangle$, also be proportional to the integral

$$F(\mathbf{q}) = \int e^{i\mathbf{q}\mathbf{r}} g'^{*}(\mathbf{r}) g(\mathbf{r}) d^3\mathbf{r}, \tag{13}$$

which appears because of the presence of the eigenvalue $g(\mathbf{r})$ of the operator $\hat{\varphi}^{(+)}(\mathbf{r})$ in (1). In this expression, q is the transferred momentum and the function $g'(\mathbf{r})$ describes the final state.[2)] When a mass $M' > M$ is created, the minimum transferred momentum (in the rest system of the ψ) is

$$q_{min} = M' - M.$$

To avoid a small phase volume, the mass M' must be appreciably greater than M, i.e., we must have $q_{min} \gtrsim M$. When this condition is satisfied, the integral (13) is very small because the spatial size of the ψ particles in our model [i.e., the size of the region where $g(r)$ is comparable with its mean] is determined by the mass m_0 which is necessarily much less than M ($m_0/M \approx 1/\bar{n} \ll 1$).

Noncoherent processes involving the disintegration of the ψ, which do not contain the formfactor, are suppressed for $n \gtrsim \bar{n}$ because the function $\tilde{g}(\mathbf{k})$ and, consequently, the single-particle density given by (10) in any reference frame, have in this model no singularities in the scaling variable

$$x = k_z \sqrt{1 - v^2}/M.$$

This is equivalent to saying that the effective multiplicity of "partons" does not increase indefinitely in any reference frame (including the system with infinite momentum) for any x ($0 \leqslant |x| \leqslant 1$), and this is in contrast to ordinary hadrons for which the distribution of soft partons has the form dx/x (see, for example, [7]). In the impulse approximation, the total cross section for inelastic processes (σ_{in}) involving the interaction of ψ particles with hadrons or leptons is given by

$$\sigma_{in} \sim \sum_n n N_n^2 \delta_n, \tag{14}$$

where N_n^2 are the normalizing factors given by (5) and

$$\delta_n = \int \delta(1 - |x_1| - \ldots - |x_n|) \prod_i \overline{\Phi^2(x_i)}\, dx_i. \tag{15}$$

These expressions are obtained from (11) and (12) by replacing the final state $|\alpha_n\rangle$ by the product of plane waves $\alpha_n(\mathbf{k}_1, \ldots, \mathbf{k}_n) = \delta(\mathbf{k}_1 - \mathbf{k}_1') \ldots \delta(\mathbf{k}_n - \mathbf{k}_n')$ and by assuming that the transverse momenta are bounded in the system in which the ψ particle moves at high velocity $v \sim c$ (as for ordinary hadrons). We have, therefore, neglected in the δ function, which corresponds to energy conservation, the small transverse momenta (which remain constant as E/M increases) in comparison with the longitudinal momenta (which increase like E/M). We then have

$$\overline{\Phi^2(x_i)} = \int \Phi^2(x_i, \mathbf{k}_i^{\perp}) d^2\mathbf{k}_i^{\perp}.$$

We also note that (15) does not take into account momentum conservation because, in this case, the vector conservation laws impose a very weak restriction on the total cross section for high multiplicity [it leads to corrections $\sim 1/n$; see also the remarks follows (7)].

Assuming that $\overline{\Phi^2(x_i)} = \text{const}$ for $x \ll 1$ [for $n \gg 1$ and sufficiently smooth $\overline{\Phi^2(x_i)}$, the integral given by (15) is determined precisely by this region of small x_i], we obtain

$$\sum_n n N_n^2 \delta_n = e^{-\bar{n}} \bar{n} I_0(2(\bar{n})^{1/2}) \approx \bar{n}^{3/4} \exp\{-\bar{n} + 2(\bar{n})^{1/2}\}. \tag{16}$$

More realistic estimates could be based on the boundedness of the Fourier components of $\tilde{g}(\mathbf{k})$ as $k \to 0$ and on (8), which gives $\overline{\Phi^2(x)} = 2x$. We then have

$$\sum_n n N_n^2 \delta_n \approx (\bar{n})^{1/3} \exp\{-\bar{n} + 3(\bar{n}/2)^{1/3}\}. \tag{17}$$

It is clear from (16) and (17) that the total cross sections for inelastic processes involving ψ-particle interactions are suppressed exponentially in $\bar{n}$ in the soliton model.

We note, for comparison with inelastic interactions of ordinary hadrons, that, if we substitute $\overline{\Phi^2} \sim 1/x$ (which is used in parton models), then there is no exponential suppression in $\bar{n}$. Instead of the quantities $2(\bar{n})^{1/2}$ and $3(\bar{n}/2)^{1/3}$, the exponentials in (16) and (17) contain simply $\bar{n}$ with the positive sign.

Of course, the foregoing ideas on the expected suppression of inelastic cross sections in the soliton model of ψ particles should, in principle, be augmented by a discussion of the validity of the quasiclassical approximation, i.e., the corrections which distinguish the exact solution of the quantum field equations from that based on the classical soliton of the Glauber coherent state. We are unable, at present, to provide a quantitative solution to this problem.

3. EIKONAL MODEL OF THE ψN INTERACTION

At high energies ($pa \gg 1$, where a is the interaction range), we can use the eikonal approximation for the small-angle ψN scattering amplitude:

$$f(\mathbf{q}) = \frac{ip}{2\pi} \int e^{i\mathbf{q}\mathbf{b}} (1 - e^{2i\delta(\mathbf{b})}) d^2\mathbf{b}, \tag{18}$$

where $\mathbf{b}$ is the impact parameter and $\mathbf{q}$ is the momentum transfer. Since the distinguishing feature of the soliton model of ψ particles is that the inelasticity of the ψN interaction processes is small, it follows that the imaginary part of the phase shift is small in comparison with the real part:

$$\delta(\mathbf{b})=\delta_2(\mathbf{b})+i\delta_1(\mathbf{b}), \quad \delta_1(\mathbf{b})\ll\delta_2(\mathbf{b}). \tag{19}$$

We then have

$$\sigma_{el}\approx\sigma_t, \quad \sigma_{in}\ll\sigma_t, \tag{20}$$

where σ_t is the total ψN-interaction cross section.

We must now consider how the predictions of the soliton model differ from the usual theoretical schemes for the ψN interaction. The ψN cross section is usually determined from photoproduction data, assuming that the vector dominance model is valid:

$$\frac{d\sigma}{dt}(\gamma N\to\psi N)=\frac{3\Gamma(\psi\to e^+e^-)}{\alpha M}\frac{d\sigma}{dt}(\psi N\to\psi N) \tag{21}$$

and that the amplitude for ψN scattering is purely imaginary:

$$\frac{d\sigma}{dt}(\psi N\to\psi N;t=0)=\frac{\sigma_t^2}{16\pi}. \tag{22}$$

Using (21) and (22), we find that $\sigma_t\approx 1$ mb and $\sigma_{el}\approx 10^{-2}\sigma_t$. In the charmed quark model, one also expects that the ratio σ_{el}/σ_t will also be small for the ψ particle.[8] Thus, for example, for the φ meson, which is similar in the quark model to the ψ boson, this ratio is ~ 0.1.

The expressions given by (19) and (20) are thus quite typical of the soliton model, and are in clear conflict with the predictions that follow from (21) and (22). We note that the violation of (21) can be expected for a relatively extensive class of models (because of the inclusion of the dependence of the $\gamma-\psi$ transition constant on the photon mass), including the charmed-quark model. However, the latter model necessarily predicts that the inelastic cross section for the disintegration of the ψ particle into charmed D mesons should be large. This means, in particular, that the cross section for the photoproduction of D mesons should be greater by a factor of approximately 30 than the cross section for the photoproduction of ψ mesons.[9,10] However, the photoproduction of D mesons has not been established experimentally, whereas the photoproduction of ψ mesons has been reliably observed.[11] Moreover, experiment reveals a small inelasticity in the photoproduction of ψ mesons.[12] As noted in[13], existing experimental data on the photoproduction of ψ mesons suggest that $\sigma_{el}\sim\sigma_t$ for the ψN interaction. These experimental results are naturally explained within the framework of the soliton model, whereas the charmed quark model requires additional assumptions even for a qualtative explanation of these facts.[13,14] One of the variants of this model, using the assumption of the dominance of nonreggeized axial exchange,[13] is in conflict with existing experimental data on the presence of the coherent peak in the photoproduction of ψ mesons on beryllium nuclei.[15]

Let us consider possible violations of (21) and (22) in the soliton model. As a first approximation, let us adopt the Gaussian parametrization of the phases

$$\delta(b)=\delta(0)e^{-b^2/a^2}, \tag{23}$$

and take the interaction length to be the value obtained from data on the photoproduction of ψ on nucleons:

$$a\approx 0.4-0.6\ \text{F}. \tag{24}$$

In the soliton model, the amplitude $f(0)$ may be purely imaginary, and (22) can be satisfied only for large phase shifts $\delta(0)\gg 1$. We then have (see[16])

$$\sigma_t=2\pi a^2\ln(2\delta(0)e^C), \tag{25}$$

where $C=0.58$ is the Euler constant. When $a=0.4-0.6\ F$, the cross section given by (25) is of the same order or even greater than the cross sections for the πN and NN interactions. Existing experimental data on the photoproduction of ψ mesons on nuclei appear to exclude such high cross sections for the ψN interaction[12] and suggest that σ_t does not exceed a few millibarns or, at any rate, is small in comparison with the cross section for the ρN interaction.

For such small cross sections, $\delta(0)\ll 1$ and we can confine our attention to only one or two leading terms in the expansion for the scattering amplitude

$$f(q)=\frac{ipa^2}{2}\sum_{n=1}^{\infty}[2i\delta(0)]^n\frac{1}{n!n}\exp\left\{-\frac{q^2a^2}{4n}\right\} \tag{26}$$

This will enable us to include the imaginary part of $f(0)$. When $\sigma_t\approx 1$ mb, and the interaction range is given by (24), we have $\delta^2(0)=0.1-0.05$ and

$$\alpha=\frac{\operatorname{Re}f(0)}{\operatorname{Im}f(0)}=\frac{2}{\delta(0)}=7-10, \tag{27}$$

i.e., (22) is violated by a factor of 50–100. The vector dominance relation (21) is then also highly violated: the ratio of the left- and right-hand parts of (21) for $\sigma_t\sim 1$ mb and $\delta^2(0)=0.1-0.05$ is of the order of 100. The considerable violation of (21) is a natural consequence of the soliton model of ψ particles. For example, the photoproduction of the ψ can be looked upon as consisting of two stages. Initially, the γ-ray photon undergoes a transition to the coherent state of the φ field with zero mass, and this then goes over into the final coherent state of mass M as a result of the interaction with the nucleon field. The photoproduction amplitude then contains the formfactor (13) in which $q_{\min}\approx M$. The square of this formfactor, present on the right-hand side of (21), leads to an important increase in the ratio of the left- and right-hand sides.

Thus, both (21) and (22) may be strongly violated in the soliton model. To determine the ψN interaction parameters, we must therefore use processes involving the production of ψ particles on nuclei.

4. COHERENT PHOTOPRODUCTION OF ψ ON NUCLEI

The quantum numbers of the ψ particles allow them to be created in a coherent fashion on nuclear targets when the latter are exposed to high-energy photons. The dependence of the cross sections for coherent photoproduction on the atomic number can, in principle, be used to determine the two parameters Im$f(0)$, which determines the absorption coefficient, and Re$f(0)$, which determines the refractive index (see, for example, [17]) for the ψN interaction. As noted in Sec. 3, the fact that the ψN interaction cross section is small indicates the validity of perturbation theory in $\delta(b)$ [Eq. (26)]. The real part of the scattering amplitude $f(0)$ is then large in comparison with the imaginary part. Let us consider in greater detail how data on coherent photoproduction cross sections can be used to distinguish between the cases of purely real and purely imaginary amplitudes.

When $A \gg 1$, so that the optical approximation is valid, the amplitude for the coherent photoproduction of ψ on a nucleus can be written in the form

$$F^{coh}(\mathbf{q}) = Af_{12}(\mathbf{q}) \int d^2\mathbf{b}\, dz \exp\{iq_L z\} \exp\{i\mathbf{q}_\perp \mathbf{b}\} \rho(\mathbf{b}, z) \times \exp\left\{-\frac{2\pi}{ip} f(0) A \int_z^\infty \rho(\mathbf{b}, z')\, dz'\right\}, \quad (28)$$

where $q_L = M^2/2p$ and f_{12} is the amplitude for the $\gamma N \to \psi N$ reaction. Let us take into account the first two terms of the expansion of $F^{coh}(\mathbf{q})$ in terms of the parameter

$$\eta = -\frac{4\pi}{ip} f(0) \rho_0 R,$$

where $\rho_0 = A/V$ is the nuclear density, which is equal to 0.17 nucleons/F^3, $V = \frac{4}{3}\pi R^3$, and $R = 1.12A^{1/3}F$. Thus, we shall use

$$F^{coh}(\mathbf{q}) = Af_{12}(\mathbf{q}) \{G_0(\mathbf{q}) + \eta G_1(\mathbf{q}) + \eta^2 G_2(\mathbf{q})\}, \quad (29)$$

where

$$G_m = \int d^2\mathbf{b}\, dz \exp\{iq_L z\} \exp\{i\mathbf{q}_\perp \mathbf{b}\} \frac{\rho(\mathbf{b}, z)}{m!} \left[\frac{A}{2\rho_0 R} \int_z^\infty \rho(\mathbf{b}, z')\, dz'\right]^m. \quad (30)$$

At very high energies, when $\xi = q_L R$ is small, we can neglect longitudinal momentum transfer in (29) and (30). The formfactors G_m then become real, and it follows immediately that the cross section for the coherent photoproduction is linear in $f(0)$ for a purely imaginary amplitude and quadratic in $f(0)$ for a purely real amplitude. For small η (which probably occurs in the ψN interaction), the determination of the phase of $f(0)$ will, in general, require coherent production data for $\xi \sim 1$.

For approximate calculations, we may suppose that the nucleus is a sphere of uniform density. In the first order in $f(0)$, we obtain the following expressions for the zero-angle coherent photoproduction cross section and the cross sections integrated over the coherent peak, respectively:

$$\left.\frac{d\sigma^{coh}}{d\Omega}\right|_{\theta=0} = |Af_{12}(0)|^2 \{f_0(\xi) + \eta_1 f_1(\xi) + \eta_2 f_2(\xi)\}, \quad (31)$$

$$\sigma^{coh} = \frac{d\sigma_{12}(\theta=0)}{dt} 8\pi^2 R^4 \rho_0^2 \{g_0(\xi) + \eta_1 g_1(\xi) + \eta_2 g_2(\xi)\}, \quad (32)$$

where $\eta_1 = \sigma_t \rho_0 R$ and $\eta_2 = \alpha\sigma_t \rho_0 R$. The functions f_i and g_i are given by

$$f_0(\xi) = 9\xi^{-6}(\sin\xi - \xi\cos\xi)^2,$$
$$f_1(\xi) = 18\xi^{-7}(\sin\xi - \xi\cos\xi)[1 - \xi\sin\xi + (\tfrac{1}{2}\xi^2 - 1)\cos\xi], \quad (33)$$
$$f_2(\xi) = 9\xi^{-7}(\sin\xi - \xi\cos\xi)[(3 - \xi^2)\sin\xi - 3\xi\cos\xi];$$
$$g_0(\xi) = \xi^{-2}[1 - \tfrac{1}{2}\xi^{-2}(\cos 2\xi + 2\xi\sin 2\xi - 1)],$$
$$g_1(\xi) = -\xi^{-2}\{\tfrac{2}{3} - \tfrac{1}{2}\xi^{-3}[2\xi\cos 2\xi + (2\xi^2 - 1)\sin 2\xi]\}, \quad (34)$$
$$g_2(\xi) = \xi^{-5}[1 + \xi^2 - 2\xi\sin 2\xi + (\xi^2 - 1)\cos 2\xi].$$

The contribution of terms containing the real part of the amplitude $f(0)$, which are proportional to f_2 and g_2, may be appreciable for $\xi \geq 1$. The real and imaginary amplitudes can then be distinguished on the basis of the different functional dependence of $f_1 g_1$ and $f_2 g_2$ on ξ. For $\xi \to 0$,

$$f_0 \to 1, \quad f_1 \to -3/4, \quad f_2 \to \xi/5;$$
$$g_0 \to 1, \quad g_1 \to -4/5, \quad g_2 \to 2\xi/9.$$

When $\xi \gg 1$,

$$f_0 \sim f_1 \sim f_2 \sim 1/\xi^4; \quad g_0 \sim g_1 \sim 1/\xi^2, \quad g_2 \sim 1/\xi^3.$$

It follows from our discussion of the properties of $F^{coh}(\mathbf{q})$ that the vanishing of f_2 and g_2 for $\xi = 0$ is a general property which is independent of the nuclear model used here. Using (29) for the coherent photoproduction amplitude, we can easily show that $g_1/g_0 \sim 1$ and $g_2/g_0 \lesssim 1/\xi$ for $\xi \gg 1$. This is again independent of the particular model of nuclear structure.

Let us consider the integrated cross section σ^{coh}. For a purely imaginary amplitude $f(0)$, the rescattering correction is a relatively slowly-varying function of ξ because the ratio g_1/g_0 is a very slowly-varying function of ξ. For example, when ξ changes from 1/3 to 3, this ratio changes from -0.8 to -0.6. At the same time, for a purely real amplitude, the rescattering correction is small both for $\xi \ll 1$ and $\xi \gg 1$, and the ratio g_2/g_0 has a maximum for $\xi \sim 1$. These properties of the rescattering correction are naturally conserved even in a more realistic model of nuclear density (for example, the Woods-Saxon model).

The parameter ξ is a function of both the atomic number A and of the energy. Figure 1 shows the ratio

$$r^{coh} = \left|\frac{\sigma^{coh}(0) - \sigma^{coh}(\eta)}{\sigma^{coh}(0)}\right|, \quad (35)$$

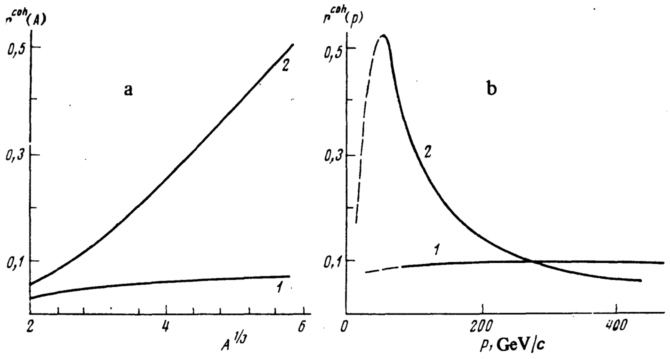

FIG. 1. a) Ratio $r^{coh}(A)$ as a function of the mass number A for $p = 54$ GeV/c. b) Ratio r^{coh} as a function of the photon momentum in the photoproduction of ψ on lead. Curves 1 correspond to $\sigma_t = 1$ mb, $\alpha = 0$; curves 2—$\sigma_t = 1$ mb, $\alpha = 6$.

as a function of the atomic number and photon energy, where $\sigma^{coh}(0)$ is the coherent photoproduction cross section when $f(0)=0$. Curves 1 correspond to a purely imaginary amplitude ($\eta_2=0$, $\sigma_t=1$ mb) and curves 2 to $\sigma_t=1$ mb, $\alpha=\mathrm{Re}f(0)/\mathrm{Im}f(0)=6$. Of course, the use of the energy dependence of r^{coh} for the determination of $f(0)$ is valid only when $f(0)$ is a slowly-varying function of energy (as compared with the longitudinal component of the transferred momentum q_L which decreases linearly with increasing γ-ray energy).

We also note that the amplitude for coherent photoproduction on heavy nuclei is a function of ξ even at photon energies ~100 GeV. It follows that, when the ψN interaction parameters are determined from the A dependence of the cross sections for the coherent photoproduction of ψ mesons on nuclei, it is important to have data on beams of "labeled" photons so that the energy of the photon producing a given event is known. Unless this is so, integration over the photon spectrum may introduce substantial uncertainties.

5. NONCOHERENT PHOTOPRODUCTION OF ψ ON NUCLEI

In order to establish whether $\sigma_{el}\sim\sigma_t$ or $\sigma_{el}\ll\sigma_t$, we can use the characteristic structure of the angular distribution of the noncoherent photoproduction cross section for ψ on nuclei. The important point here is that the cross section for the noncoherent photoproduction on a nucleus outside the first diffraction peak is determined by the rescattering of ψ in the final state and is proportional to σ_{el}.

Consider the amplitude for the noncoherent photoproduction of ψ on a nucleus when the latter either remains in the ground state, or is excited, or breaks up, but without the production of a new particle. In this case, we can use the Glauber formalism,[18] generalized to particle production processes by a number of workers (see the review in[19]). The photoproduction amplitude will be written in the form

$$F_{fi}(\mathbf{q})=\frac{ip}{2\pi}\int d^2\mathbf{b}\, e^{i\mathbf{qb}}\langle f|\Gamma(\mathbf{b},\mathbf{r}_1,\ldots,\mathbf{r}_A)|i\rangle, \tag{36}$$

where

$$\Gamma(\mathbf{b},\mathbf{r}_1,\ldots,\mathbf{r}_A)=\sum_{l=1}^{A}\Gamma_{12}(\mathbf{b}-\mathbf{s}_l)\exp(iq_Lz_l)\prod_{j\neq l}[1-\theta(z_j-z_l)\Gamma(\mathbf{b}-\mathbf{s}_j)].$$

In these expressions, $\mathbf{s}_l$ is the projection of the nucleon coordinate $\mathbf{r}_l$ onto the plane perpendicular to the initial momentum $\mathbf{p}$ and Γ_{12} and Γ are the photoproduction and ψ, N scattering amplitudes in the impact-parameter representation

$$\Gamma_{12}(\mathbf{b})=\frac{1}{2\pi ip}\int d^2\mathbf{q}_\perp \exp(-i\mathbf{q}_\perp\mathbf{b})f_{12}(\mathbf{q}_\perp). \tag{37}$$

$\Gamma(\mathbf{b})$ is thus expressed in terms of the elastic ψN scattering amplitude $f(\mathbf{q})$. Finally, we can use the following parametrization for the amplitudes:

$$f_{12}(\mathbf{q}_\perp)=f_{12}(0)\exp(-b_\gamma\mathbf{q}_\perp^2/2),\quad f(\mathbf{q})=f(0)\exp(-b_\psi\mathbf{q}_\perp^2/2). \tag{38}$$

In the case of identical slopes $b_\gamma=b_\psi=b$, the differential noncoherent photoproduction cross section can be written in the form ($\mathbf{q}^2R^2\gg1$):

$$\frac{d\sigma^{n.c}}{d\Omega}=\sum_{f\neq i}|F_{fi}(\mathbf{q})|^2=|f_{12}(0)|^2\sum_{n=1}^{A}B_n\sigma_{el}^{n-1}e^{-b\mathbf{q}^2/n}, \tag{39}$$

where $B_1\to A$ for $\sigma_t\to0$, $q_L^2\ll q_\perp^2$. The coefficients B_n depend on the nuclear parameters and on σ_t. As a rule, they decrease relatively rapidly with increasing n so that, for $q^2\lesssim b^{-1}$, the main contribution is provided by the first term. As $\mathbf{q}^2$ increases, there is an increase in the relative magnitude of terms containing powers of σ_{el}. It is clear from the structure of (39) that the dependence of $d\sigma^{n,c}/3\Omega$ on $\mathbf{q}^2$ can, at least in principle, be used to determine σ_{el} and b when σ_t is known (for example, from experiments on noncoherent photoproduction).

When σ_t is small, so that the parameter $\eta_1=\sigma_t\rho_0R$ is also small, we can use the expansion of $\sigma^{n,c}$ in powers of η_1:

$$\frac{d\sigma^{n.c}}{d\Omega}=A|f_{12}(0)|^2\Big\{(1-\eta_1K_2)\exp(-b_\gamma\mathbf{q}^2)-\eta_1K_2\exp\{-(b_\gamma+b_\psi)\mathbf{q}^2/2\}$$
$$+\eta_1K_2\frac{\sigma_{el}}{\sigma_t}\frac{b_\psi}{b_\psi+b_\gamma}\exp\{-b_\psi b_\gamma\mathbf{q}^2/(b_\psi+b_\gamma)\}+O(\eta_1^2)\Big\}, \tag{40}$$

where

$$K_2=\frac{1}{2}\frac{A\rho_0}{R}\int T^2(\mathbf{b})\,d^2\mathbf{b},\quad T(\mathbf{b})=\int_{-\infty}^{\infty}\rho(\mathbf{b},z)\,dz.$$

In deriving (40) from (36), we have neglected the effect of correlations between nucleons in the nucleus. Estimates given in[20] do, in fact, indicate that this effect is small.

The slope of the curves corresponding to the third term in braces in (40) is proportional to σ_{el} and is always less than the slope of the curve corresponding to the first two terms for any ratio of b_ψ to b_γ. When $b_\gamma=b_\psi$, which occurs in the vector dominance model, this slope is smaller by a factor of 2 than the slopes corresponding to the first two terms.

Figure 2 shows graphs of

$$r^{n,c}(\mathbf{q}^2)=\frac{1}{A|f_{12}(0)|^2}\frac{d\sigma^{n.c}}{d\Omega} \tag{41}$$

for lead with $b_\gamma=b_\psi=4$ GeV^{-2}, $\sigma_t=1$ mb when σ_{el}

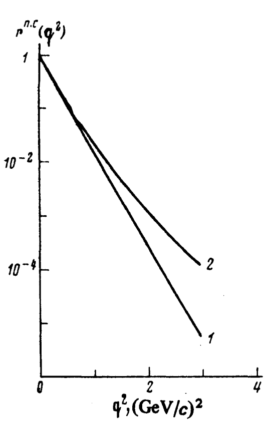

FIG. 2. Ratio $r^{n,c}(\mathbf{q}^2)$ as a function of $\mathbf{q}^2$ for the photoproduction of ψ on lead. Curve 1—$\sigma_t=1$ mb, $\sigma_{el}=0$; curve 2—$\sigma_{el}=\sigma_t=1$ mb.

$=0.01\sigma_t$ (curve 1) and $\sigma_{el}=\sigma_t$ (curve 2). The coefficient K_2 is set equal to 3/4, which is valid for a sphere with uniform density. It is clear that the two curves are, in fact, different for transferred momenta $\mathbf{q}^2 \gtrsim 1$ GeV^2.

Measurement of noncoherent photoproduction cross sections for ψ particles on heavy nuclei for $\mathbf{q}_2 \approx 1-3$ GeV^2 will therefore be decisive for verifying whether $\sigma_{el} \sim \sigma_t$. When this experiment is performed, it will be important to select events in which the photoproduction of ψ particles is not accompanied by the production of other particles (other than nucleons from the breakup of the nucleus). At the same time, if the number of events involving the production of other fast particles greatly predominates, this will mean that the inelastic cross section for the ψN interaction is large and $\sigma_{in} \gg \sigma_{el}$.

6. CONCLUSIONS

In summary, we may conclude that the above soliton model of ψ bosons provides acceptable qualitative predictions for interactions of ψ particles with hadrons and leptons. These predictions can be subjected to experimental verification on existing accelerators in the case of ψ-hadron interactions. Existing data on ψ-hadron interactions are insufficient for a quantitative comparison with the above theory. However, it seems to us that measurements of the cross section for the photoproduction of ψ particles on nucleons and nuclei, and the information on the ψN interaction that can be deduced therefrom, are in agreement with the predictions of the soliton model. In contrast to the popular quark models of ψ particles based on the hypothesis of a new quantum number, i.e., charm, the soliton model does not encounter any difficulties in explaining the observed small inelasticities in ψ production processes, or difficulties connected with the lack of experimental evidence for charmed particles, for which the production cross sections are predicted by charmed quark models to be greater by two orders of magnitude than the cross sections for the production of ψ.

The problem of the physical nature of ψ particles is still experimentally unresolved, and time will show whether the soliton model is an adequate representation of the information relating to ψ particles. Whatever the outcome, we should like to emphasize that analysis of the coherent state constructed on the basis of the soliton-like solution of the classical field equation leads theoretically to the existence of heavy particles with the properties of the quasiclassical objects whose decay was considered in[1,2] and whose interaction with ordinary particles is discussed above. Verification of the existence of such states would be direct evidence for the importance of nonlinear field equations with weak coupling and spontaneous symmetry violations in the physics of elementary particles. Finally, we note that the soliton model of ψ particles which we have used in this paper is technically incomplete, in particular, in relation to such questions as the inclusion of spin and of isotopic spin. The method used to construct the coherent state of the physical pion field with definite isotopic spin, which was applied to multiple production of pions in[21,22], seems to us to be a means of removing these difficulties.

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1)The multiparticle wave functions $\Phi_n(\mathbf{k}_1, \ldots, \mathbf{k}_n)$ of the exact solution of the quantum equations in the Schrödinger representation should include δ functions which ensure the conservation of momentum. In the present case, the momentum δ function does not appear because the particular coherent state that we are considering is an approximate (quasiclassical) solution and is, therefore, not an eigenstate of the momentum operator (momentum is, nevertheless, conserved on the average). When $n \gg 1$, kinematic correlations due to momentum conservation have very little effect on the particle distributions (energy conservation provides a more stringent restriction). The approximate functions Φ_n can therefore be used to calculate the cross sections for $n \gg 1$.

2)The formula (13) is obvious if the final state is coherent with eigenfunction $g'(\mathbf{r})$. It is also valid for other final states although $g'(\mathbf{r})$ may not then be the eigenvalue of the operator $\hat{\varphi}^{(+)}(\mathbf{r})$ but the sum of a series of eigenvalues (this follows from the possibility of expanding an arbitrary state in terms of coherent states). The condition for the validity of (13) is that the convergence of this series is rapid enough.

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Electron-electron weak interaction in atoms and ions

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We discuss parity-nonconservation effects in two-electron atoms and ions, arising as the result of electron-electron neutral weak currents. Calculations are presented for the energy levels and transition probabilities for excited states of two-electron ions, and crossing of levels of opposite parity is observed in the region of charge $Z\sim 37$, which is the most suitable for observation of these effects. In the vicinity of this crossing the quantities characterizing the degree of parity nonconservation are of the order $\sim 10^{-8}$.

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The observation of neutral weak currents[1] has led to intensive searches for electron-nuclear weak interactions in atoms.[2–4] All methods of detection of weak interactions between an electron and a nucleus in an atom at low energies are based on the assumption of nonconservation of parity in neutral currents. The electron-electron weak interaction in atoms should have, apparently, the same order of magnitude as the electron-nuclear interaction. In observation of electron-electron weak interactions it is necessary to consider processes in which the electron-nuclear weak interaction is suppressed. Such processes are the emission of photons from atomic levels which have orbital angular momenta l greater than unity.[5]

Because of the centrifugal barrier, the wave functions of the electrons go to zero as r^l as $r\to 0$, where r is the distance from the center of the nucleus. The weak interaction, which does not conserve parity, has zero range and is proportional to $\mathbf{a}\cdot\mathbf{p}$, where $\mathbf{a}$ is some axial vector composed of the spins of the interacting particles and $\mathbf{p}$ is the momentum operator. The product of the initial and final wave functions of the electrons goes to zero as $r^{l_1+l_2}$, where l_1 and l_2 are the orbital angular momenta at the beginning and end of the process. The momentum $\mathbf{p}$ removes one power of r. Therefore after integration over the nuclear volume the matrix element of the weak-interaction potential turns out to be proportional to $(R/a)^{l_2+l_1-1}$, where R and a are the radii of the nucleus in the atom. For interaction with the nucleus the matrix elements between S and P states turn out to be nonvanishing as $R/a\to 0$. The matrix elements of the electron-electron interaction include integration over the entire volume of the atom and do not have parametric smallness, which depends on the magnitude of the orbital angular momenta. Therefore the matrix elements of the transitions $P\to P$, $D\to S$, and so forth for the electron-nuclear weak interaction are suppressed in comparison with $S\to P$ transitions by at least a factor $R/a\sim 10^5$, while the matrix elements of electron-electron transitions have their previous value.[1)] This statement remains valid even for large nuclear charges Z, where relativistic effects are important and it is necessary to use Dirac Coulomb functions.

The weak interaction U_W, which is invariant with respect to time reversal and which does not conserve spatial parity, can be constructed only from the product of vector and axial currents.[2–4] The electron-electron interaction contains only two independent relativistic invariants:

$$U_W=2^{-1/2}Gg(\gamma_\mu)_1(\gamma_\mu\gamma_5)_2+2^{-1/2}Gh(\sigma_{\mu\nu}q_\nu)_1(\gamma_\mu\gamma_5)_2+(1\rightleftarrows 2), \tag{1}$$

where $G=1.0\times 10^{-5}m_p^{-2}$, m_p is the proton mass, g and h are certain constants, and the subscripts 1 and 2 refer to the first and second interacting electrons. The identity of the electrons is taken into account by antisymmetrization of the wave functions of the initial and final states.

In the nonrelativistic approximation interaction (1) takes the form

$$U_W=2^{-1/2}Gg(\sigma_1-\sigma_2)(\mathbf{v}_1^+-\mathbf{v}_2^+)+2^{-1/2}G(g+h)[\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2]\times(\mathbf{v}_1^--\mathbf{v}_2^-)+(1\rightleftarrows 2), \tag{2}$$

where $\mathbf{v}_i^\mp=(\mathbf{p}_i\mp\mathbf{p}_i')/2m$; σ_i, $\mathbf{p}_i$, and $\mathbf{p}_i'$ are the Pauli spin matrices and the initial and final momenta of the i-th electron. The second term of (1) contributes only to the second term of Eq. (2), while the first term of (1) contributes to both terms of Eq. (2). Therefore for simplicity we have carried out all calculations by setting $h=0$, $g=1$.

In coordinate space the interaction (1) takes the form

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