Spatial structure in a nonequilibrium superconductor

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It is shown that if a superconducting sample is exposed to microwave radiation, then a transition into a spatially homogeneous state of the electron system sets in at a certain critical pump level.

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In a superconductor pumped by some source, a stationary nonequilibrium state is established in the electron system and is described by a nonequilibrium distribution function $n(\varepsilon)$. This function, in turn, determines the order parameter. If the pump source, which can be a sound wave, laser light, a microwave signal, etc., is spatially homogeneous in intensity and if the superconducting sample satisfies the same homogeneity requirement, then the resultant nonequilibrium state can quite naturally be assumed to be also homogeneous. The influence of such a nonequilibrium state that is homogeneous in the coordinates on the order parameter was first considered by Éliashberg.[1]

In principle, however, it can be assumed that a system of two equations—the kinetic equation and the self-consistency equation—for the two quantities $n(\varepsilon)$ and for the modulus of the order parameter—has stationary spatially-inhomogeneous solutions. This can occur, for example, in the case when the local decrease in the density of the number of quasiparticles in a certain region of space leads to an increase of the order parameter, which in turn decreases the number of quasiparticles. These two processes will be self-maintaining, and the instability will cease, while the attainment of the stationary regime will be determined by the nonlinearity.

It is shown in the present paper that this is precisely the situation in a superconducting film or in a sufficiently long narrow channel in the case of exposure to electromagnetic radiation of frequency $\omega \ll \Delta$. What turn out to be spatially dependent are the electron distribution and the modulus of the order parameter. The phase of the order parameter is immaterial.

The critical equation for the nonequilibrium increment to the distribution function $n'(\xi)$ assumes in the limit $\omega \ll \Delta$ the form $(n = n_F + n')$[2]

$$\gamma n' - \frac{\xi}{\Delta} D \frac{\partial^2 n'}{\partial Z^2} = \alpha \frac{\omega}{2T} \Delta \left[\frac{\theta(\xi^2 - 2\Delta\omega)}{(\xi^2 - 2\Delta\omega)^{1/2}} - \frac{1}{(\xi^2 + 2\Delta\omega)^{1/2}} \right], \tag{1}$$

where γ is the reciprocal energy-relaxation time, $\alpha = D(e/c)^2 A_\omega^2$, and $\xi = (\varepsilon^2 - \Delta^2)^{1/2}$ is the energy of the quasiparticles in the normal metals. The BCS self-consistency condition near T_c takes the form of the Ginzburg–Landau equation with right-hand side

$$\frac{T_c - T}{T} \Delta - \frac{7\zeta(3)}{8\pi^2} \frac{\Delta^3}{T^2} = 2N, \quad N = \int_\Delta^\infty n' \, d\xi. \tag{2}$$

In the approximation $\omega \ll \Delta$, the value of N is proportional to the density of the number of the quasiparticles. We have left out of (2) the terms with derivatives with respect to the coordinate, since they must be taken into account only together with the small quantity γ, as was done in (1).

The solution of (1) and (2) for the stationary spatially homogeneous state is given in[1] and is such that the radiation increase the order parameter. We seek the solutions in the form

$$\Delta(\mathbf{r}) = \Delta_0 + \Delta_1(\mathbf{r}), \quad n'(\mathbf{r}, \varepsilon) = n_0'(\varepsilon) + n_1(\mathbf{r}, \varepsilon), \quad \Delta_1 \ll \Delta_0, \quad n_1 \ll n_0', \tag{3}$$

where Δ_0 and n_0' are spatially homogeneous nonequilibrium quantities. As to Δ_1 and n_1, we assume that they are proportional to e^{ikz}. Linearizing (1) and (2) with respect to Δ_1 and n_1 (in view of the singularity in (1), it is more convenient to integrate first with respect to ξ), we obtain the dispersion equation

$$\frac{14\zeta(3)}{\pi^2} \frac{\Delta^2 \gamma}{\alpha \omega T} = f\left[\frac{Dk^2}{\gamma} \left(\frac{2\omega}{\Delta} \right)^{1/2} \right], \tag{4}$$

$$f(p) = \int_0^\infty \frac{2 + px}{(1+px)^2} \left[\frac{1}{(x^2+1)^{1/2}} - \frac{\theta(x^2-1)}{(x^2-1)^{1/2}} \right] dx$$

$$= \begin{cases} 3p, & p \ll 1 \\ \dfrac{1}{p}\left(\ln 2p - \dfrac{\pi}{2} + 1 \right), & p \gg 1. \end{cases}$$

At a certain $p_0 \sim 1$, the function f reaches a maximum $f(p_0) = f_0 \sim 1$. It can be shown that so long as

$$\alpha < \alpha_c = \frac{14\zeta(3)}{\pi^2 f_0} \frac{\Delta^2 \gamma}{\omega T}, \tag{5}$$

only solutions that attenuate in time are possible for the variables Δ_1 and $n_1 \sim e^{-\varkappa t}$, i.e., $\varkappa > 0$. In the opposite case $\alpha > \alpha_c$, solutions appear with $\varkappa < 0$. As the pump intensity increases, the instability sets in first at $\varkappa = 0$, $\alpha = \alpha_c$, and

$$\frac{Dk^2}{\gamma} \left(\frac{2\omega}{\Delta} \right)^{1/2} = p_0 \sim 1. \tag{6}$$

If the irradiation intensity and the frequency are fixed then the instability sets in, with rising temperature, at

$$\frac{T_c - T}{T} \sim \left(\frac{\Delta}{T} \right)^2 \sim \frac{\alpha}{\gamma} \frac{\omega}{T}, \quad \frac{Dk^2}{\gamma} \sim \left(\frac{\alpha}{\gamma} \frac{T}{\omega} \right)^{1/4}. \tag{7}$$

It is assumed here that $\Delta \gg \omega$. Combining this with the condition for the applicability of the approximation lin-

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ear in the intensity in the kinetic equation (1),[2] $\alpha/\gamma \ll (\omega/\Delta)^{1/2}$, we obtain the condition under which our analysis is valid:

$$\omega/T \ll \alpha/\gamma \ll (\omega/T)^{1/2}. \tag{8}$$

When varying (1) and (2) we have assumed that α does not depend on the order parameter and the vector potential $\mathbf{A}_\omega$ is homogeneous in space. Since the order parameter is statically modulated, preservation of the electroneutrality calls for the appearance of a vector potential $\mathbf{A}_1(\mathbf{k}, \omega) \sim \Delta_1(\mathbf{k})\mathbf{A}_\omega$ and a scalar potential $\varphi(\mathbf{k}, \omega) \sim \mathbf{A}_\omega \mathbf{k}\Delta_1(\mathbf{k})$. But since $\mathbf{k}$ is small, the scalar potential can be neglected, and the corrections to the vector potential should be obtained from the equation for the current

$$\mathbf{j}_\omega = \frac{i\omega\sigma}{c}\mathbf{A}_\omega - \frac{\pi}{2c}\frac{\Delta^2\sigma}{T}\mathbf{A}_\omega.$$

However, as follows from (5), instability sets in at temperatures such that the penetration of the alternating field into the sample is subject to the skin effect, $T\omega \gg \Delta^2$. For this reason, the corrections to the vector potential are also small.

If the intensity exceeds its critical value α_c, then the amplitude of the resultant inhomogeneous state increases with time and assumes a stationary value when account is taken of the nonlinear effects. It can be shown, by integrating the system (1) and (2) up to terms of order Δ_1^3 inclusive, that in the right-hand side of the dispersion equation (4) there appears a term $\beta\Delta_1(\mathbf{k})\Delta_1(-\mathbf{k})$, where $\beta > 0$. Thus, the nonlinearity contributes to the transition at $\alpha < \alpha_c$. This situation is analogous to a first-order phase transition, and in fact a real transition will take place at $\alpha \lesssim \alpha_c$ jumpwise into a state that is strongly inhomogeneous in the amplitude Δ and has a characteristic wave vector (6). It is difficult to make any definite assumptions concerning the properties of the resultant state (whether it is superconducting, normal, or resistive), since this calls for exact knowledge of the solution of the system (1), (2).

The physical picture consists in the following. Assume, as follows from (6), that the characteristic inhomogeneity length, whose role can be assumed by the diffusion scale $L \sim 10^{-2}$ cm, is smaller than the dimensions of the sample (one can use as this dimension, for example, the length of the neck of a superconducting bridge). Then, at a specified irradiation intensity, as the temperature is increased the superconductivity is stimulated[1,2] and the nonequilibrium state remains homogeneous up to a certain point C defined by (7), as shown in Fig. 1 (the dashed curve represents the equilibrium dependence of the order parameter on the temperature), where an inhomogeneity is produced jumpwise. The relative increase of the order parameter at the point C is

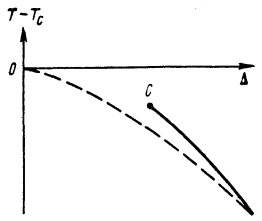

FIG. 1.

$$\frac{\delta\Delta}{\Delta} \sim \left(\frac{\gamma\omega}{\alpha T}\right)^{1/2} \ln\frac{\alpha T}{\gamma\omega} \ll 1. \tag{9}$$

The formation of the inhomogeneity depends substantially on the pumping method. If the excitation is by microwaves or by transverse sound, variation of the density of the quasiparticles with respect to $\Delta_1(k)$ is negative. To the contrary, in excitation by longitudinal sound, i.e., by a scalar potential, we have $\partial N/\partial\Delta_1 > 0$, and in this case there is no transition to the inhomogeneous state. The experimental plot[3] of Δ against the temperature agrees well with the theoretical one calculated for a homogeneous state in the temperature region where $\omega \lesssim \Delta$.

Interest attaches to experiments on laser irradiation of superconducting samples, where a resistive state is reached at a sufficiently strong signal.[4,5] It is perfectly possible that this state is the consequence of the formation of an inhomogeneity of the type considered above. In the low-temperature limit $T \ll \Delta$, at not too intense a pump, we can use for the nonequilibrium electron distribution function the effective-chemical-potential approximation[6] $n = \exp[(\mu - \varepsilon)/T]$. It can be shown that so long as this approximation is valid, the system is stable to small spatial variations of the chemical potential and of the modulus of the order parameter, in contradiction to the results of[7].

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