of such a turbulence in the long-wavelength region of the spectrum in resonance with the beam is small in the ratio of the corresponding phase volumes.

$$\Delta=\left(\frac{T_e}{mv_0^2}\right)^{1/2}\frac{T_e}{T_{\perp b}}\frac{\Delta v}{v_0}\left(\frac{n_0T}{W}\right)^{3/2}$$

($T_{\perp b}$ is the transverse temperature in the beam). Assuming that $\Delta\ll E_0^2/16\pi W$ we shall in the present paper not take into account the effect of the trapped plasmons on the dynamics of the beam.

2) According to[18] the development of the collapse leads to the excitation of rather strong sound turbulence in the plasma. However, the conversion of plasma noise in resonance with the beam into density fluctuations which are produced by sound does not contribute greatly to the total energy balance as the characteristic growth rate of such a process cannot exceed the growth rate of the modulational instability $\gamma_M\sim\omega_p(mW/Mn_0T)^{1/2}$, and hence remains considerably smaller than ν_{eff}.

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The establishment of a stationary turbulence spectrum due to induced scattering of waves by particles

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We consider the problem of the evolution of the turbulence spectrum, taking into account the processes of creation, destruction, and induced scattering of waves. We show that any initial distribution of waves relaxes to a stationary distribution. We estimate the relaxation time.

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In studying plasma turbulence one often has to deal with the following equation for the occupation numbers $n(\mathbf{k}, t)$:

$$\frac{\partial}{\partial t}n=2\gamma_{\mathbf{k}}n+n\int A(\mathbf{k};\mathbf{k}')n(\mathbf{k}';t)d^3\mathbf{k}'+\varepsilon_{\mathbf{k}}. \tag{1}$$

The problems of the spectra of the turbulence excited by beam or parametric plasma heating (see, e.g., [1–4]) and also some problems in non-linear optics,[5,6] in particular, reduce to the solution of this equation. The first term on the right-hand side of Eq. (1) describes the induced emission and absorption processes of waves (depending on the sign of the growth rate $\gamma_{\mathbf{k}}$), and the second the processes of induced scattering of waves by particles. The scattering probability is characterized by the kernel $A(\mathbf{k}, \mathbf{k}')$. The actual form of the kernel is unimportant for what follows. Essential is only that the number of quanta is conserved in the scattering. A formal expression of this fact is the antisymmetry of the kernel with respect to the interchange of the arguments $\mathbf{k}$ and $\mathbf{k}'$:

$$A(\mathbf{k};\mathbf{k}')=-A(\mathbf{k}';\mathbf{k}).$$

The third term on the right-hand side of (1) is the intensity of the thermal noise source.

We assume that Eq. (1) has a stationary solution and we consider the problem of how this stationary solution

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is established. It is possible in a number of cases (see[2,4]) to find the stationary solution (we denote it by $n_\infty(\mathbf{k})$) analytically. From the definition of $n_\infty(\mathbf{k})$ it follows that

$$n_\infty\left[2\gamma_{\mathbf{k}}+\int A(\mathbf{k};\mathbf{k}')n_\infty(\mathbf{k}')d^3\mathbf{k}'\right]+\varepsilon_{\mathbf{k}}=0.$$

Hence we find the instability growth rate $\gamma_{\mathbf{k}}$ and we substitute $\gamma_{\mathbf{k}}$ into Eq. (1). As a result we get

$$\frac{\partial}{\partial t}n=-\varepsilon_{\mathbf{k}}\frac{n-n_\infty}{n_\infty}+n\int A(\mathbf{k};\mathbf{k}')[n(\mathbf{k}';t)-n_\infty(\mathbf{k}')]d^3\mathbf{k}'.$$

We multiply both sides of this equation by $(n-n_\infty)/n$ and integrate over $\mathbf{k}$. Using the antisymmetry of the kernel $A(\mathbf{k},\mathbf{k}')$ we have

$$\frac{\partial}{\partial t}I=-\int\varepsilon_{\mathbf{k}}\frac{(n-n_\infty)^2}{nn_\infty}d^3\mathbf{k}, \tag{2}$$

where

$$I=\int\left(n-n_\infty-n_\infty\ln\frac{n}{n_\infty}\right)d^3\mathbf{k}. \tag{3}$$

The right-hand side of Eq. (2) vanishes only when the spectrum $n(\mathbf{k},t)$ coincides with the stationary one. For any other distribution of waves it is less than zero.[1)] In the non-stationary case the integral I therefore necessarily decreases with time. We note now that this integral is bounded from below as for any values $x>0$ the inequality $x-1-\ln x\geqslant 0$ holds. The minimum of the integral I is reached for the stationary distribution of waves.

It follows from what we have said that for any initial condition the solution of Eq. (1) tends to a stationary state (provided, of course, that a stationary state exists). The approach to a stationary state for small deviations from it has been shown earlier in[4].

Condition (2) enables us to obtain a useful estimate for the maximum number of quanta existing at any moment of the change to the stationary state. To find this we write down the following relation:

$$I\equiv N-N_\infty-\int n\frac{n_\infty}{n}\ln\frac{n}{n_\infty}d^3\mathbf{k}\leqslant I_0, \tag{4}$$

where the quantity I_0 corresponds to the initial distribution of quanta while N and N_∞ denote, respectively, the total number of quanta at time t and in the stationary state (we assume that the number of quanta in the stationary state is finite). For the integral occurring in Eq. (4) we can write down the following chain of inequalities:

$$\int n\frac{n_\infty}{n}\ln\frac{n}{n_\infty}d^3\mathbf{k}\leqslant\int n\left[\max_{\mathbf{k}}\left(\frac{n_\infty}{n}\ln\frac{n}{n_\infty}\right)\right]d^3\mathbf{k}\leqslant\frac{1}{e}\int n\,d^3\mathbf{k}=\frac{N}{e}. \tag{5}$$

Combining Eqs. (4) and (5) we find that

$$N\leqslant\frac{e}{e-1}(I_0+N_\infty). \tag{6}$$

We turn now to an estimate of the characteristic relaxation time τ. It is natural to introduce this time as follows:

$$\tau=\int\left(n-n_\infty-n_\infty\ln\frac{n}{n_\infty}\right)d^3\mathbf{k}\Big/\int\varepsilon_{\mathbf{k}}\frac{(n-n_\infty)^2}{nn_\infty}d^3\mathbf{k}. \tag{7}$$

A simple study of Eq. (7) at the maximum shows that the quantity τ is a maximum in the final stage of the relaxation process when n and n_∞ are close to one another. Therefore

$$\tau_{max}\approx\int n_\infty\left(\frac{n}{n_\infty}-1\right)^2d^3\mathbf{k}\Big/2\int\varepsilon_{\mathbf{k}}\left(\frac{n}{n_\infty}-1\right)^2d^3\mathbf{k}\leqslant\frac{1}{2}\max_{\mathbf{k}}\frac{n_\infty}{\varepsilon_{\mathbf{k}}}. \tag{8}$$

It is clear from Eq. (8) that for a given spectral function n_∞ the time τ_{max} is inversely proportional to the power of the thermal noise source. In fact, τ_{max} is equal to the time during which the noise source "pumps" into each region of $\mathbf{k}$-space just as many quanta as should be there in the stationary state. Due to the low intensity of the noise source τ_{max} is usually considerably longer than the characteristic time for the growth of oscillations due to the instability, which equals $\gamma_{\mathbf{k}}^{-1}$. We can thus distinguish two stages in the relaxation process. Initially the nonlinear stabilization of the instability is reached fast (after a time of the order of 10 $\gamma_{\mathbf{k}}^{-1}$) and a "rough" condition for balance between pumping and damping is ensured to hold. In the second (longer) stage the details of the distribution of the waves over the spectrum become established.

In conclusion we note that all the results given here can easily be extended to the case when scattering may lead to the transformation of the initial waves into waves of another kind. An example of such a process is, say, the scattering of Langmuir waves by ions in the plasma in which they are changed into electromagnetic waves. The generalization of Eq. (2) to the case of several kinds of waves $(n_i(\mathbf{k};t),\ i=1,2,\dots)$ has the following form

$$\frac{\partial}{\partial t}\sum_i I_i=-\sum_i\int\varepsilon_i(\mathbf{k})\frac{(n_i-n_{\infty i})^2}{n_in_{\infty i}}d^3\mathbf{k},$$

where the quantities I_i are introduced by using Eq. (3), viz.,

$$I_i=\int\left(n_i-n_{\infty i}-n_{\infty i}\ln\frac{n_i}{n_{\infty i}}\right)d^3\mathbf{k}.$$

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1)To avoid confusion we point out that $\varepsilon_{\mathbf{k}}$, $n(\mathbf{k},t)$, and $n_\infty(\mathbf{k})$ are positive definite functions. The introduction of negative occupation numbers has a sense only for waves with a negative energy.[7] In the present paper such waves are not considered.

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Charge movement in crystalline helium

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The mobilities of the carriers in crystalline helium are determined by investigating the diode volt-ampere characteristics and the temperature dependence of the currents induced by a tritium source. The diode current is found to be proportional to the square of the voltage in a comparatively narrow range of voltages and temperatures. In the range of high voltages, the form of the volt-ampere characteristics is determined by the dependence of the drift velocity of the charges on the electric field strength. For direct measurement of the carrier drift velocity, a three-electrode time-of flight method is employed. The temperature dependences of the positive and negative carrier mobilities are measured at different molar volumes. The dependence of the carrier drift velocity on the electric field strength is also measured. The data are compared with the results of existing theoretical studies.

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The first experiments on the movement of charges in solid helium were made by Shal'nikov[1] and were continued in Refs. 2 and 3. In these studies specially grown helium crystals served as the first object of the investigation. (An attempt at current measurement in polycrystalline samples obtained by the blocked capillary method showed that the currents in this case were not amenable to measurement.[4]) In Refs. 1–3, the helium crystals were grown from the liquid phase at constant pressure in a cylindrical glass container. The growth rate was determined by the temperature gradient along the container axis. The growth process could be monitored visually. The decrease in the induced diode current during the solidification process was the criterion of the quality of the crystal sample. Thus, in the solid phase, the current amounted at most to 2–3% of the current in liquid helium for the worst samples and reached 60% for the better samples. Usually, the better samples were obtained at a growth rate of ~5 μm/sec.

The described method of obtaining the crystals was used by Mezhov-Deglin in the study of the thermal conductivity of solid He^4.[5] The record values of thermal conductivity measured by him, up to 50 W/cm-deg, and the absence of an "annealing" effect, indicate that the samples obtained under these conditions were very nearly perfect single crystals. A direct verification of this fact was obtained in Refs. 6 and 7 by x-ray analysis.

The purpose of the present work was the study of the process of charge transfer in the He^4 crystals obtained by the described method. The research followed two basic directions:

1) measurement of the currents induced in the diode;

2) measurement of the charge velocity by a three-electrode time-of-flight method.

CONSTRUCTION OF THE APPARATUS

The construction of the apparatus used in the present research and the method of growing the crystals were described in detail in Refs. 2 and 5. The crystals were grown in containers of two types: glass (diameter 10 mm) and metal (diameter 40 mm). Inside each of them, depending on the goal of the experiment, was placed a diode or triode electrode system. The construction of the glass container with a small triode inside is shown in Fig. 1a. Rings of ferrochrome (alloy N47KhB) were

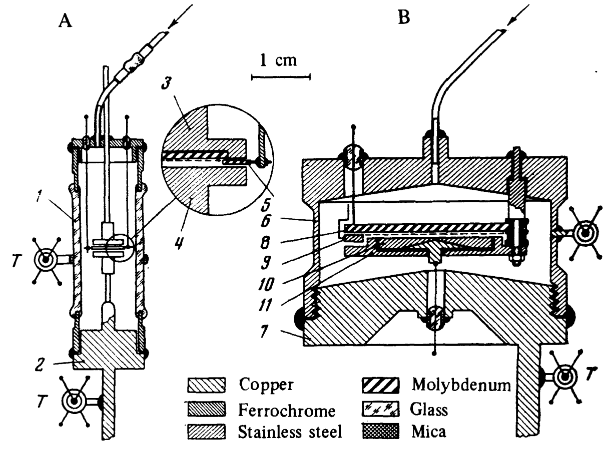

FIG. 1. Construction of containers and triodes. 1, 6—housing; 2, 7—bottom with cold conductor; 3, 8—β-active electrode; 4, 10—collector; 5, 9—grid; 11—guard ring; T—thermometers.

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