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Interaction between a beam with a supercritical initial velocity and finite-amplitude waves

A. S. Bakaĭ, S. S. Krivulya, I. N. Onishchenko, and V. D. Fedorchenko

Physico-technical Institute, Ukrainian Academy of Sciences
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Results are presented of an investigation of the instability of finite-amplitude waves in a system consisting of an electron beam with a supercritical velocity and a bounded plasma. It is shown that the beam with the supercritical velocity ($v_b > v_c \approx \omega_p / k_\perp$) can transfer up to 60% of its energy to the wave. Results are presented of a computer simulation of the amplification process and compared with the results of experiment.

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The instability of waves of finite amplitude in a system consisting of an electron beam with a supercritical velocity and a bounded plasma was discovered theoretically in[1] and then observed experimentally in[2]. The theoretical analysis shows that beams with supercritical velocities, in the course of their interaction with an amplifiable wave, transfer to the wave a considerably higher energy than do beams of the same density, but with subcritical velocities, as a result of which beams with supercritical velocities are more efficient means of wave amplification. Therefore, the investigation of the development of the instability of waves with finite amplitudes in such systems is of interest both for plasma physics and for practical application.

The present report contains the results of a further study of the process of interaction of finite-amplitude waves with beams whose initial velocities exceed the critical velocity, which is equal to the maximum phase velocity of the wave in the bounded plasma.

1. It is well known that waves with infinitely small amplitudes are not intensified by a beam in a bounded plasma if the initial velocity of the beam exceeds the maximum possible phase velocity of the plasma waves. This can easily be seen from the dispersion equation for the waves. In the case of magnetized electrons ($\omega_H > \omega_p$, where ω_H is the electron cyclotron, and ω_p the electron plasma, frequency), the electron motion can be assumed in the description of the potential oscillations to be one-dimensional. In this case the dispersion equation has the following form (see[3]):

$$\frac{\omega_b^2}{(\omega - k_\parallel v_b)^2} + \frac{\omega_p^2}{\omega^2} = 1 + \frac{k_\perp^2}{k_\parallel^2}, \qquad (1)$$

where v_b is the beam velocity; $\omega_b^2 = 4\pi e^2 n_b / m$, n_b being the beam density; e, m are the electron charge and mass; $k_\parallel$ and $k_\perp = \pi/d$ are the longitudinal and transverse components of the wave vector, d denoting the transverse dimensions of the plasma.

This equation does not possess complex solutions when $v_b > v_c \approx \omega_p / k_\perp$. This means that waves with infinitely small amplitudes are not intensified by a beam in a bounded plasma if the initial velocity of the beam exceeds the maximum possible phase velocity, v_c, of the plasma waves.

As shown in[1], the system, which is stable in the linear approximation, loses its stability if the amplitude of an initial perturbation exceeds some critical value at which the wave begins to capture the beam electrons. The value of the critical amplitude depends on the velocity of propagation of the bare wave in the system.

As is well known, the characteristic time, τ, during which the velocity of a particle can change significantly under the influence of the field of a wave with amplitude φ_0 is equal in order of magnitude to

$$\tau \sim k_\parallel^{-1} (e\varphi_0/m)^{-1/2}, \qquad (2)$$

if the initial particle velocity is not very far from the phase velocity of the wave. Therefore, if the amplitude of the bare wave in the course of the wave's excitation increases for a period of time much shorter than τ, then the particle velocities do not have time to change appreciably during this period, and the beam can be considered to be monoenergetic. Since the velocities of the beam particles at the initial moment exceed the

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phase velocity of the wave, the particles will, as a result of the interaction with the wave, be slowed down on the average, transferring energy to the wave. The capture of the particles will occur at the initial moment if the amplitude of the bare wave exceeds the value determined by the relation

$$e\varphi_c = {}^1/_2 m(v_b - v_c)^2. \tag{3}$$

An appreciable intensification of the wave and the subsequent capture of the beam electrons occur at somewhat smaller initial amplitudes, but then the beam will be nonmonoenergetic by the time the capture of the beam electrons begins.

In an experiment, the regime of instantaneous excitation of the bare wave is attained when a monoenergetic beam is injected into a plasma in which a plasma wave of the requisite amplitude has been excited from external sources.

Another regime of excitation of the bare wave—an adiabatic regime in which the amplitude of the wave increases for a period of time much longer than τ—is also possible. In this case the particle velocities have time to change appreciably during the wave-excitation time. It can be shown that, up to the moment of capture, the adiabatic invariant of the particles,

$$I = \frac{k_\parallel m}{2\pi} \int\limits_x^{x+2\pi/k_\parallel} v\,dx, \tag{4}$$

is conserved. Hence, all the beam particles have one and the same total energy

$${}^1/_2 mv^2 + e\varphi(\xi) = W(I, t), \quad \xi = x - v_{ph} t. \tag{5}$$

The relations (4) and (5) determine the phase trajectories of the particles, and it follows from them that the capture of the beam particles (virtually all at the same time) occurs when the wave amplitude attains the value

$$\varphi_{c1} = \frac{\pi^2 m}{16e}(v_b - v_{ph})^2. \tag{6}$$

In a bounded system with a beam injected from without, the adiabatic excitation of the wave can be achieved in a slow spatial intensification of the wave along the length of the system.

Let us estimate the energy that can be lost by a beam with a supercritical velocity upon its capture by a wave. The velocity of a captured particle varies in the interval $2v_{ph} - v_b < v < v_b$, the mean velocity of the particle being equal to v_{ph}. Initially, when the phases of the particles are not highly mixed, the upper bound of the energy that can be given up by the particles in the course of their retardation is estimated by the value

$$\Delta\mathscr{E} = {}^1/_2 mn_b[v_b^2 - (2v_{ph} - v_b)^2] = 2mn_b v_{ph}(v_b - v_{ph}). \tag{7}$$

In the case of strong phase mixing of the particles, when the mean velocity of the particles that have interacted is equal to v_{ph},

$$\Delta\mathscr{E} = {}^1/_2 mn_b(v_b^2 - v_{ph}^2). \tag{8}$$

The actual maximum value of the energy transferable by the beam to the wave, a value which determines the maximum attainable value of the wave amplitude:

$$\max\varphi = \max\,(8\pi\Delta\mathscr{E}/k_\parallel^2)^{1/2}, \tag{9}$$

lies between the values determined by (7) and (8).

From (7) and (8) it can be seen that, as in the case of wave intensification by a beam with a subcritical velocity, the energy that can be lost by the beam is proportional to the difference $v_b - v_{ph}$. In the case of a beam with a subcritical velocity, $v_b - v_{ph} \leq (n_b/n_0)^{1/3} v_{ph}$, and the fraction of energy losable by the beam is comparatively small. The velocity of a supercritical beam is limited only by the magnitude of the initial wave amplitude φ_0. As follows from (6), in this case $v_b - v_{ph} \leq (16e\varphi_0/\pi^2 m)^{1/2}$. Consequently, the energy losses of a beam with a supercritical velocity can exceed by a considerable factor the energy losses for the case of a beam with a subcritical velocity.

Comparing (7) and (8), we see that the upper and lower bounds of the energy that can be lost by the beam differ by roughly a factor of two. In reality, not all the beam particles can be captured in the course of the evolution of the wave, and then in (8) the mean beam density n_b should be replaced by the value of the density of the captured particles.

2. Since the analytical description of the processes of wave evolution meets with great difficulties, it is possible to investigate the instability of an electron beam in a bounded plasma in the case of $v_b > v_c$ and a large initial amplitude of the perturbation only in a computer simulation of the process. We used the method of partial simulation,[4] when the plasma-electron oscillations are assumed to be linear and analyzed analytically, while the nonlinear motion of the beam particles is simulated by macroparticles. Applying the harmonic-analysis method to the Poisson equation, and using the dimensionless units of[5] with the formal replacement $k \to k_\parallel$, we obtain for the dimensionless amplitude, ε, and the phase, α, of a perturbation that increases with the dimensionless distances ζ the following equations:

$$\frac{d\varepsilon}{d\zeta} = \frac{1}{N}\sum_{i=1}^{N} \sin 2\pi\tau_i(\zeta_i, \tau_i^0),$$
$$\varepsilon\left(\frac{d\alpha}{d\zeta} + \Delta\right) = \frac{1}{N}\sum_{i=1}^{N} \cos 2\pi\tau_i(\zeta_i, \tau_i^0), \tag{10}$$

while the equation of motion of the macroparticles can be written in the form

$$\frac{d^2\tau}{d\zeta^2} = -\frac{1}{2\pi}\varepsilon\sin(2\pi\tau + \alpha). \tag{11}$$

Notice that, in contrast to[4,5], Eqs. (10) and (11) were solved for large initial amplitudes ε_0 and large detunings Δ. The dimensionless quantities entering into Eqs. (10) and (11) are similar to those given in[5], and have the form

$$\varepsilon = \frac{E(z)}{[4\pi n_b m v_b^2 (n_b v_b / n_0 v_{ph})^{1/3} (v_b / v_{ph})]^{1/2}},$$
$$\zeta = \frac{\omega_p}{v_b}\left(\frac{n_b v_{ph}}{n_0 v_g}\right)^{1/3} z, \quad \tau = -\frac{\omega}{2\pi}\left(t - \frac{z}{v_b}\right), \tag{12}$$

where $E(z)$ is the perturbation amplitude:

$$E(z, t) = E(z) \exp\{i[\omega(z/v_b - t) + \alpha(z)]\}.$$

Equations (10) and (11) were integrated on an electronic computer by the Runge-Kutta method for $N = 100$ particles. As a result, we have obtained curves of the amplitude function $\varepsilon(\zeta)$ for different values of the initial amplitude ε_0 and detuning Δ. Some of them are shown below in Fig. 3. The results of the computer simulation are discussed and compared with the experimental results below.

The experiments were performed on an installation consisting of a plasma cylinder of diameter 2.2 cm and length 100 cm located in a conducting casing of diameter 5 cm. The entire system was located in a uniform longitudinal magnetic field having an intensity of up to 1000 Oe.

At one end of the installation is located a hf plasma source operating in the continuous regime. The source of the hf power is a magnetron operating at a frequency of 2400 MHz with an adjustable power of between 20 and 150 W. If we choose the magnetic field such that the frequency of the generator is close to the electron cyclotron frequency in this field, then a hf discharge is easily ignited in the source and a plasma flows out from it along the magnetic lines of force, forming a plasma cylinder with a uniform density along its radius and length, with the exception of small regions at the ends. By varying the rate of gas inflow, the power inflow from the magnetron, and the magnetic field, we can vary the plasma density within quite broad limits.

The above-described source allows us to vary independently the density of the plasma, whereas in the production of a plasma through binary collisions of the electrons of a beam with a residual gas the variation of the beam parameters or the presence in the system of a hf field of sufficiently large amplitude entails the variation of the plasma density.

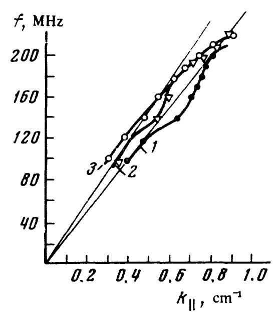

FIG. 1. Dispersion dependences: 1) plasma waveguide with a beam with $v_b = 1.6 \times 10^7$ m/sec; 2) plasma waveguide with a beam with $v_b = 1.9 \times 10^7$ m/sec; 3) plasma waveguide without a beam.

At the other end of the installation is located an electron gun operating in the continuous regime and producing an electron beam of annular cross section, the diameter and thickness of the ring being respectively 16 mm and 2 mm. The energy of the beam electrons can be varied from 0 to 2500 eV, the current up to 10 mA. Located in front of the electron gun is a high-transmission wire gauze. By feeding a hf signal from an external source to the gauze, we can excited electromagnetic waves in the plasma cylinder. The variation of the wave amplitude along the plasma column can be monitored with the aid of a movable probe, which is a radial antenna (not in contact with the plasma) matched with a transmission line. Beyond the plasma source there is a collector for the reception of the electron beam and beyond it, a multigrid energy analyzer. The results of the measurement of the energy distribution functions of the beam electrons are given in[2].

Usually, the operating parameters were as follows: the power of the magnetron was 50 W, the pressure of the working gas was $2-3 \times 10^{-5}$ Torr, the magnetic-field intensity was 900 Oe, the plasma density was 7×10^8 cm^{-3}, and the plasma-electron temperature was in the range 5–10 eV. The magnetic-field intensity was chosen from the condition for cyclotron resonance with the magnetron frequency, while the plasma density was chosen such that the condition $\omega_H > \omega_p$ was fulfilled. Under this condition the electrons are magnetized, and their motion can be assumed to be one-dimensional, which allows us to use the one-dimensional theory in the description of the results of the experiment.

As can be seen from (6) and (7), the phase velocity of the excitable wave is an important dynamical characteristic. This quantity has been measured by comparing the phase of a signal emanating from the plasma and picked up by the probe with the phase of the pedestal signal for different initial beam velocities and incoming-signal frequencies. The measured dispersion curves are shown in Fig. 1. The curves 1) and 2) correspond to beams with subcritical velocities (the thin lines represent the straight lines $w = kv_{b1,2}$). Because of the fact that in this case the beam (unstable) branch of the oscillations is intensified, it is not possible to record the branch consisting of the stable space-charge waves. "Sags" can be seen in the curves 1) and 2) at the places of intersection of the dispersion curves of the beam branch and the space-charge wave branch.

When the beam velocity is higher than the critical velocity, the external signal propagates, attenuating slightly, in the plasma, and we register the dispersion of the space-charge waves (the curve 3). The same dispersion curve is also obtained in the absence of a beam, which indicates a weak influence of the beam *on the dis*persion of the space charge waves.

In the subsequent experiments, the operating frequency (the frequency of the incoming signal) was chosen such (180 MHz) that it lay in the region of maximum intensification in the case of beams with subcritical veloc-

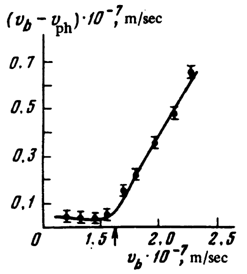

FIG. 2. Dependence of $(v_b - v_{ph})$ on v_b. At the point marked by the arrow $v_b = v_c$.

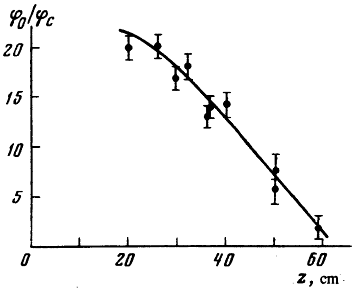

FIG. 4. Dependence of the distance over which the maximum of the wave amplitude is attained on the initial signal amplitude.

ities. Since, as can be seen from (7) and (8), the energy that can be lost by the beam is determined by the difference between the initial velocity of the beam and the phase velocity, we measured this quantity for different initial beam velocities. The results of the measurements are shown in Fig. 2. Here the arrow indicates the value of the critical beam velocity. We see that, for $v_b < v_c$, the value $v_b - v_{ph} \approx 5\times10^{-2} v_{ph}$ and does not depend on v_b. A verification showed that this value of $v_b - v_{ph}$ is in good agreement with the formulas for the linear dispersion laws for a bounded beam-plasma system.[3,6] For $v_b > v_c$, the wave belongs to the space charge branch, and the difference $v_b - v_{ph}$ increases in proportion to v_b, as was to be expected.

In[2] the threshold external-signal amplitude values, at which a supercritical ($v_b > v_c$) electron beam becomes unstable, were found, and it was shown that the critical value of the amplitude of the external signal is determined by the relation (3) (see the comparison of the experimental and theoretical results in[2]). The object of the present paper is to investigate the amplification of waves by electron beams with supercritical velocities, to determine the maximum wave amplitudes in the system, and to compare the results with those of a computer simulation experiment.

At small amplitudes of the incoming signal there occurs the usual beam instability, but as the initial beam velocity is increased and v_b approaches v_c, the increment decreases, and there is virtually no wave amplification when $v_b > v_c$. At large amplitudes of the input signal the character of the dependences is preserved, but the critical velocity at which amplification of the wave ceases increases in accordance with Eq. (3). Correspondingly, the maximum values of the wave amplitude in the system increase. Furthermore, as the initial amplitude increases, the point at which the wave amplitude attains its maximum moves toward the beginning of the system.

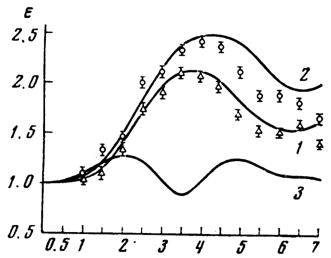

FIG. 3. Dependence of the wave amplitude on distance—comparison of the computer calculation with experiment. The initial wave amplitude is fixed; $v_{ph} = 1.8\times10^7$ m/sec. The curve 1) is for $v_b = 1.15 v_{ph}$; 2) for $v_b = 1.3 v_{ph}$; 3) for $v_b = 1.5 v_{ph}$. The experimental points: o) are for $v_b = 2.4\times10^7$ m/sec; Δ) for $v_b = 2\times10^7$ m/sec.

To determine the conditions under which the beam-energy losses are maximal, we measured the dependences of the wave amplitude along the length of the system for different amplitudes of the input signal and different beam velocities. The results of these measurements were compared with the computer simulation data. In Fig. 3 we show the experimental and theoretical dependences of the wave amplitude on distance. It can be seen that the experimental results and the results of the computer simulation are in good agreement. It follows from the experiments and the computer calculations that the maximum value of the wave amplitude virtually does not change if the initial amplitude of the signal, when every other parameter is fixed, exceeds the critical value. However, this maximum value of the amplitude is attained at different distances from the beginning of the system. As can be seen from Fig. 4, the distance from the beginning of the system to the point where the wave amplitude is a maximum is roughly inversely proportional to the initial value of the signal amplitude.

The measurements with the aid of the movable probe give relative values of the wave amplitude along the length of the installation. To obtain the absolute values of the wave amplitude, we used a second (probing) electron beam. This beam has a diameter of 2 mm and is transmitted along the axis of the system. The beam current is between 200 and 300 μA, so that it has virtually no influence on the process of amplification of the wave by the first beam, whose current was 4 mA. If, to start with, we take the velocity of the test beam to be such that the beam is not captured by the wave being amplified by the first beam and then reduce the velocity of the test beam, then from the variation of the amplitude of modulation of the test-beam current to the collector at the frequency of the wave being amplified we can accurately determine the velocity at which the test beam goes over from the uncaptured into the captured state (the modulation increases discontinuously during this transition). Since the phase velocity of the wave in the system has been measured, and the beam velocity at which the beam goes over into the captured state has been found, we can compute from the relation (3) the value of the amplitude at which the beam is captured. This will be the value of the wave amplitude at the point

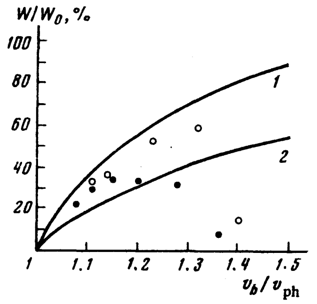

FIG. 5. The energy losses of the beam as a function of the beam velocity: 1) computed from the formula (7); 2) computed from the formula (8). The measured maximum values of the energy stored in the wave: (●) initial amplitude of the wave was 17.5 V; o) 25 V.

of its maximum. The error was made when the critical beam velocity is determined in this way is not worse than 5%. The maximum value of the wave amplitude in our experiments was 140 V. It was obtained at an initial signal amplitude of 25 V, a beam velocity of 2.4×10^7 m/sec, and a beam current of 4 mA.

Values of the energy stored in the wave field, as computed from the measured values of the wave-field amplitude, are shown in Fig. 5. The two series of points differ in the initial amplitude of the wave. The continuous curves in the same figure (the curves 1 and 2) represent the upper and lower values of the beam-energy losses, which are determined by the relation (7) and (8). The energy losses of the beam and the wave-field energies have been divided by the initial beam energy. The upper values of the beam-energy losses, (7), are estimated under the assumption that the cpatured beam particles are completely coherent, while the lower values, (8), correspond to the case when all the beam particles are captured, but their phases are completely mixed up, so that the phase distribution is uniform. It can be seen from Fig. 5 that, when the amplitudes $\varphi_0 = 17.5$ and $\varphi_0 = 25$ V exceed the critical values (in the first case this occurs when $v_b < 1.2 v_{ph}$; in the second, when $v_b < 1.37 v_{ph}$), the maximum values of the energy transferable by the beam to the wave lie between the curves 1) and 2). This is natural, since initially the degree of coherence of the motion of the particles is quite high: they all begin to interact with the wave when $v > v_{ph}$ and, consequently, the initial phase spread $\Delta\theta < \pi$. As the beam velocity increases, the initial amplitudes approach their critical values, and the maximum values of the wave energy decrease. When, however, the initial amplitudes become less than the critical values ($\varphi_0 = 17.5$ V for $v_b > 1.2 v_{ph}$ and $\varphi_0 = 25$ V for $v_b > 1.37 v_{ph}$), the maximum values of the energy drop below the values determined by the curve 2. This occurs because in this case not all the particles of the beam are captured by the wave, so that not all of them transfer their energy to it. Upon further increase of the beam velocity, the fraction of captured particles decreases to zero and the amplification of the wave ceases.

Notice that in our experiments the energy stored in the wave at the point of its maximum value constitutes about 60% of the beam energy in the case of a comparatively small initial amplitude of the wave, which is roughly five times greater than in the case of a beam with a subcritical velocity.

The performed investigation of the regimes of wave amplification in a bounded plasma by a monoenergetic beam at different values of the initial beam velocity and bare wave amplitude allows us to choose the regimes of amplification such that the energy transferable by the beam to the wave is maximal and the maximum is attained at the requisite place in the system. We see that, as expected, beams with supercritical velocities are more effective means of amplifying waves in a bounded plasma than subcritical beams.

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