curved crystalline diffraction gratings for x rays which may have considerable advantages over cylindrical ones bent by the mechanical method of Johann, Cauchois, *et al.* (horizontal focusing) and by the method of Kunzle *et al.* (vertical focusing). These questions will be the subject of further investigations by us.

The author wishes to express her sincere thanks to Academician G. Nadzhakov for interesting discussions of the work and for a number of useful suggestions.

[1]F. Twyman, Prism and Lens Making, London, 1952, p. 318.
[2]F. W. Preston, Trans. Opt. Soc. London **23**, 144 (1922).
[3]F. Batajczyk, Feingerätetechnik **15**, 445 (1966).
[4]F. Ratajczyk, Feingerätetechnik **16**, 254 (1967).
[5]G. S. Nadjakov and E. G. Nikolova, Doklady Bulg. Sci. **19**, 193 (1966).
[6]G. S. Nadjakov and E. G. Nikolova, Doklady Bulg. Sci. **19**, 273 (1966).
[7]E. G. Nikolova, Doklady Bulg. Sci. **28**, 897 (1975).
[8]S. P. Tomishenko and S. W. Krieger, Theory of Plates and Shells, McGraw (Russian Transl., Nauka, 1966, p. 61).

Translated by N. G. Anderson

Resistance of pure aluminum and of weak solutions of Mg, Zn, and Ga in Al in the region 2–40°K

M. N. Klopkin, G. Kh. Panova, and B. N. Samoĭlov

I. V. Kurchatov Institute of Atomic Energy
(Submitted April 26, 1976; resubmitted August 17, 1976)
Zh. Eksp. Teor. Fiz. **72**, 550–556 (February 1977)

The temperature and concentration dependences of the resistivity and of the transverse magnetoresistance of polycrystalline aluminum and weak solutions of magnesium, zinc, and gallium in aluminum were investigated experimentally at temperatures 2–40°K in magnetic fields up to 50 kOe. Deviations from the Matthiessen rule and an anomalous behavior of the magnetoresistance were observed. It is noted that the magnetoresistance depends on the type of impurity atom and has a nonmonotonic temperature dependence. It is shown that the observed anomalies can be attributed to anisotropy of the scattering by phonons and by impurity atoms.

PACS numbers: 72.15.Gd, 72.15.Qm

1. INTRODUCTION

Kagan *et al.*[1,2] have shown that the observed anomalies in the behavior of the resistivity and magnetoresistance of irregular metals are of like nature and have a common explanation. The principal feature of that theory is that the character of the distribution of the nonequilibrium electrons on the Fermi surface in metals without impurities has a sharply pronounced anisotropy. The latter is connected with umklapp processes, with nonsphericity of the Fermi surface, and with anisotropy of the phonon spectrum. It was suggested[1,2] that both the impurities and the magnetic field make the distribution of the nonequilibrium electrons isotropic. This explains the anomalous dependence of the resistivity and of the magnetoresistance on the temperature and on the impurity concentration.

The singularities of the temperature and concentration dependences of the resistivity of metals with impurities have by now been investigated experimentally in considerable detail.[3,4] As to measurements of the magnetoresistance, notice should be taken here of the interesting and unexpected results obtained in[5–8] for aluminum and indium. Until recently, however, there were no investigations of the magnetoresistance of aluminum with definite impurities. Investigations in strong magnetic fields were carried out either on samples of unequal purity, with unidentified purities,[6–8] or on samples having defects of the vacancy or dislocation type, obtained by quenching or by irradiation.[9–11] The reason is that introduction of a sufficient amount of impurities into a pure metal contradicts the requirement that the electron have a large free path, and the latter must be satisfied if the conditions of strong magnetic fields are to be obtained ($\omega\tau \gg 1$, where ω is the cyclotron frequency and τ is the relaxation time).

We have investigated the temperature and concentration dependences of the resistivity ρ and of the transverse magnetoresistance ρ_H of aluminum and of weak solutions of Mg, Zn, and Ga in aluminum at temperatures 2–40 °K and in magnetic fields up to 50 kOe. The high purity of the initial aluminum ($R_{295\,K}/R_0 = 5900$) and the possibility of working in fields up to 50 kOe have enabled us to investigate weak solutions of Mg, Zn, and Ga in Al under conditions of sufficiently strong magnetic fields ($\omega\tau > 2$).

SAMPLES AND EXPERIMENTAL PROCEDURE

We investigated samples of pure Al (99.999%) with resistance ratio $R_{295\,K}/R_0 = 5900$ and weak solutions of Mg, Zn, and Ga in Al. The characteristics of the samples are listed in Table I (R_0 and ρ_0 are the resistance and resistivity in the absence of a magnetic field at $T = 2$ °K).

 0038-5646/77/4502-0287$02.40

TABLE I.

Sample number	Introduced impurity, at.%	R_{295K}/R_0	ρ_0, nΩ·cm	Sample number	Introduced impurity, at.%	R_{295K}/R_0	ρ_0, nΩ·cm
1	–	5900	0.46	6	0.035 Ga	290	9.31
2	0.003 Zn	1630	1.66	7	0.0013 Mg	2540	1.06
3	0.0075 Zn	1050	2.66	8	0.043 Mg	153	17.60
4	0.042 Zn	266	9.83	9	0.155 Mg	39	68.05
5	0.010 Ga	900	3.01				

The concentration c of the impurities introduced in alloys of Al with Mg and Zn was determined from the ratio of the initial impurity and solvent-metal masses, while in alloys of Al with Ga they were determined by chemical analysis. The geometric factor was not measured. The absolute value of the resistivity was calculated from the formula

$$\rho(c,T)=\rho_{Al}(295\,\mathrm{K})\frac{R(c,T)}{R(c,295\,\mathrm{K})-R(c,2\mathrm{K})}$$

where $\rho_{Al}(295\ \mathrm{K})$ was taken to be 2.66 μΩ-cm from[12].

The samples were made in the form of wires of 1 mm diameter and ~350 mm length from ingots by pressure extrusion. The wires were next wound on a cylinder of 12 mm diameter at a pitch of 2 mm. The samples were then annealed at 300 °C in air for two hours.

Figure 1 shows the dependence of the residual resistivity ρ_0 on the impurity-atom content for weak solutions of Mg, Zn, and Ga in Al. The resistivity at 2 °K is designated ρ_0. It is seen that in the investigated concentration region the residual resistivity of all alloys depends linearly on the impurity content. We note that the increase of the residual resistivity is the largest for the alloy Al:Mg (0.42 μΩ-cm/at.%). For Al–Ga and Al–Zn alloys its values are 0.25 and 0.21 μΩ-cm/at.%, respectively. These results are in good agreement with the data of[12].

The investigations were performed in a setup in which measurements could be made in the range 1.5–40 °K in a superconducting-solenoid field up to 60 kOe. The magnetic field was uniform along the sample within ~0.1%. The magnetic-field measurement accuracy was ~0.2%.

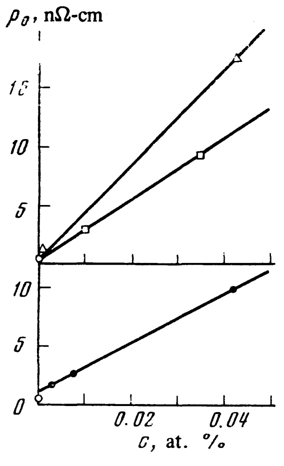

FIG. 1. Dependence of the residual resistivity of aluminum on the introduced impurity content: ○—initial aluminum, Δ–Al–Mg, ●—Al–Zn, □—Al–Ga.

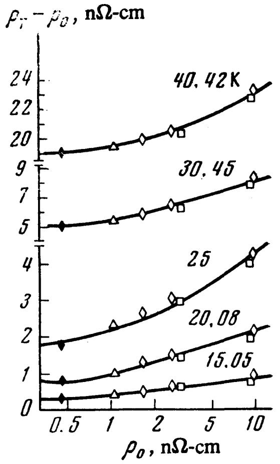

FIG. 2. $\rho_T-\rho_0$ as a function of ρ_0 at fixed temperatures: ♦—initial aluminum, Δ—Al–Mg, ◇—Al–Zn, □—Al–Ga.

The resistances of the samples and of the thermometer were measured by a standard null method using R-348 potentiometers (sensitivity 2×10^{-8} V, measurement current through samples 0.2–0.5 A). The resistance measurement accuracy at low temperatures ranged from 3% for the pure samples to 0.2% for alloys. At high temperatures, the accuracy was 0.5% and was determined mainly by the constancy of the temperature.

The temperature was measured with a "scientific instruments" model 2G (1.5–100 K) germanium resistance thermometer. This thermometer was calibrated in magnetic fields up to 50 kOe in a vacuum adiabatic calorimeter.[13,14]

The sample resistance was measured at fixed temperatures 2; 10.06; 15.05; 20.08; 25; 30.65 and 40.42 °K. At each temperature point, with the exception of 40.42 °K, the measurements were made in magnetic fields 0; 5; 10; 20; 30; 40 and 50 kOe, while at T = 40.42 °K they were made in fields 0, 30, and 50 kOe.

RESISTIVITY

We measured the resistivity of pure aluminum and of weak solutions of Zn and Ga (2–40 K) and Mg (2–100 K) in aluminum with different impurity-atom contents (9 samples). Figure 2 shows the temperature-dependent part of the resistivity $\rho_T-\rho_0$ as a function of the residual resistivity for five fixed temperatures. It is seen that at $\rho_0>1$ nΩ-cm the value of $\rho_T-\rho_0$ is practically independent of the number of impurities. At $\rho_0>1$ nΩ-cm, the Matthiessen rule is violated and $\rho_T-\rho_0$ increases with increasing ρ_0. Good agreement is observed between our results and those cited in[7,15].

The deviation from the Matthiessen rule (DMR) manifests itself most strongly in the temperature dependence of the relative change of the alloy resistivity

$$\frac{\Delta\rho}{\rho_0}=\frac{\rho(c,T)-\rho(c,0)-\rho(0,T)}{\rho(c,0)},$$

where $\rho(c,T)$ is the total resistivity of the alloy, $\rho(c,0)\equiv\rho_0$ is the residual resistivity due to scattering of the electrons by the impurities, and $\rho(0,T)$ is the pure-metal resistivity due to scattering by phonons. Figure

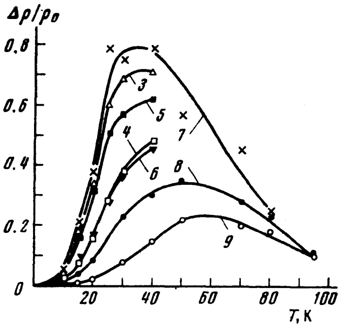

FIG. 3. Deviation from the Matthiessen rule, referred to ρ_0, for samples No. 3–9 (the numbers are marked on the curves).

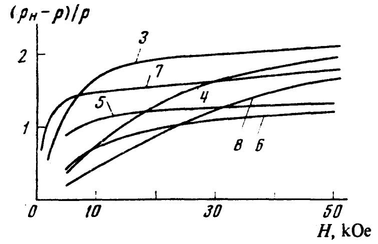

FIG. 5. Transverse magnetoresistance of aluminum at $T = 2\,^\circ$K for samples 3–8 (numbers on the curves).

3 shows the temperature dependence of the relative change of the resistivity of samples of different compositions. A strong nonlinear dependence of $\Delta\rho/\rho_0$ on the impurity concentration is observed, together with a monotonic variation with temperature. All curves are characterized by the presence of a maximum in the region 35–60 °K, and the maximum decreases with increasing impurity content of the atoms, while its position shifts into the region of higher temperatures.

The change of the character of the temperature dependence of the resistivity following introduction of impurities or application of a magnetic field is demonstrated in Fig. 4. It is seen that above 25 °K the resistivity of pure aluminum (sample No. 1) varies like T^5. Introduction of a relatively large number of impurity atoms (~0.04 at.%) leads to a $\rho \propto T^3$ dependence. For alloys with lower impurity content, the power law ranges from T^3 to T^5. When a transverse magnetic field is applied, the temperature-dependent part of the resistivity of the investigated samples depends little on the impurity content and increases with temperature more like T^3 (more accurately, $T^{3.3}$).

Thus, the resistivity of aluminum in a transverse magnetic field and the resistivity of aluminum with sufficiently large impurity content have nearly cubic temperature dependences. The DMR observed in a zero magnetic field is suppressed to a considerable degree by a strong magnetic field.

MAGNETORESISTANCE

The transverse magnetoresistance of polycrystalline aluminum and of weak solutions of magnesium, gallium, and zinc in aluminum was measured in the temperature range 2–40 °K and in magnetic fields up to 50 kOe (samples 1–7).

Figure 5 shows the dependence of $(\rho_H - \rho)/\rho$ on the magnetic field for six alloys: Al–Zn (0.0075 and 0.042 at.%), Al–Ga (0.01 and 0.035 at.%), and Al–Mg (0.0013 and 0.043 at.%). It is seen that at 2 °K, when the resistance is due to scattering by impurities, this quantity depends not only on the content but also on the type of impurity. In Al–Zn alloys the resistance in a 50-kOe field is increased by 3 times, and in Al–Ga alloys by only 2.3 times.

The temperature dependence of the quantity $(\rho_H - \rho)/\rho$ of all alloys in a 50 kOe field is shown in Fig. 6. For Al and Al–Mg and Al–Ga alloys, it first increases with increasing temperature, goes through a maximum between 20 and 30 °K. With increasing impurity content, the maximum decreases in magnitude and shifts towards lower temperatures.

For Al–Zn alloys, a more complicated temperature dependence is observed in the magnetoresistance. With increasing temperature, the value of $(\rho_H - \rho)/\rho$ first decreases, and then increases and goes through a maximum. Here, too, the value of the maximum decreases with increasing impurity content, while the alloy with the maximum zinc content has no maximum at all and $(\rho_H - \rho)/\rho$ decreases monotonically with increasing temperature.

We note that the maxima of $(\rho_H - \rho)/\rho$ occur at lower temperatures than the maxima in the DRM (cf. Figs. 3 and 6).

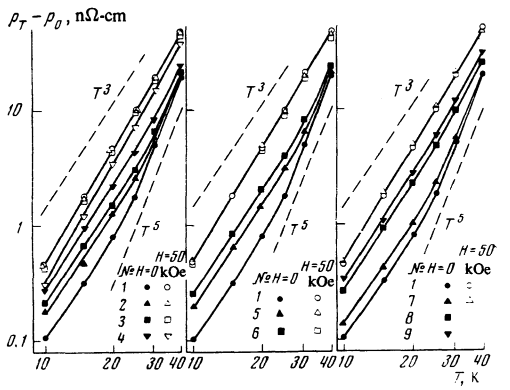

FIG. 4. Temperature-dependent part of the resistivity in a zero magnetic field and in a 50 kOe field, for samples 1–9.

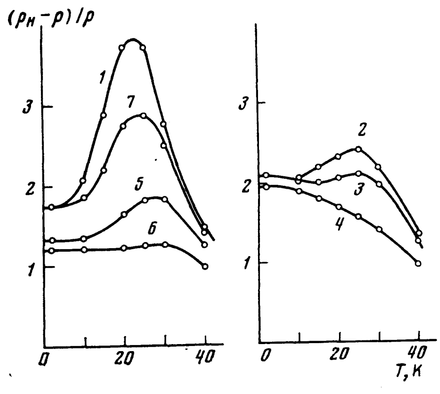

FIG. 6. Value of $(\rho_H - \rho)/\rho$ of samples 1–7 in a field 50 kOe as a function of the temperature.

DISCUSSION OF RESULTS

In the alloys investigated by us the masses of the matrix atoms and of the impurity atoms did not differ by more than a factor of 2. Therefore, the DMR due to the deformation of the phonon spectrum as a result of the difference between the atom masses cannot play a decisive role in these systems.

The nonlinear dependence of the DMR on the impurity concentration and the presence of maxima in the temperature dependence of the DMR can be explained on the basis of the theory of Kagan and Zhernov.[1] The appearance of a maximum in the temperature dependence of the DMR and its shift towards higher temperatures with increasing impurity occur, according to[1], as a result of the change of the anisotropy of the electron distribution function. In a pure metal this anisotropy is caused by the anisotropies of the phonon and electron spectra and by the umklapp processes in the electron-phonon interaction. Introduction of the impurities suppresses the anisotropy of the distribution function via elastic scattering of electrons by the impurity atoms. This unusual interference between the anisotropic phonon and impurity scattering of the electrons gives rise to strong nonlinear dependences of the resistivity on the temperature and on the impurity concentration.

The magnetic field causes the distribution function to become isotropic, and this suppresses these interference effects and makes the temperature-dependent part of the resistance in a magnetic field independent of the impurity content.[2]

It is known that the magnetoresistance of a metal depends both on the geometry of the Fermi surface and on the scattering processes. Owing to the smallness of the impurity-atom concentrations (<0.2%), the change in the geometry of the Fermi surface in the samples investigated by us can be neglected.

The anomalies observed by us in the value of $(\rho_H-\rho)/\rho$ (nonmonotonic temperature dependence, dependence of $(\rho_H-\rho)/\rho$ on the type of impurity) can be explained as being due to features of the scattering of electrons by impurities and phonons, namely by anisotropy of this scattering. The dependence of $(\rho_H-\rho)/\rho$ on the type of impurity can therefore be attributed to differences between anisotropy of the scattering by the impurity atoms.

The temperature dependence of the magnetoresistance can be attributed, as follows from[2], to the anisotropy of the electron–phonon scattering. In alloys, the anisotropy of the electron–phonon scattering is partially suppressed by scattering by impurities, which leads to a decrease of $(\rho_H-\rho)/\rho$. We note that the existence of a minimum of $(\rho_H-\rho)/\rho$ as a function of the temperature was predicted for an impurity that scatters anisotropically. Such a minimum was observed by us experimentally in the alloys of Al with Zn.

Our investigations have thus led to the following results:

1) The magnetoresistance was observed to be dependent of the type of impurity atom.

2) A monochromatic temperature dependence of the quantity $(\rho_H-\rho)/\rho$ was observed. It manifests itself more clearly for pure aluminum. With increasing impurity content, the maximum of $(\rho_H-\rho)/\rho$ decreases and the position of the maximum shifts towards higher temperatures. In investigations of the Al–Zn alloy, a more complicated temperature dependence was observed, having the minimum predicted in[2].

3) The DMR observed in the resistance in a zero field has a strong nonlinear dependence on the impurity content. Superposition of a strong magnetic field suppresses the DMR. This indicates that the decisive role in the onset of the DMR is the isotropization of the nonequilibrium distribution of the electrons in scattering by impurities.

The $\rho \propto T^5$ dependence observed in pure aluminum goes over into a dependence close to T^3 when a large number of impurity atoms is introduced or when a sufficiently strong magnetic field is applied.

5) The observed anomalies in the temperature and concentration dependences of the resistance and of the magnetoresistance are qualitatively explained with the aid of the conclusions of[1,2].

In conclusion, the authors are deeply grateful to Yu. M. Kagan, A. P. Zhernov, and V. N. Flerov for a discussion of the results, to N. A. Chernoplekov for interest in the work, and to A. A. Shikov and V. V. Stepanov for assistance.

[1]Yu. Kagan and A. P. Zhernov, Zh. Eksp. Teor. Fiz. 60, 1832 (1971) [Sov. Phys. JETP 33, 990 (1971)].
[2]Yu. Kagan and V. N. Flerov, Zh. Eksp. Teor. Fiz. 66, 1374 (1974) [Sov. Phys. JETP 39, 673 (1974)].
[3]J. Bass, Adv. Phys. 21, 431 (1972).
[4]M. R. Cimberle, G. Bobel, and C. Rizzuto, Adv. Phys. 23, 639 (1974).
[5]A. D. Bronnikov, E. P. Vol'skiĭ, V. T. Petrashov, L. I. Razgonov, and V. S. Tsoĭ, Zh. Eksp. Teor. Fiz. 67, 722 (1974) [Sov. Phys. JETP 40, 1451 (1975)].
[6]E. S. Borovik, V. G. Volotskaya, and N. Ya. Fogel', Zh. Eksp. Teor. Fiz. 45, 46 (1963) [Sov. Phys. JETP 18, 34 (1964)].
[7]F. R. Fickett, Phys. Rev. B3, 1941 (1971).
[8]Yu. N. Tszyan, V. V. Eremenko, and O. G. Shevchenko, Zh. Eksp. Teor. Fiz. 57, 1923 (1969) [Sov. Phys. JETP 30, 1040 (1970)].
[9]K. Boning, H. J. Fenzl, E. Olympios, J. M. Welter, and H. Wenzl, Phys. Status Solidi 34, 395 (1969).
[10]W. Kesternich, H. Ullmaier, and W. Shilling, Philos. Mag. 31, 471 (1975).
[11]V. N. Morgun, V. I. Khotkevich, G. A. Zaĭtsev, and L. V. Sidorkina, Fiz. Metal. Metalloved. 39, 655 (1975).
[12]F. R. Fickett, Crygonics 11, 349 (1971).
[13]A. A. Shikov, G. Kh. Panova, B. N. Samoĭlov, and N. A. Chernoplekov, Preprint IAE-2603, Moscow, 1976.
[14]G. Kh. Panova, B. N. Samoĭlov, N. A. Chernoplekov, and A. A. Shikov, Preprint IAE-2565, Moscow, 1975.
[15]A. D. Caplin and C. Rizzuto, Aust. J. Phys. 24, 309 (1971).

Translated by J. G. Adashko