Effective surface tension of solids

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An investigation of the Twyman effect in optical glasses, quartz glass, and quartz crystals in the form of thin platelets 1–0.1 mm thick showed that it resulted in a bending two orders of magnitude larger than in previous observations. This makes it possible to carry out accurate measurements and to establish the physical laws governing the effect. It was found that when they bend, plane-parallel platelets ground on one side with abrasive powder and polished on the other side assume an exact spherical form, the polished side being concave. The radius of curvature depends quadratically on the thickness of the platelets and is independent of their dimensions and shape. The concept of an effective surface tension is introduced for solids and a phenomenological theory of it is presented.

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1. INTRODUCTION

In 1905 F. Twyman observed an interesting phenomenon involving the bending of thin glass platelets polished on one side and ground on the other[1]; this was later given the name of the Twyman effect.[2] However, in these and later investigations[3,4] neither the form of the bending, nor the exact physical law to which the phenomenon is subject were established.

In electrostatic electrometer investigations[5,6] we independently came upon an analogous phenomenon in very thin glass platelets,[7] and this resulted in our taking a deeper interest in the effect and undertaking a detailed investigation of it.

2. EXPERIMENTAL RESULTS

In experiments with thin platelets of optical and quartz glass 1–0.1 mm thick it was established that when ground on one side and polished on the other they bend and assume the shape of an accurately spherical surface, the polished surface being concave. For a given type of glass the radius of this surface depends on the thickness of the platelet according to a strictly defined law. Preliminary experiments with thin platelets made from quartz crystals by the same technology showed that bending is also observed in them, the form of the curved surface depending on the orientation of the crystal face concerned—in the case of the (0001) face it is strictly spherical, as in the case of optical glasses

The radius of curvature of the platelets was determined by measuring the diameters of the Newton rings produced by multiple reflection of monochromatic light between the polished spherical surface of the platelet and a glass optical flat or a spherically concave standard with a suitable radius of curvature. This interference method allows the radius of curvature of different spherical segments of a glass platelet to be determined.

In the experiments, strict reproducibility of the spherical bending was found in platelets of different shape, size, and thickness made of optical glasses. Tests showed that the radius of the spherical surface of the platelets does not depend either on their shape or their size, but only on the thickness of the platelets and the modulus of elasticity of the material concerned. In addition, the method of measuring the Newton interference rings showed that the radius of curvature is strictly constant for all segments of the platelet surface, and the bending is consequently strictly spherical.

The results on the dependence of the radius of curvature of platelets R on their thickness h for round platelets of diameter $2r = 15$ mm made of quartz glass and of the optical glasses K8, BK6, and TF5 are shown in the figure on a double-logarithmic scale (straight lines 1–4). The straight line 5 is drawn from the experimental data for round platelets with the same diameter $2r = 15$ mm made from a quartz crystal ((0001) face). As is evident from Fig. 1, the variation of R with h is strictly linear on a logarithmic scale, the slope of the straight lines being the same for all the optical glasses investigated and for the (0001) face of a quartz crystal. From these experimental curves it was established that the $R(h)$ dependence is a quadratic parabola for $E = \text{const.}$:

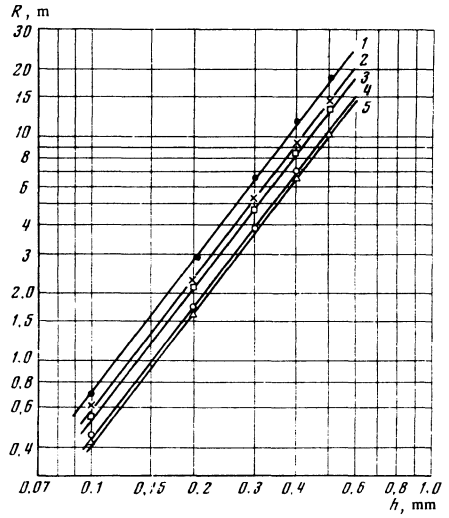

FIG. 1. The dependence of the radius of curvature R of platelets on their thickness h: 1—quartz glass, 2—K8 glass, 3—BK6, 4—TF5, 5—quartz crystal ((0001) face).

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$$R=ah^2, \tag{1}$$

the constant a depending on the material. Its value is given below:

Material:	Quartz glass	K8	BK6	TF5	Quartz crystal
$a\cdot 10^{-5}$, cm^{-1}	7.0	5.8	5.3	4.3	4.2

3. THEORETICAL CONCLUSIONS

We suppose that the observed effect is caused by a difference in the internal stresses in the thin layer of the surface which have been differently treated. Since the effect of these stresses is analogous to that of a true surface tension, we are justified in terming it the effective surface tension α_{eff}. When the two surfaces of a thin glass platelet are differently treated, an excess surface energy U_s referred to one surface of the platelet arises which is proportional to its surface area S:

$$U_s=\alpha_{\text{eff}}S. \tag{2}$$

When the platelet bends, the change in energy ΔU_s is proportional to the change in the area of the platelet ΔS:

$$\Delta U_s=\alpha_{\text{eff}}\Delta S. \tag{3}$$

The coefficient α_{eff} is determined from the condition that the total energy of the platelet is a minimum or that the effective force producing the deviation of the platelet from the equilibrium curvature is equal to zero. The total energy U of the platelet can be represented as the sum of the elastic deformation energy of the platelet U_e and the energy of the polished surface U_s. The energy U_s can be represented as the sum of the energy $U_{s,\text{const}}$ in the unbent state and the change in energy ΔU_s due to the bending and to the change in surface area by ΔS determined from (3):

$$U_s=U_{s,\text{const}}+\Delta U_s. \tag{4}$$

The total energy of the platelet U will be equal to

$$U=U_{s,\text{const}}+\Delta U_s+U_e. \tag{5}$$

For the energy of elastic deformation U_e of a spherically curved round platelet with a constant radius of curvature R

$$U_e=\frac{E}{1-\mu}\frac{S_c h^3}{12}x^2, \tag{6}$$

can easily be derived from Eq. (47) in the book of Timoshenko and Krieger (Ref. 8, page 61), where x is the curvature of the platelet, E is Young's modulus, μ is the Poisson coefficient, $S_c=\pi r^2$ is the area of the neutral central plane of the platelet.

From the condition that the total energy U, defined by formula (5), should be a minimum, we obtain on differentiating with respect to x:

$$\frac{dU}{dx}=0=\alpha_{\text{eff}}\frac{d\Delta S}{dx}+\frac{E}{1-\mu}\frac{S_c h^3}{6}x. \tag{7}$$

From simple geometrical considerations it is evident that $\Delta S/S_m=-h/R=-hx$, or

$$\Delta S=-S_c hx. \tag{8}$$

On substituting (8) in (7) we get

$$\frac{dU}{dx}=0=-\alpha_{\text{eff}}S_c h+\frac{E}{1-\mu}\frac{S_c h^3}{6}x,$$

from which

$$\alpha_{\text{eff}}=\frac{1}{6}\frac{E}{1-\mu}h^2x=\frac{1}{6}\frac{E}{1-\mu}\frac{h^2}{R}. \tag{9}$$

Finally, on comparing (9) with the experimentally derived law (1) for the dependence of R on the platelet thickness h we get:

$$\alpha_{\text{eff}}=\frac{1}{6a}\frac{E}{1-\mu}. \tag{10}$$

As is evident from (10), the effective surface tension α_{eff} we introduced above depends only on the material characteristics E, μ, and a, and not on the geometrical dimensions of the samples investigated, and it is consequently a material constant. It can easily be determined by measuring the radius of spherically curved think platelets made of different optical glasses and (0001) quartz crystals using formula (9). The values obtained in this way are given in Table I together with the values of other physical constants for the optical glasses and (0001) quartz plates investigated by us.

For greater simplicity we have so far assumed that α_{eff} = const. and does not depend on the curvature $x=1/R$. It is possible that $\alpha_{eff}=f(1/R)=f(x)$, leading to more complicated theoretical conclusions and requiring more refined experimental investigations.

4. CONCLUSION

The effective surface tension quantity introduced in this paper therefore possesses real physical and technical meaning and can probably be used in analyzing phenomena in which surface forces play a significant role, e.g., in the stressing of thin filaments, platelets, etc., and in other cases.

Particularly interesting results are obtained in the case of crystalline platelets. On the one hand, they suggest a new method of investigating anisotropy in crystals, and on the other, they offer a new method of making

TABLE I.

Material	$E\cdot 10^{-11}$, dyn/cm^2	μ	$\alpha_{eff}\cdot 10^{-5}$, dyn/cm
Quartz glass	5.9	0.33	2.10
K8	8.1	0.209	2.94
BK6	7.0	0.232	2.87
TF5	5.4	0.244	2.77
Quartz crystal ((0001) face)	7.3	0.135	3.35

curved crystalline diffraction gratings for x rays which may have considerable advantages over cylindrical ones bent by the mechanical method of Johann, Cauchois, *et al.* (horizontal focusing) and by the method of Kunzle *et al.* (vertical focusing). These questions will be the subject of further investigations by us.

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Resistance of pure aluminum and of weak solutions of Mg, Zn, and Ga in Al in the region 2–40°K

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The temperature and concentration dependences of the resistivity and of the transverse magnetoresistance of polycrystalline aluminum and weak solutions of magnesium, zinc, and gallium in aluminum were investigated experimentally at temperatures 2–40°K in magnetic fields up to 50 kOe. Deviations from the Matthiessen rule and an anomalous behavior of the magnetoresistance were observed. It is noted that the magnetoresistance depends on the type of impurity atom and has a nonmonotonic temperature dependence. It is shown that the observed anomalies can be attributed to anisotropy of the scattering by phonons and by impurity atoms.

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1. INTRODUCTION

Kagan *et al.*[1,2] have shown that the observed anomalies in the behavior of the resistivity and magnetoresistance of irregular metals are of like nature and have a common explanation. The principal feature of that theory is that the character of the distribution of the nonequilibrium electrons on the Fermi surface in metals without impurities has a sharply pronounced anisotropy. The latter is connected with umklapp processes, with nonsphericity of the Fermi surface, and with anisotropy of the phonon spectrum. It was suggested[1,2] that both the impurities and the magnetic field make the distribution of the nonequilibrium electrons isotropic. This explains the anomalous dependence of the resistivity and of the magnetoresistance on the temperature and on the impurity concentration.

The singularities of the temperature and concentration dependences of the resistivity of metals with impurities have by now been investigated experimentally in considerable detail.[3,4] As to measurements of the magnetoresistance, notice should be taken here of the interesting and unexpected results obtained in[5–8] for aluminum and indium. Until recently, however, there were no investigations of the magnetoresistance of aluminum with definite impurities. Investigations in strong magnetic fields were carried out either on samples of unequal purity, with unidentified purities,[6–8] or on samples having defects of the vacancy or dislocation type, obtained by quenching or by irradiation.[9–11] The reason is that introduction of a sufficient amount of impurities into a pure metal contradicts the requirement that the electron have a large free path, and the latter must be satisfied if the conditions of strong magnetic fields are to be obtained ($\omega\tau \gg 1$, where ω is the cyclotron frequency and τ is the relaxation time).

We have investigated the temperature and concentration dependences of the resistivity ρ and of the transverse magnetoresistance ρ_H of aluminum and of weak solutions of Mg, Zn, and Ga in aluminum at temperatures 2–40 °K and in magnetic fields up to 50 kOe. The high purity of the initial aluminum ($R_{295\,\mathrm{K}}/R_0 = 5900$) and the possibility of working in fields up to 50 kOe have enabled us to investigate weak solutions of Mg, Zn, and Ga in Al under conditions of sufficiently strong magnetic fields ($\omega\tau > 2$).

SAMPLES AND EXPERIMENTAL PROCEDURE

We investigated samples of pure Al (99.999%) with resistance ratio $R_{295\,\mathrm{K}}/R_0 = 5900$ and weak solutions of Mg, Zn, and Ga in Al. The characteristics of the samples are listed in Table I (R_0 and ρ_0 are the resistance and resistivity in the absence of a magnetic field at $T = 2$ °K).

