

Mechanisms that limit the amplitudes of parametrically excited spin waves in ferrites

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An experimental procedure was developed to permit measurement of the phase shift of parametrically excited spin waves, as well as the real and imaginary parts of nonlinear susceptibility. A study of these characteristics has established the role played by different mechanisms governing the amplitude of the spin waves beyond the parametric-excitation threshold. The phase and self-modulation limitation mechanisms, the hard excitation mechanism, and the mechanism of nonlinear damping due to three-magnon coalescence and three-magnon splitting of spin waves. A comparative estimate is made of the efficiencies of the various mechanisms that limit the spin-wave amplitude.

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INTRODUCTION

At present there are two known principal nonlinear mechanisms that determine the amplitude of parametrically excited spin waves (PSW) beyond the threshold of their parametric excitation.^[1] The first is the phase mechanism that limits the PSW amplitude and is the result of nonlinear interactions of the spin waves with one another; it leads to a deviation of the phase shift $\psi_k = \varphi_k + \varphi_{-k}$ of a pair of PSW, relative to the pump phase, from the optimal value $\psi_k = \pi/2$. Here φ_k and φ_{-k} are the individual phase shifts of the PSW that make up a parametric wave pair with equal but opposite momentum directions. In the case of the phase mechanism of limitation^[1] we have

$$\sin \psi_k = \zeta^{-1} = h_{thr}/h; \quad (1)$$

ζ is the supercriticality and is equal to the ratio of the amplitude of the microwave field h acting on the sample to the threshold field value h_{thr} at which parametric instability sets in.

Another limitation mechanism is nonlinear damping, as a result of which the PSW relaxation frequency γ_k becomes dependent on the number N_k of the PSW:

$$\gamma_k = \gamma_k^0 + \eta N_k, \quad (2)$$

where η is the nonlinear damping constant and is determined by the three-wave PSW interaction processes. Depending on the PSW wave number k , the quantity η receives contributions from various three-wave processes. For waves with $k \geq k_{3h}$, hard excitation of PSW is possible, a process for which $\eta = \eta_1 < 0$. At $k = k_{3m} > k_{3h}$ there exists, besides the hard excitation, also a process of nonlinear three-magnon coalescence of two PSW into one new spin wave, having a combined energy and a combined wave vector. Finally, at $k \geq k_{3sp}$, three-magnon splitting of one PSW into two new waves with half the energy becomes possible. The coefficient of the nonlinear damping for waves with $k = k_{3m}$ and $k \geq k_{3sp}$ will be designated η_2 and η_3 , respectively.

In the case of parallel pumping of spin-waves instability,^[2] there exists a one-to-one relation between the

constant magnetic field H_0 applied to the sample and the value of the wave vector of the thus excited PSW:

$$k^2 = (H_s - H_0)/D; \quad (3)$$

D is the exchange constant and H_s is the field of the minimal threshold of the spin-instability at which PSW are excited as $k \rightarrow 0$. Relation (3) enables us to determine the limits within which various nonlinear damping processes are turned on in terms of constant magnetic fields whose values can be easily determined in experiment. In the case of yttrium iron garnet (YIG) at a pump frequency $\omega = 2\pi \cdot 9.4$ GHz, hard excitation of the spin waves (and a nonlinear-interaction constant $\eta = \eta_1$ associated with this process) exists at $H_0 \leq H_{3h} = H_s - 200$ Oe, three-magnon coalescence ($\eta = \eta_2$) occurs in fields $H_0 \leq H_{3m} = H_s - 550$ Oe, and three-magnon splitting ($\eta = \eta_3$) takes place at $H_0 \leq H_{3sp} = H_s - 930$ Oe.

In the presence of nonlinear damping, the PSW phase is no longer described by relation (1). By the methods described in^[1] it is easy to obtain at $\eta \neq 0$ the relation

$$\sin \psi_k = \frac{1}{\zeta} \left(1 + \frac{\pm \{1 + (\zeta^2 - 1) [(S/\eta)^2 + 1]\}^{1/2} - 1}{(S/\eta)^2 + 1} \right); \quad (4)$$

S is the nonlinear four-wave PSW interaction constant and determines the effectiveness of the phase mechanism that limits the PSW amplitude (see^[1]):

$$S = 2\pi g^2 \left(\frac{\omega_M}{\omega} \right) \left[\frac{\omega_M}{\omega} (N_z - 1) + \left\{ 1 + \left(\frac{\omega_M}{\omega} \right)^2 \right\}^{1/2} \right], \quad (5)$$

where g is the gyromagnetic ratio for the electron spin, ω is the pump frequency, $\omega_M = 4\pi g M_0$, M_0 is the saturation magnetization of the sample, and N_z is the demagnetizing factor of the sample in the direction of application of the constant magnetic field. The upper and lower signs in front of the curly brackets in (4) are taken at $\eta > 0$ and $\eta < 0$, respectively.

It is seen from (4) that the dependence of PSW phase on the supercriticality is strongly influenced by the effectiveness of the nonlinear damping, which can be characterized by the ratio η/S . At $\eta/S = 0$, Eq. (4) goes over into (1). At $\eta/S < 0$, a jump of the PSW phase

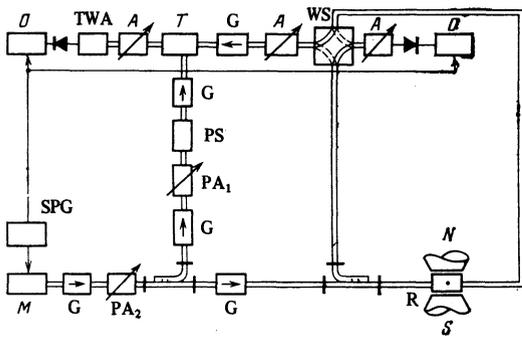


FIG. 1. Block diagram of experimental setup: M—magnetron, SPG—generator of square-wave synchronization pulses: G—waveguide gate, PA—precision attenuator, PS—phase shifter, T—waveguide tee junction, TWA—traveling-wave-tube amplifier, O—oscilloscope, A—attenuator, WS—waveguide switch, R—resonator with investigated sample.

should be observed near the threshold. On the other hand, if $\eta \geq S$, the situation is reversed and the PSW phase deviates very little from the optimal value $\pi/2$ with increasing supercriticality. All this gives grounds for hoping that a study of the experimental functions $\psi_h(\xi)$ will yield information on how various spin-wave limitation mechanisms operate beyond the parametric excitation threshold, and also on the effectiveness of these mechanisms.

EXPERIMENTAL SETUP AND MEASUREMENT PROCEDURE

1. A brilliant method of determining the change of the PSW phase with changing supercriticality is described in the literature^[3] and consists of varying the pump-signal phase rapidly in comparison with the PSW relaxation time $1/\gamma_h^0$. We have developed another, more accurate method, based on measurements of the phase and amplitude of the signal in a resonator containing the investigated ferrite.

A block diagram of the experimental setup is shown in Fig. 1. The amplitude and phase of the oscillations in the resonator were measured by comparing the signal passing through the resonator with a reference signal. At a pump amplitude below the threshold value ($h < h_{thr}$) the signal cancels out to zero. At $h > h_{thr}$, a magnetization $m_z = \chi h$ is produced in the sample; it varies with the pump frequency and is directed along the constant magnetic field; $\chi = \chi' - i\chi''$ is the nonlinear magnetic susceptibility of the ferrite in the case of parallel pumping of the spin-wave instability. The magnetization m_z produces in the resonator an additional microwave reaction magnetic field $h_r(m_z)$, which changes in turn the amplitude and phase of the combined self-consistent field of the resonator h relative to the pump amplitude and phase. These changes can be measured with a phase shifter PS and an attenuator PA₁ located in the reference channel, by canceling out the mismatch signal that appears as a result of the excitation of the PSW. It is obvious that if we know the change of the amplitude and the phase of the field in the resonator, we can obtain information on the microwave reaction magnetic

field $h_r(m_z)$, and consequently on the phase of the variable longitudinal magnetization m_z , which is shifted 180° relative to ψ_h .

2. We express ψ_h in terms of the experimentally determined changes of the amplitude of the phase of the microwave magnetic field of the resonator. To this end, we write down the equations for the coupled oscillations of the resonator electromagnetic field and of the ferrite magnetization^[4]:

$$m_z = (\chi' - i\chi'')h, \quad h(\omega^2 - \omega_0^2 + i\omega^2/Q_L) = Am_z + B; \quad (6)$$

A is a coefficient that appears in the solution of the problem of excitation of resonator oscillations by a variable magnetization^[5]:

$$A = 4\pi\omega^2 p, \quad (7)$$

p is the sample filling factor in the resonator:

$$p = \int_{\text{sample}} h^2 dv / \int_{\text{res}} h^2 dv,$$

ω_0 and Q_L are the unperturbed (i. e., in the absence of PSW) natural frequency of the resonator and its loaded Q. The forced oscillations of the system (6) are due to the external force B, an explicit expression for which is not given here because the form of B does not influence the final formulas.

We shall consider henceforth the case when the natural frequency of the resonator is tuned at low power ($h < h_{thr}$) to resonance with the pump frequency, i. e., $\omega = \omega_0$.

From (6) we can obtain for the self-consistent microwave magnetic field in the resonator

$$h = B / \left[i \left(\frac{\omega^2}{Q_L} + A\chi'' \right) - A\chi' \right]. \quad (8)$$

In accordance with the foregoing, the amplitude and phase of h depend on χ according to (8). The change of the phase of h under the influence of the PSW will be designated by φ , while the change of the amplitude will be designated by $a = |h_0|/|h|$, where h_0 is the value of h at $\chi = 0$ and is proportional to the value of the reference signal. From (8) we easily obtain

$$\text{tg } \psi_h = \text{ctg } \varphi - 1/a \sin \varphi. \quad (9)$$

In the derivation of (9) we took into account the relation that follows from the first equation of the system (6):

$$\text{tg } \psi_h = -\chi''/\chi'. \quad (10)$$

Expression (9) admits of simple geometrical interpretation, which is shown in Fig. 2. According to this figure, the self-consistent field in the resonator is the sum of the unperturbed field h_0 and the reaction field $h_r(m_z)$, shifted 90° in phase relative to m_z .

With the aid of the installation shown in Fig. 1 and relation (9) it is possible to determine experimentally

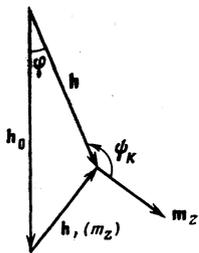


FIG. 2. Geometrical interpretation of formula (9) ($|h_0|/|h|=a$). h_0 —unperturbed field of the resonator; $h_1(m_z)$ is the sample reaction field due to the sample magnetization m_z ; h —perturbed self-consistent field of the resonator.

the PSW phase ψ_k . In fact, the quantities φ and a needed for this purpose are given respectively by the phase shifter PS and the precision attenuator PA₁. The other instability characteristics, such as χ' and χ'' , can also be determined with the aid of this installation, using the following relations:

$$\chi' = -aA \sin \varphi, \quad \chi'' = aA \sin \varphi \tan \psi_k. \quad (11)$$

3. The measurements were performed at a pump frequency 9370 MHz in a pulsed regime. The field H_s at $H_0 \parallel [100]$ was 1680 Oe. The pulse duration was 200 μ sec and the repetition frequency 50 Hz. We investigated single-crystal YIG spheres of diameter 1–4.15 mm. The cavity resonator was a segment of a standard rectangular 3-cm band waveguide, operating in the H_{012} mode, and with a loaded $Q_L \approx 10^3$. The samples were placed at the center of the resonator, in the antinode of the microwave magnetic field parallel to the constant magnetic field. To measure the amplitude and phase of the microwave oscillations in the resonator, an opening was provided in the end phase of the resonator, through which about 1% of the power incident on the resonator was guided to the measuring circuit.

The measurement accuracy is determined mainly by the accuracy of the precision attenuators PA₁ and PA₂, which are graduated in decibels, and of the phase shifter PS. The attenuator graduation accuracy was 0.1 dB, and that of the phase shifter 0.3°. The accuracy with which the phase ψ_k of the PSW is measured depends on the absolute value of ψ_k . As $\psi_k \rightarrow 90^\circ$, the error was maximal and amounted to 6°. The accuracy could be greatly improved (to 1–2°) by using the measurement scheme "in reflection" instead of "in transmission." The transition from one measurement scheme to the other was effected with a waveguide switch (element WS in Fig. 1). The rotation produced by the PSW in the phase of the reflected signal, which we shall designate by α , is connected with ψ_k by the relation

$$\text{ctg}(\varphi + \psi_k) = \sin \alpha / (b - \cos \alpha), \quad (12)$$

where b is the ratio of the moduli of the coefficients of reflection from the resonator without and with the PSW. It is seen from (12) that to use the system "in reflection" it is necessary to know the phase shift φ measured in the system "in transmission." An estimate, with the aid of (11), of the accuracy with which the susceptibility

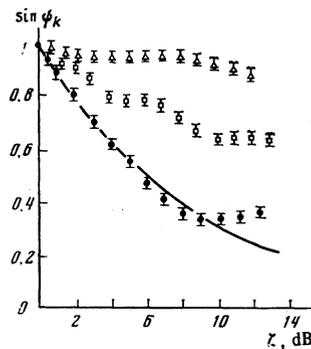


FIG. 3. Dependence of PSW phase on the supercriticality when the crystal is magnetized along the [100] axis (\bullet), the [110] axis (\square) and the [111] axis (Δ), $H_0 = H_s - 50$ Oe. Sample—YIG sphere of 2 mm diameter. Solid line—theoretical curve constructed in accordance with formula (1).

χ' is determined leads to an error $\leq 5 \times 10^{-4}$.

The magnitude of the supercriticality was determined from measurements, with the aid of the attenuators PA₁ and PA₂, of the relative power passing through the resonator with the ferrite. The supercriticality in decibels is in this case $\zeta[\text{dB}] = (D_{2\text{thr}} - D_2) + (D_{10} - D_1)$, where $D_{2\text{thr}}$ are the readings of the attenuator PA₂, corresponding to the instability threshold; D_{10} and D_1 are respectively the readings of the attenuator PA₁ below the instability threshold and at a power above threshold, when the attenuator PA₂ is in the position D_2 . The supercriticality was measured accurate to 0.2 dB.

EXPERIMENTAL RESULTS AND DISCUSSION

1. The results of the PSW phase measurements are shown in Figs. 3 and 4. Figure 3 corresponds to the case $k < k_{3n}$, i. e., when all the nonlinear three-wave processes are forbidden ($\eta = 0$), and the only mechanism that limits the PSW amplitude, according to the theoretical concepts,^[1] is the phase mechanism of limitation. It is seen that when the crystal is magnetized along the [100] axis, experiment agrees quite satisfactorily with theory up to supercriticalities $\zeta \sim 8$ –10 dB (formula (1) or formula (4) at $\eta = 0$). It can be assumed that the discrepancy at $\zeta \geq 10$ dB is due to the appearance in the

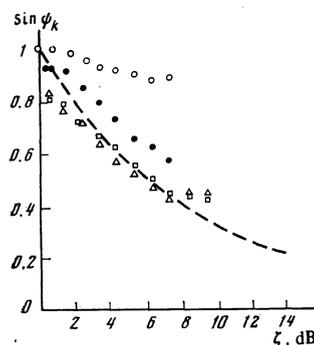


FIG. 4. Dependence of the PSW phase on the supercriticality at different values of the constant magnetic field: Δ — $H_0 = H_s - 500$ Oe, \square — $H_0 = H_s - 600$ Oe, \bullet — $H_0 = H_s - 550$ Oe = H_{3m} , \circ — $H_0 = H_s - 1000$ Oe $< H_{sp}$. Sample—YIG sphere of 2mm diameter, $H_0 \parallel [100]$. Dashed line—theoretical plot of (1).

sample of secondary PSW pairs not accounted for by the simple theory^[11] used in the derivation of expressions (1) and (4). The agreement between theory and experiment is much worse for a crystal magnetized along the [110] axis, and particularly along [111]. In the latter case, at a supercriticality 8 dB, the deviation of the phase ψ_k from the optimal value $\pi/2$ is only $\sim 15^\circ$, as against the $\sim 70^\circ$ which follows from the theory. The most substantial difference between parametric excitation of spin waves in a YIG sphere magnetized along the axis [110] and [111], on the one hand, in the case of magnetization along [100], on the other, is the presence of strong self-modulation oscillations of the magnetization at $H_0 \parallel [111]$ and $H_0 \parallel [110]$.^[12] This circumstance makes it possible to suggest that these oscillations exert an influence on the process of establishment of the PSW amplitude beyond the excitation threshold. In this case, besides the phase mechanism, there exists also a strong self-modulation mechanism, heretofore not considered in the theory, which limits the PSW amplitude. The existence of this mechanism can be understood within the framework of a known effect, namely the suppression of the additional absorption in ferrites by modulation of the constant magnetic field.^[6] In this case the self-modulation of the magnetization, via the change of the internal magnetic field in the crystal, leads to a change in the natural frequency of the spin waves at a rate of the same order as the relaxation rate of these waves. Consequently, each of the PSW is at parametric resonance with the pump only for a short time, during which the amplitudes of these PSW cannot reach their maximum value, after which the phase mechanism of amplitude limitation becomes effective. At $H_0 \parallel [111]$, the self-modulation limitation mechanism does not depend on the constant magnetic field or on the PSW wave vector. In this case the experimental relations obtained at $H_0 < H_c$ are analogous to those marked by triangles in Fig. 3.

Owing to the self-modulations of the magnetization, distorts the initially flat top of the pulse passing through the resonator so that it is impossible to use in the experiment a reference signal to cancel out the entire pulse simultaneously. Cancellation of different parts of the pulse in the experiment yielded different values of a and φ , but the phase ψ_k of the PSW pair remains constant within the limits of the experimental accuracy. This indicates that the self-modulation of the magnetization constitutes in the main changes of the magnetization amplitude m_z , but not of its phase relative to the pump.

2. Owing to the strong influence of the self-modulation, the role of the nonlinear-damping mechanisms was determined with the sample magnetized along the [100] axis, when the self-modulation does not limit the PSW amplitude. Figure 4 shows the results of investigations of the PSW phase at different constant magnetic fields corresponding to the action of various nonlinear damping of the PSW in addition to the phase mechanism. All the plots shown in Fig. 4 can be explained with the aid of relation (4). At $k > k_{3h}$ ($H_0 < H_{3h}$) a jump of ψ_k from the optimal value occurs immediately past the PSW instability threshold. This jump is due to the presence of nonlinear negative damping, $\eta < 0$, takes place in a field $H_0 \approx H_{3h}$, and increases with decreasing field. At

$H_0 = H_{3m}$, the jump of ψ_k has a resonant minimum, as is clearly seen from Fig. 4: the plots marked by the triangles and squares respectively were obtained in fields 50 Oe above and below the field H_{3m} corresponding to the curve that can be drawn through the black circles in the same figure. In fields $H_0 \leq H_{3m}$, the mechanism of nonlinear damping due to coalescence of two PSW comes into play. The probability of this mechanism increases resonantly in a field $H_0 = H_{3m}$, because the state density for which the coalescence process is allowed increases in this field.^[11] Since the three-magnon coalescence makes a positive contribution to the nonlinear damping, the experimentally observable resonant decrease of the change of the PSW phase with increasing ζ near the field H_{3m} becomes understandable. It is interesting to note that even in the case of a maximum probability of the three-magnon coalescence at $H_0 = H_{3m}$, the total nonlinear damping $\eta = \eta_2$ of the PSW remains less than zero near the instability threshold, meaning that the negative nonlinear damping due to the hard excitation of the PSW cancels out completely the nonlinear positive damping, due to the three-magnon coalescence, which was assumed earlier, in the initial theory developed by Gottlieb and Suhl for the stationary state of PSW beyond the excitation threshold,^[17] to be the principal mechanism that limits the PSW amplitude.

After turning on the three-magnon splitting mechanism ($H_0 < H_{3sp}$), the jump of the PSW phase decreases as $\zeta \rightarrow 1$ and vanishes completely at $H_0 \lesssim 700$ Oe—the total nonlinear damping becomes positive: $\eta = \eta_3 > 0$.

3. From the experimental plots of $\psi_k(\zeta)$ we can determine with the aid of relation (4) the quantity η/S , which characterizes the effectiveness of the nonlinear damping processes in YIG crystals. It turns out that the effectiveness of the nonlinear damping depends on the supercriticality. This is well seen from the relation represented by the dark circles on Fig. 4. At $\zeta < 1$ dB we have $\eta_2 > 0$, and at $\zeta > 2$ dB the nonlinear damping becomes positive, leading to a smaller rotation of the PSW phase away from the optimal value than called for by formula (1), which describes the pure phase mechanism of limitation. The dependence of η/S on the constant magnetic field (i. e., for PSW with different wave vectors k) as $\zeta \rightarrow 1$ is shown in Fig. 5. It is seen that in the field region $H_{3sp} < H_0 < H_{3h}$ the nonlinear damping is negative and amounts to $|\eta| \approx 0.3S$, and its absolute value decreases by about one-half near the field H_{3m} .

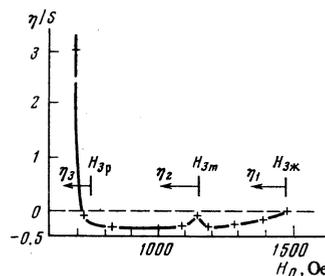


FIG. 5. Dependence of the effectiveness of the nonlinear damping η/S on the constant magnetic field as $\zeta \rightarrow 1$. Sample—YIG sphere of 2 mm diameter, $H_0 \parallel [100]$.

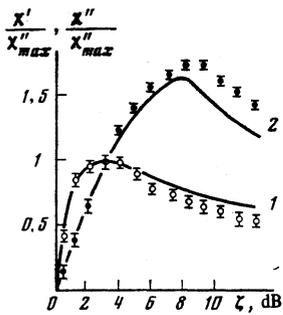


FIG. 6. Dependence of the relative nonlinear susceptibilities in parallel pumping of spin-wave instabilities on the supercriticality for a YIG sphere of 2mm diameter, $H_0 \parallel [100]$. Solid lines 1 and 2—theoretical calculation for χ' and χ'' , respectively^[1]; $H_0 = H_s - 50$ Oe.

At $H_0 < H_{3sp}$ the nonlinear damping becomes very large and positive, indicating that in this field region the principal mechanism that limits the PSW is the splitting of the spin waves. The ratio η/S decreases here with increasing supercriticality. Whereas at $\zeta < 1$ dB we have $\eta \approx 3S$, at $\zeta = 3$ to 7 dB we have $\eta \approx S$.

4. An investigation of the function $\psi_h(\zeta)$ at $H_0 > H_s$ and $H_0 \parallel [100]$ has shown that in this case the phase limitation mechanism is the principal one that limits the PSW amplitude, although the phase shift is somewhat smaller here than the theoretical value, possibly because the spin wave spectrum becomes less dense at $H_0 > H_s$. For example, at $H_0 = H_s + 50$ Oe and $\zeta = 13$ dB, the phase shift is $\psi_h \approx 70^\circ$ in place of the $\sim 80^\circ$ that follows from the theory. There is also a substantial deviation from the measurements at $H_0 < H_s$, namely, excitation of the second group of the PSW was not observed at $H_0 > H_s$.

5. At the present time there is no unanimity in the literature concerning the behavior of the susceptibility, in the case of parallel pumping of spin-wave instability, as a function of the supercriticality.^[1,8] The experimental $\chi'(\zeta)$ and $\chi''(\zeta)$ curves obtained by use are shown in Figs. 6 and 7. In the absence of nonlinear damping (Fig. 6), the agreement between the experimental result and the theory of the phase mechanism of limitation is quite satisfactory, and there is no gap between the instability threshold and the appearance of nonzero values of the real part χ' of the instability. Moreover, in the presence of negative nonlinear damping, a jump of χ' may even be observed as $\zeta \rightarrow 1$ (see curves 1 and 2 of Fig. 7). In the case of a strong nonlinear positive damping (curve 3 in the same figure). The derivative $\partial\chi'/\partial\zeta$ vanishes as $\zeta \rightarrow 1$, and this leads inevitably in the experiment to the existence of the aforementioned gap, the size of which depends on the sensitivity of the setup. In our case the sensitivity threshold of the apparatus with respect to χ' ($\sim 10^4$) was reached in fields $H_0 < H_{3sp}$ at a supercriticality $\zeta \approx 0.8$ dB. The presence of jumps of χ' and of zeroes of the derivative as $\zeta \rightarrow 1$ and at $\eta > 0$ is in good agreement with the theory of the phase mechanism of limitation if account is taken of nonlinear damping, thus offering one more argument in favor of this theory.

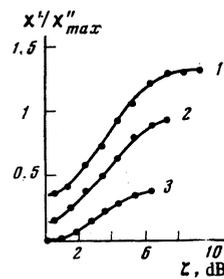


FIG. 7. Dependence of the real part of the nonlinear susceptibility in the case of parallel pumping on the value of the instability for three values of the constant magnetic field: 1) $H_0 = H_s$ and 500 Oe, 2) $H_0 = H_{3m}$, 3) $H_0 = H_s - 1000$ Oe. Sample—YIG sphere of 2 mm diameter, $H_0 \parallel [100]$.

CONCLUSIONS

The amplitude of parametrically excited spin waves, in the case of parallel pumping of the spin-wave instability, is determined by various nonlinear mechanisms, depending on the magnitude of the spin wave vector and on the direction of the magnetization relative to the crystal axes. The most universal is the phase mechanism that limits the amplitude, which is present for all spin waves at any crystal orientation. The self-modulation mechanism is connected with self-oscillations of the magnetization and is present only in the presence of these self-oscillations. When a single-crystal YIG sphere is magnetized along the [100] axis, the self-modulation limitation mechanism does not come into play. Negative nonlinear damping due to hard excitation of spin waves leads to a jump of the amplitude of these waves after passing through the excitation threshold. This damping cancels to a considerable degree the effect of the positive nonlinear damping that is caused by the three-magnon coalescence of the spin wave. Nonlinear damping due to three-magnon splitting of spin waves, on the contrary, suppresses the hard excitation of the spin waves; the constant of this damping is three times larger than the constant of the phase mechanism of limitation.

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- ¹V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk 114, 609 (1974) [Sov. Phys. Usp. 17, 896 (1975)].
- ²Ya. A. Monosov, Nelineinyi ferromagnitnyi rezonans (Nonlinear Ferromagnetic Resonance), Nauka, 1971.
- ³L. A. Prozorova and A. N. Smirnov, Zh. Eksp. Teor. Fiz. 67, 1952 (1974) [Sov. Phys. JETP 40, 970 (1975)].
- ⁴B. A. Auld, J. Appl. Phys. 34, 1629 (1963).
- ⁵A. G. Gurevich, Ferrity na SVCh (Ferrites at Microwave Frequencies), Fizmatgiz, 1960.
- ⁶T. S. Hartwick, E. R. Peressini, and M. E. Weiss, Phys. Rev. Lett. 6, 176 (1961).
- ⁷P. Gottlieb and H. Suhl, J. Appl. Phys. 33, 1508 (1962).
- ⁸Ya. A. Monosov, V. V. Surin, and V. A. Shakhunov, Zh. Eksp. Teor. Fiz. 65, 1905 (1973) [Sov. Phys. JETP 38, 951 (1974)].

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